Experimental Verification of Models of Nonlinear Stream Aquifer Transients

S. R. Workman[1] and Sergio E. Serrano[2]

Abstract: The phenomenon of stream-aquifer interaction was investigated using a reduced scale sand tank model coupled with mathematical modeling using the Boussinesq equation. The effect of highly fluctuating transient stream level on the subsequent propagation of hydraulic transients in the adjacent aquifer was quantified using solutions to the linearized and the nonlinear Boussinesq equation. The semigroup solution of the linearized equation and the temporal-partial decomposition solution of the nonlinear equation were investigated subject to a transient, large-amplitude, periodic boundary condition. The models were verified with a limited numerical solution to the nonlinear Boussinesq equation and compared and assessed as to their ability to reproduce measured water table hydrographs at different locations in the laboratory model. Results indicated that the linearized solution and the nonlinear model coincided well with the numerical solution and with the observed hydrograph. Using decomposition the derivation of a solution to the nonlinear equation is simpler than that of a linear one.

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Introduction

Alluvial valley aquifers are hydraulically connected to their adjacent channels and exchange flow through the streambed (Perkins and Koussis 1996; Barlow et al. 2000). The connection causes water levels in these systems to fluctuate with respect to the other. If the stream stage increases over a short period of time, a flow reversal between the channel and aquifer will occur as a result of a change in the hydraulic gradient (Workman et al. 1997). A flood wave is then propagated into the aquifer and increases bank storage. While the stream is returning to normal flows, the bank storage is released. Alluvial valley aquifers pose some interesting boundary conditions. One side of the flow domain is the river source/sink, which is fluctuating. As a result of the alluvial valley formation, the other side of the flow domain is the valley wall, which is a no flow boundary condition.

The ground-water level rise in the great Bend Prairie aquifer of the Kansas area was studied by Sophocleous (1991) and has been attributed to the pressure waves from stream flooding that propagate in translatory motions through numerous high transmissivity and high hydraulic diffusivity buried channels. Whiting and Pomeranets (1997) studied the saturated flow response in an aquifer due to changes in the water level in the stream channel that cuts across the aquifer. They used the finite-element method to solve the boundary value problem and have deduced that the banks can store and later release large volumes of stream water. Flood wave propagation is affected by hydraulic conductivity, specific yield, and the length of time flow reversal conditions (Cartwright et al. 2003).

Analytical solutions to the governing ground-water differential equations have been shown to accurately predict water table profiles subject to smooth boundary conditions. The Boussinesq equation is a nonlinear partial differential equation that describes horizontal, transient flow in an unconfined aquifer and its solution has important hydrological application in the study of unconfined groundwater flow, stream-aquifer interaction, and bank storage effects. However, with Dupuit assumptions of negligible vertical flow components, negligible capillary pore forces, and small changes in transmissivity, the Boussinesq equation can be linearized (Van de Giesen et al. 1994; Tallaksen 1995). The analytical solution to the linearized form of the Boussinesq equation was developed by Govindaraju and Koelliker (1994) to study the water fluxes from the stream to the aquifers for any arbitrary shaped flood stage hydrograph. They showed that analytical solutions to the nonlinear equations are not very accurate as compared to the numerical solutions. Guo (1997) developed a new solution technique using the Boltzmann transformation to reduce the Boussinesq equation to a nonlinear ordinary differential equation and the flow between the reservoir and unconfined aquifer is calculated using the Newton–Raphson method.

Hornberger et al. (1970) used a fourth-order Runge–Kutta numerical technique to solve the nonlinear Boussinesq equation. Fouss and Rogers (1992) developed an iterative computer model that used a numerical solution to the Boussinesq equation to predict subsurface water movement and water table profiles between two adjacent subsurface drains. Previous studies have indicated that the Boussinesq equation can be used to simulate the operation of drainage systems for design purposes (Skaggs 1976, 1991; Lorre et al. 1994; Serrano and Workman 1998). Approximate analytical solutions to the nonlinear Boussinesq equation that accounts for transient boundary conditions are available in the literature (Rasmussen and Anderson 1959). However, since exact

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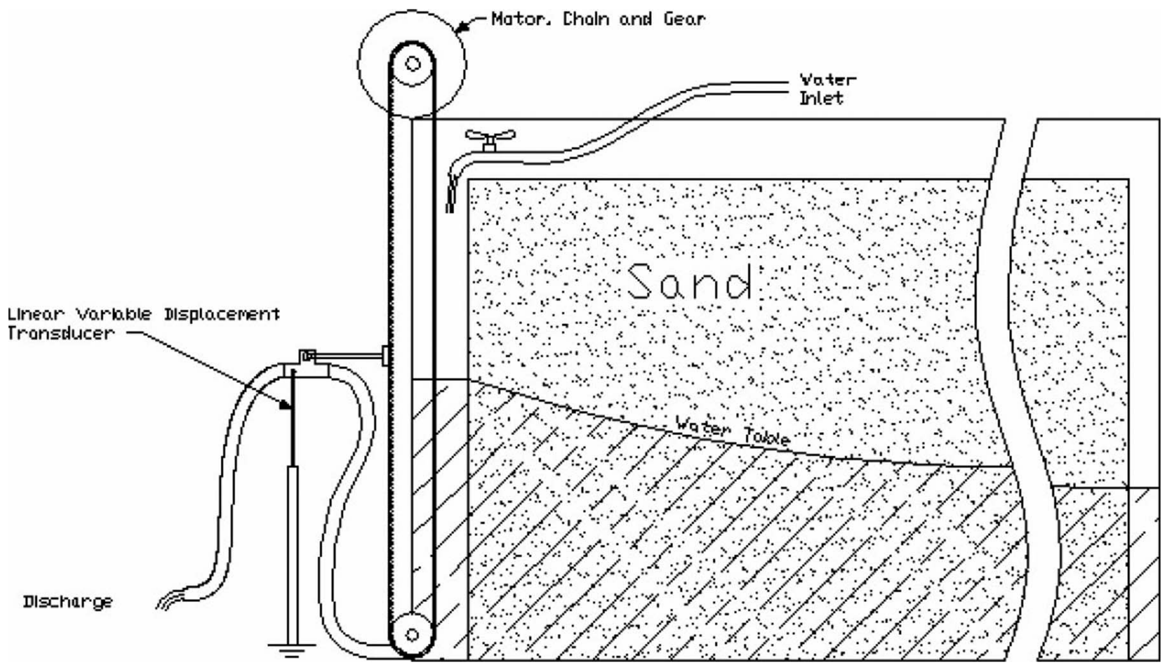

Fig. 1. Illustration of laboratory model

solutions do not exist, most researchers solve the flow problems using available numerical methods.

Serrano et al. (2007) developed new analytical solutions for the nonlinear Boussinesq equation using temporal-partial and spatial-partial decomposition series. In this paper a laboratory verification of the linearized and nonlinear Boussinesq equations is conducted. The models are tested as to their ability to reproduce experimentally measured heads in a sand tank subject to a sinusoidal transient boundary condition. A laboratory model was constructed to evaluate lateral subsurface flow at the Biosystems and Agricultural Engineering Laboratory of the University of Kentucky (see Fig. 1). For the linearized Boussinesq equation, a semigroup solution is used. For the nonlinear Boussinesq equation, the Adomian's method of decomposition is used to derive an approximate analytical solution (Serrano et al. 2007). Both solutions are compared with a simple numerical solution and with experimental data.

Decomposition is now being used to solve deterministic, stochastic, linear, or nonlinear equations in various branches of science and engineering (Adomian 1991, 1994; Srivastava and Singh 1999; Biazar et al. 2003; Wazwaz 2000; Wazwaz and Gorguis 2004). It is one of the few systematic methods available to solve nonlinear differential equations. In ground-water flow problems, this method has been extensively used (Serrano 1992, 1995a,b, 1997, 2003; Serrano and Adomian 1996; Adomian and Serrano 1998), to obtain analytical, and sometimes closed-form solutions to linear, nonlinear, and stochastic problems. A short introduction of the method to ground-water systems can be found in Serrano (1997, 2001). The method has been shown to be systematic, robust, and sometimes capable of handling large variances in the controlling hydrogeological parameters.

Analytical Solution to Transient Ground-Water Flow Equation

The governing equation for one-dimensional, lateral, unconfined ground-water flow with Dupuit assumptions is the Boussinesq equation (Bear 1979)

$$\frac{1}{S}\frac{\partial}{\partial x}\left(Kh\frac{\partial h}{\partial x}\right) = \frac{\partial h}{\partial t}, \quad 0 \leqslant x \leqslant l_x, \quad 0 \leqslant t \tag{1}$$

where h=hydraulic head [L]; K=hydraulic conductivity [LT^{-1}]; S=specific yield; x=horizontal coordinate [L]; t=time coordinate [T]; and l_x=horizontal dimension of the aquifer [L]. Without loss of generality, in this study we do not consider recharge from rainfall since our focus is on the effect of boundary conditions. We consider a long and thin aquifer bounded by a river on one side and an impervious boundary on the other. The boundary and initial conditions are

$$h(0,t) = H_1(t)$$

$$\frac{\partial h(l_x,t)}{\partial x} = 0$$

$$h(x,0) = h_i(x) = y_0 \tag{2}$$

where $H_1(t)$=time-fluctuating head at the left boundary [L], such that $H_1(0)=y_0$; and y_0=initial head across the aquifer [L].

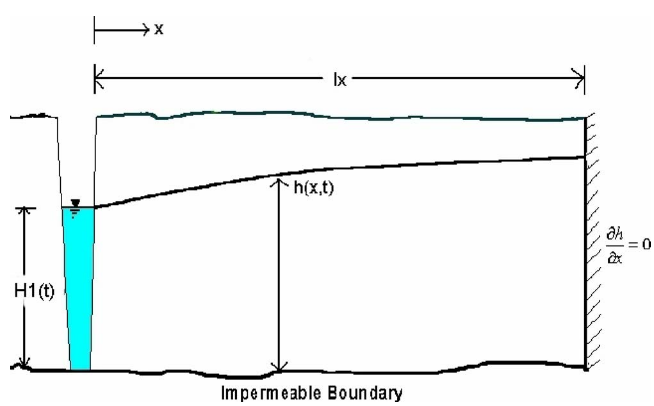

Fig. 2. Idealized cross section for mathematical model

Fig. 2 depicts the idealized cross section of the model under consideration. A smooth transient left boundary condition was considered as

$$H_1(t) = y_0 + A \sin\left(\frac{2\pi t}{B} + C\right) \tag{3}$$

where A, B, and C=constants.

We first attempt a semigroup solution of the linearized equation. The governing Eq. (1) is linearized and is expressed as

$$\frac{\partial h}{\partial t} - \frac{T}{S}\frac{\partial^2 h}{\partial x^2} = 0 \tag{4}$$

where T=aquifer transmissivity $[L^2T^{-1}]$ defined as the product of the hydraulic conductivity times an average saturated thickness. The solution may be expressed as the summation of two components (Serrano and Unny 1987)

$$h(x,t) = V(x,t) + W(x,t) \tag{5}$$

where $W(x,t)$=primary solution; and $V(x,t)$=secondary solution. Substituting Eq. (5) into Eq. (4) we get

$$\frac{\partial W}{\partial t} - \frac{T}{S}\frac{\partial^2 W}{\partial x^2} = -\left(\frac{\partial V}{\partial t} - \frac{T}{S}\frac{\partial^2 V}{\partial x^2}\right) \tag{6}$$

subject to

$$W(0,t) = H_1(t) - V(0,t) = W_0 \tag{7}$$

$$\frac{\partial W(l_x,t)}{\partial x} = -\frac{\partial V(l_x,t)}{\partial x} \tag{8}$$

$$W(x,0) = h_i(x) - V(x,0) \tag{9}$$

We choose a smooth function $V(x,t)$ such that the boundary conditions in Eq. (7) become zero

$$V(x,t) = H_1(t) \tag{10}$$

The solution to Eq. (6) is (Serrano and Unny 1987)

$$W(x,t) = J_t W_0 - \int_0^t J_{t-\tau}\frac{\partial H_1(\tau)}{\partial \tau}d\tau \tag{11}$$

where the semigroup operator associated with Eq. (6), J_t, is given as

$$J_t f = \sum_{n=1}^{\infty} b_n(f)\phi_n(x)M_n(t) \tag{12}$$

The Fourier coefficients, b_n, are given as

$$b_n(f) = \frac{2}{l_x}\int_0^{l_x} f \sin(\lambda_n \xi)d\xi \tag{13}$$

the eigenvalues, λ_n, are

$$\lambda_n = \left(\frac{2n-1}{2l_x}\right)\pi n = 1,2,3 \ldots . \tag{14}$$

the basis function, $\phi_n(x)$, is given as

$$\phi_n(x) = \sin(\lambda_n x) \tag{15}$$

and

$$M_n(t) = e^{-\lambda_n^2 Tt/S} \tag{16}$$

From this solution, the solution to Eq. (6) is obtained as

$$W(x,t) = -\sum_{n=1}^{\infty} b_n(1)\phi_n(x)I_n(t) \tag{17}$$

where

$$I_n(t) = \int_0^t M_n(t-\tau)\frac{\partial H_1}{\partial \tau}d\tau \tag{18}$$

In terms of h, the solution reduces to

$$h(x,t) = H_1(t) - \sum_{n=1}^{\infty} b_n(1)\phi_n(x)I_n(t) \tag{19}$$

We now attempt a decomposition, defined as a temporal-partial solution scheme, sometimes referred to as the t-partial solution (Adomian 1994). The governing Eq. (1) is expressed as

$$\frac{\partial h}{\partial t} = \frac{K(h_0+h_1+\ .\ .)}{S}\frac{\partial^2(h_0+h_1+\ .\ .)}{\partial x^2} + \frac{K}{S}\left(\frac{\partial(h_0+h_1+\ .\ .)}{\partial x}\right)^2 \tag{20}$$

Defining the operator $L_t=\partial/\partial t$, we can write Eq. (20) as

$$h = L_t^{-1}\frac{K(h_0+h_1+\ .\ .)}{S}\frac{\partial^2(h_0+h_1+\ .\ .)}{\partial x^2} + \frac{K}{S}L_t^{-1}\left(\frac{\partial(h_0+h_1+\ .\ .)}{\partial x}\right)^2 \tag{21}$$

Again the solution is expressed as the series $h=h_0+h_1+h_2+\ldots$, where the first term satisfies

$$\frac{\partial h_0}{\partial t} - \frac{Kh_0}{S}\frac{\partial^2 h_0}{\partial x^2} = 0 \tag{22}$$

subject to

$$h_0(0,t) = H_1(t), \quad \frac{\partial h_0(l_x,t)}{\partial x} = 0, \quad h_0(x,0) = h_i(x) = y_0 \tag{23}$$

The solution to Eq. (22) is the linearized solution to Eq. (1), as given by Eq. (19), when the transmissivity is taken as $Kh_0(0) = Ky_0$, that is

$$h_0(x,t) = H_1(t) - \sum_{n=1}^{\infty} b_n(1)\phi_n(x)I_n(t) \tag{24}$$

The second term is given by

$$h_1 = L_t^{-1}\left(\frac{Kh_0}{S}\left(\frac{\partial^2 h_0}{\partial x^2}\right) + \frac{K}{S}\left(\frac{\partial h_0}{\partial x}\right)^2\right) \tag{25}$$

Using Eq. (23) it becomes

$$h_1 = \frac{2K}{Sl_x^2}L_t^{-1}\left(h_0(x,t)(H_1(t)-H_1(0)) + \frac{2}{l_x^2}((H_1(t)-H_1(0))(x-1))^2\right) \tag{26}$$

Hence, a two-term approximation to Eq. (20) is given by

$$h(x,t) = H_1(t) - \sum_{n=1}^{\infty} b_n(1)\phi_n(x)I_n(t) + \frac{2K}{Sl_x^2}L_t^{-1} \times \left(h_0(x,t)(H_1(t) - H_1(0)) + \frac{2}{l_x^2}((H_1(t) - H_1(0))(x-1))^2 \right) \tag{27}$$

Although the algebraic difficulty in deriving additional terms in the series increases, it is known that the convergence rate of a decomposition series is usually high and that only a few terms are needed in order to assure an accurate model. From the hydrologic point of view, an accurate model is more important than a mathematical closed-form solution. For a rigorous mathematical discussion on the convergence problem of decomposition series, the reader is referred to Abbaoui and Cherruault (1994), Cherruault (1989), and Cherruault et al. (1992). It is also important to mention the rigorous mathematical framework for the convergence of decomposition series developed by Gabet (1992, 1993, 1994). He connected the method of decomposition to well-known formulations where classical theorems (e.g., fixed point theorem, substituted series, etc.) could be used. For a discussion on the convergence of decomposition series of convection-diffusion equations, including a theorem with proof, see Serrano (1998). For additional comparisons between exact and truncated decomposition solutions, see Serrano and Adomian (1996).

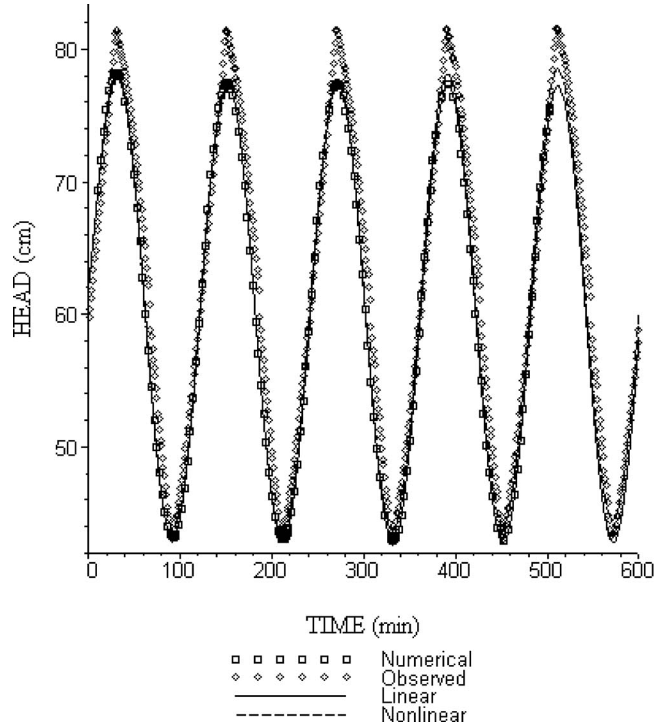

Fig. 3. Comparison between linear solution [Eq. (19)], temporal-partial nonlinear solution [Eq. (27)], numerical solution, and observed temporal head at $x=50$ cm

Experimental and Numerical Verification

To verify the solutions and to test the assumptions behind the Boussinesq equation as to its ability to reproduce measured hydraulic transients, a laboratory model was constructed to evaluate lateral subsurface flow at the Biosystems and Agricultural Engineering Laboratory of the University of Kentucky (see Fig. 1). The model was constructed of 12.7 mm thick Plexiglas and was 12 m long, 1.2 m tall, and 0.3 m wide. It was filled with a fine, uniform sand that had a d_{50} of 0.28 mm and a coefficient of uniformity of 2.57. An average hydraulic conductivity over the length of the tank of $K=34$ mm/min was determined using methods of Skaggs (1976). The porosity of the sand was $n=0.41 \pm 0.02$ $\text{mm}^3 \times \text{mm}^{-3}$. For the simulations, the specific yield, S, was taken as equal to the porosity. Water retention, entrapped air, and hysteresis of wetting and drying was neglected, which is reasonable for the sand. Water level sensors were installed along the base of the tank at 0, 0.1, 0.25, 0.5, 1, 2, 4, and 6 m from either end to measure hydraulic head. The sensors were connected to a data logger that could be programmed to measure the hydraulic head at chosen intervals.

A large drain tube on one of the outlet reservoirs was secured to a chain that could move vertically along the outlet reservoir. Water continuously flowed into the outlet reservoir with excess water flowing out the drain tube to maintain the desired boundary water table height. The chain was wrapped around a sprocket moved by a servo drive/motor connected to a computer via an analog to digital input-output (I/O) board and expansion board. The motorized lift mechanism was capable of raising and lowering the drain tube based on input from the computer. The vertical drain tube position was measured with a linear variable differential transformer (LVDT). A data file of positions (voltages) was used to position the servo drive/motor as height was monitored by the LVDT.

Experiments were developed to impose transient boundary conditions on the laboratory model. The water table of the sand tank was raised to a uniform height of 0.6 m. Once static conditions were established, the left boundary was subjected to changing conditions, while the right boundary was left freely so as to simulate a no flux condition. To evaluate the ability to model transient conditions, a sinusoidal pattern was applied. The test started with the water table at steady state at 0.6 m, then the left boundary was raised by 12 mm/h until it reached a height of 0.96 m. From there, the system began to lower at a rate of 12 mm/h until it reached a height of 0.24 m, and then it began to rise again and repeat the process. Results were graphed to show hydraulic head as a function of time at locations of 0.5, 1, and 2 m from the boundary. The periodic sine wave had the constants $A=0.2475$ m, $B=120$ min, and $C=0$. A transmissivity value of $T=Ky_0$ was used.

The linearized and nonlinear solutions in the previous section were compared with a standard explicit finite difference solution and with the observed heads at fixed locations from the left boundary. Fig. 3 shows a comparison between the linear solution [Eq. (19)], the temporal-partial nonlinear solution [Eq. (27)], the numerical solution, and the observed temporal head at $x=50$ cm. Fig. 4 shows a comparison between the linear solution [Eq. (19)], the temporal-partial nonlinear solution [Eq. (27)], the numerical solution, and the observed temporal head at $x=200$ cm.

The Boussinesq equation assumes a nearly horizontal flow, negligible vertical gradients, and neglects capillary and seepage effects. The use of a linearized Boussinesq equation further requires a constant transmissivity. The aquifer model in this experiment tends to improve its conformance to these requirements as distance from the left boundary increases. As x increases, hydraulic gradients and head amplitude decrease, transmissivity tends to a constant, and thus the performance of the linear and nonlinear Boussinesq models increase. This is easy to see from

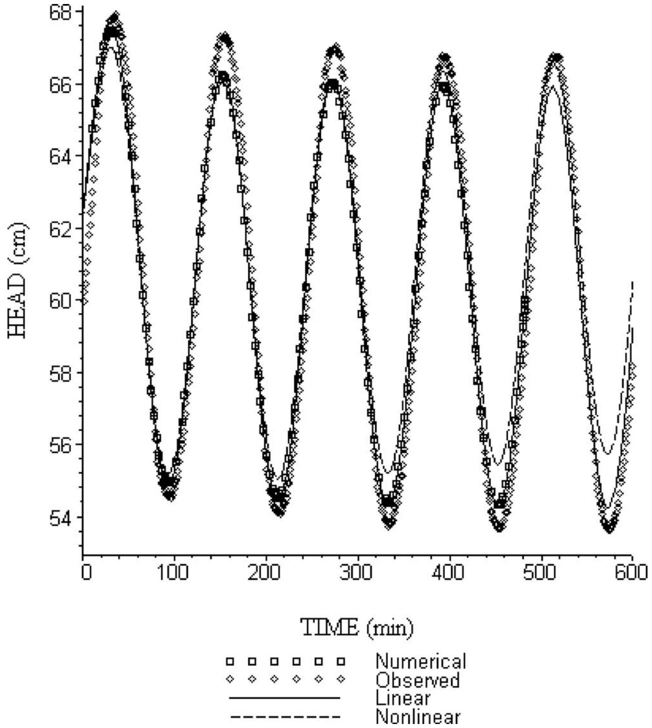

Fig. 4. Comparison between linear solution [Eq. (19)], temporal-partial nonlinear solution [Eq. (27)], numerical solution, and observed temporal head at $x = 200$ cm

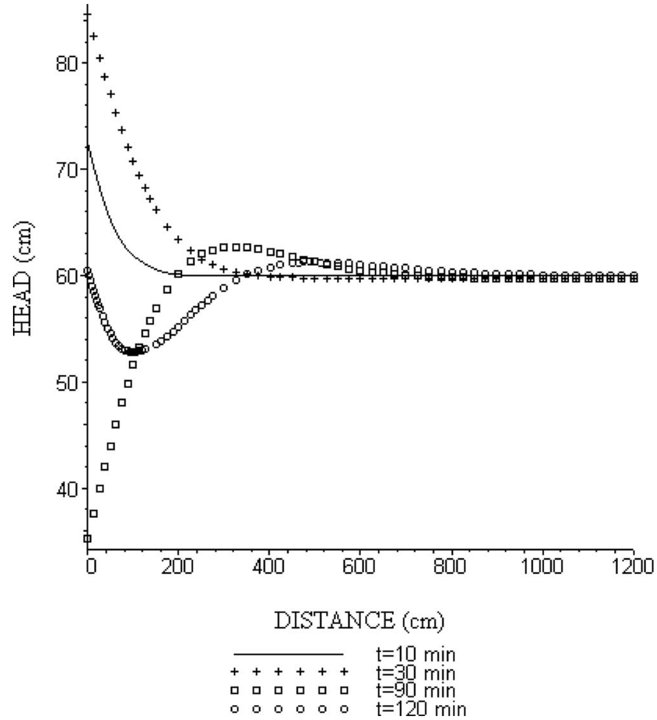

Fig. 5. Simulated nonlinear spatial distribution of heads at different times, according to Eq. (27)

Figs. 3 and 4: the maximum difference between observed and simulated heads at $x = 50$ is about 4 cm (Fig. 3), whereas the maximum difference between observed and simulated heads at $x = 200$ is about 2 cm. With these provisions, both the linearized and nonlinear solutions appear to compare well with the numerical solution and with the observed hydrographs. They reproduce well the damping of the hydraulic transient as distance increases, exemplified as a decrease in amplitude with distance. The minor differences in the head peaks are due to the use of the Boussinesq equation as a model, the effect of linearization, and possible errors in measurement. These differences increase as head amplitude and hydraulic gradients increase. In the model this occurs as the observer approaches the left boundary. In this study we focus on one-dimensional solutions to the Boussinesq equation. Hunt (2005) has presented a solution to the two-dimensional equation.

The time-partial nonlinear model appears to be a simple model for potential applications to the investigation of the effect of high-amplitude flood hydrographs on adjacent aquifer heads. Using decomposition, the derivation of a nonlinear solution is easier than an analytical one using traditional analytical methods, such as Fourier series. Fig. 5 shows a simulation of the spatial distribution of heads at different times. It is interesting to observe the "whipping" effect as the hydraulic transient propagates into the aquifer. Fig. 6 illustrates the temporal distribution of hydraulic heads damping at different locations in the aquifer.

Summary and Conclusions

The phenomenon of stream aquifer interaction was investigated using a reduced scale sand tank model coupled with mathematical modeling using the Boussinesq equation. The effect of highly fluctuating transient stream level on the subsequent propagation of hydraulic transients in the adjacent aquifer was quantified using solutions to the linearized and the nonlinear Boussinesq equation. A periodic analytic function was used to represent the time variability and high fluctuation in the left boundary condition. The solutions tested included a semigroup solution to the linearized Boussinesq equation and a nonlinear decomposition solution series expansion. The solutions were verified with a standard explicit finite difference solution and experimentally with measured hydraulic heads at a reduced sand tank model. The results indicated that the linearized solution and the temporal-partial nonlinear solutions compared well with the numerical so-

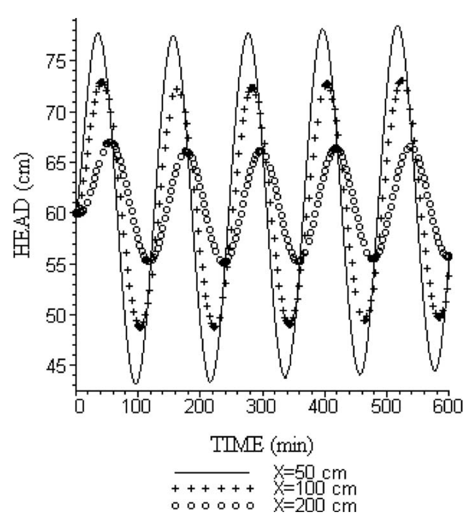

Fig. 6. Simulated nonlinear temporal distribution of heads at different distances, according to Eq. (27)

lution and with the experimental data. The nonlinear solution appears as a simple model for practical simulations.

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