Homework 2, Math 8061 Fall 2015

Due 9/16. Answer at least 3 of the following.

(1) Show that the following sets are topological manifolds with boundary. Find $\partial M$ for each one.
   (a) $M = \{(x, y, z, w) \in \mathbb{R}^4 \mid x^2 + y^2 + z^2 + w^2 \leq 1\}$
   (b) $M =$The set of polynomials of the form $ax^2 + bx + c$, $a, b, c \in \mathbb{R}$, that has a real root.
   (c) $M =$The surface $x = (5 + s \cos(t/2)) \cos t, y = (5 + s \cos(t/2)) \sin t, z = s \sin(t/2)$ with $s \in [-1, 1]$ and $t \in [0, 2\pi]$.

(2) Let $O(3)$ be the set of $3 \times 3$ matrices $A$, such that $||Av|| = ||v||$ for every vector $v \in \mathbb{R}^3$. Consider the map $F_A : S^2 \to S^2$ by matrix multiplication. Prove that $F_A$ is a smooth map for all $A \in O(3)$ with respect to the smooth structure on $S^2$ from stereographic projections.

(3) Prove that, over the set of smooth manifolds, being diffeomorphic is an equivalence relation. Consider connected subsets of $\mathbb{R}$ (or intervals) with the standard smooth structure. Show that there are four diffeomorphism equivalence classes.

(4) Let $M_1, \ldots, M_k$ be smooth manifolds without boundaries and $N$ be a smooth manifold with boundary. Prove that the product $M_1 \times \cdots \times M_k \times N$ is a smooth manifold with boundary, where the boundary is $M_1 \times \cdots \times M_k \times \partial N$. 

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