Topology Homework 07

- Remember to reference all resources used. Due end of class on Thursday, 4/2.

1. Determine whether the given topological space is compact.
   (a) $\mathbb{R}$ with the Sorgenfrey topology, that is, the topology with basis being the collection $\{[a, b)\}_{a < b}$.
   (b) $\mathbb{Z}$ with the finite-closed topology.
   (c) $(X, \tau)$ where $\tau \subset \tau_1$ and $(X, \tau_1)$ is compact.

2. Let $(X, \tau)$ be a compact topological space. If $\{F_j : j \in J\}$ is a family of closed subsets of $X$ such that any finite subfamily has nonempty intersection, prove that
   \[ \bigcap_{j \in J} F_j \neq \emptyset. \]
   This is called the finite intersection property and it is actually equivalent to compactness.

3. Let $(X, \tau)$ be a $T_2$ topological space. If $A$ is a compact subset and $x \notin A$, prove that there exist disjoint open sets $U, V$ such that $x \in U$ and $A \subset V$. Furthermore, prove that for $A, B$ disjoint compact subsets, there exist disjoint open sets $G, H$ such that $A \subset G$ and $B \subset H$. 