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Stochastic Hydrology and Hydraulics

Development of the instantaneous unit hydrograph using stochastic differential equations

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Abstract: Recognizing that simple watershed conceptual models such as the Nash cascade of n equal linear reservoirs continue to be reasonable means to approximate the Instantaneous Unit Hydrograph (IUH), it is natural to accept that random errors generated by climatological variability of data used in fitting an imprecise conceptual model will produce an IUH which is random itself. It is desirable to define the random properties of the IUH in a watershed in order to have a more realistic hydrologic application of this important function. Since in this case the IUH results from a series of differential equations where one or more of the uncertain parameters is treated in stochastic terms, then the statistical properties of the IUH are best described by the solution of the corresponding Stochastic Differential Equations (SDE's). This article attempts to present a methodology to derive the IUH in a small watershed by combining a classical conceptual model with the theory of SDE's. The procedure is illustrated with the application to the Middle Thames River, Ontario, Canada, and the model is verified by the comparison of the simulated statistical measures of the IUH with the corresponding observed ones with good agreement.

Key words: Instantaneous Unit Hydrograph, Conceptual models, Stochastic differential equations.

1 Introduction

The Instantaneous Unit Hydrograph (IUH) concept has been, and continues to be one of the most widely used rainfall-runoff forecasting and design tools in the field of hydrology. Many recent authors and applied manuals offer the IUH as a fundamental element of analysis (Karlsson and Yakowitz, 1987; Sorooshian, 1983; Linsley et al., 1982; Bras and Rodriguez-Iturbe, 1985). Behind this concept is the inherent assumption that the watershed can be treated as a linear system whose impulse response function is a unique (deterministic), time invariant, linear transformation.

Parallel to the development of the IUH concept has been the proliferation of conceptual models to represent the streamflow process in watersheds (see for example Singh (1988) for an extensive summary of these models). Each model visualizes the watershed as a particular combination of imaginary hydraulic structures (linear or non-linear planes, channels and reservoirs) which replicate the stages in the streamflow generation. These are indeed simplifications of the complex hydrologic processes occurring in watersheds, but many of these models have proven most useful in the difficult task of streamflow synthesis for various applications. The idea that the watershed could be represented as a system consisting of n linear reservoirs in cascade was introduced by Nash (1957). When the input to this model is a unit impulse, the output could be interpreted as a conceptual IUH, and in fact this model has proven to be a simple and effective means to approximate the IUH of a watershed (Huggins and Burney, 1982). In an extensive numerical simulation study Dooge (1977) demonstrated that the Nash cascade of linear reservoirs performed equally or better than other more complex deconvolution procedures such as transform methods and optimization methods.

Recognizing that the price to be paid for the simplicity of adopting a conceptual model, such as the Nash cascade, to approximate the IUH results in substantial errors in the estimation of the shape of this function, the authors believe that there is a need to evaluate the uncertainty associated with fitting such a model. If in addition the climatological and streamflow information used in fitting the model comes from uncertain and variable data, then the derived IUH is also subject to a degree of uncertainty which should be evaluated. Thus the IUH is not a unique watershed function, but rather a random function which should be characterized in statistical terms.

Consider for example a study (Serrano et al., 1985) where a Nash cascade was used to detect the changes in the IUH features over a long period of time which could be attributed to agricultural and land drainage. After a detailed procedure to select suitable pairs of effective rainfall-storm runoff hydrographs from over 30 years of record in order to fit several Nash cascade models, it was found that the fitted IUH's did not present any particular trend over time in either the peak magnitude, the time base or the parameters n and K of the cascades. Since however there was a wide range of variability in the values of n and particularly K, there is sufficient evidence to believe that these parameters should be treated as random quantities reponding to climatological and model uncertainty.

The Nash cascade results from a set of ordinary differential equations, one for each reservoir, where the parameters n, and K play an important role. If the parameters are treated as stochastic quantities, then the resulting equations are stochastic differential equations (SDE's) whose solution will describe the random properties of the IUH.

Stochastic methods have been used in combination with conceptual models since 1970. Early contributions (Moran, 1971; Bernier, 1970; Quimpo, 1971, 1973; Weiss, 1973; Unny and Karmeshu, 1984; Unny, 1987; Koch, 1985) concentrated on the development of conceptual models subject to stochastic inputs. Bodo and Unny (1987) formulated a model which considered the evaporation and precipitation as random inputs and solved the corresponding SDE.

In the present article the authors propose a different type of stochastic model, where the parameters, rather than the inputs, are defined as random quantities. Specifically we study the Nash cascade of linear reservoirs when the most uncertain parameter, K, is random. It is assumed that the excess rainfall hyetograph and direct runoff hydrograph used to calculate the parameters are subject to measurement and fitting errors; the mathematical model for the IUH is subject to a varied degree of uncertainty or error generated by the assumptions made in the development of the differential equations and the inherent simplifications in the streamflow process; the measured data is subject to the same degree of uncertainty as simulated data; and the IUH with a stochastic parameter is itself stochastic.

The above assumptions imply that both the observed and the model IUH are stochastic, and therefore the verification of the model may be done through the comparison of the statistical properties of the observed IUH with the corresponding simulated ones. If corresponding observed and model statistical measures are similar in magnitude, then it is concluded that the model replicates the IUH of the watershed (see Serrano and Unny, 1987). A verified model of the random IUH of a watershed may constitute a more realistic and general streamflow forecasting tool.

The objective of the present article is to introduce a methodology to derive and analize an IUH as a random function by developing and solving the SDE governing the random IUH when a conceptual model, such as the Nash cascade of equal reservoirs, subject to one random parameter is used.

2 Development of the stochastic instantaneous unit hydrograph

As stated in section 1, we assume that the streamflow generation process in a watershed can be reasonably represented by a Nash cascade of n equal linear reservoirs. The differential equation governing the outflow rate from any reservoir i results from the combination of the continuity equation with a linear storage-outflow relationship (Nash, 1957):

$$K\frac{dO_i}{dt} + O_i = O_{i-1}, \tag{1}$$

subject to $O_i(0) = 0, i = 1, 2,..., n$, where O_i is the outflow rate from reservoir $i, i = 1,..., n$ (m^3/s); t is the time coordinate; and K is the storage coefficient (hr). Eq. (1) is a linear first order ordinary differential equation whose forcing function is the outflow rate from the previous reservoir in the cascade. If the forcing function of the differential equation for the first reservoir, O_1, is a unit impulse (the Dirac's delta function), then the outflow rate from the last reservoir, O_n, can be interpreted as a conceptual IUH.

Consider the case when in Eq. (1) the parameter K exhibits a significant degree of uncertainty. This situation arises when fitting Nash cascades using several pairs of homogeneous effective hyetograph-storm runoff hydrograph of a watershed, and finding that each cascade has a completely different value in the parameter K, without showing any particular trend. If it is assumed that the parameter could be expressed as $K = \bar{K} + K'(\omega)$, where $\bar{K}$ represents the mean storage coefficient and $K'(\omega)$ a zero-mean random variable with ω the probability variable, then the outflow rate from the first reservoir in Eq. (1) is governed by

$$(\bar{K} + K')\frac{dO_1}{dt} + O_1 = \delta(t), \tag{2}$$

where $\delta(\cdot)$ is the Dirac's delta function and ω has been omitted for convenience. On dividing through by $\bar{K}$ and placing the terms containing K' on the right side, Eq. (2) becomes

$$\frac{dO_1}{dt} + \frac{O_1}{\bar{K}} = \frac{\delta(t)}{\bar{K}} - \frac{K'}{\bar{K}}\frac{dO_1}{dt}, \tag{3}$$

subject to $O_1(0) = 0$. The impulse response function of Eq. (3) is equal to $e^{-t/\bar{K}}$, $t > 0$ and then its solution could be written as

$$O_1(t) = e^{-t/\bar{K}}O_1(0) + \frac{1}{\bar{K}}\int_0^t e^{-(t-s)/\bar{K}}\delta(s)ds - \frac{K'}{\bar{K}}\int_0^t e^{-(t-s)/\bar{K}}\frac{dO_1}{ds}ds, \tag{4}$$

where K' is outside the integral since it is a random variable (i.e., not a random process dependent on time). After applying the initial condition and solving the first integral in the right side,

$$O_1(t) = \frac{1}{\bar{K}}e^{-t/\bar{K}} - \frac{K'}{\bar{K}}e^{-t/\bar{K}}\int_0^t e^{s/\bar{K}}\frac{dO_1(s)}{ds}ds. \tag{5}$$

This equation contains O_1 in the right side. Therefore we expand this term as an infinite series (see Serrano, 1988) $O_1 = \sum_{j=1}^{\infty} O'_j$. Thus Eq. (5) becomes

$$O_1(t) = \frac{1}{\bar{K}}e^{-t/\bar{K}} - \frac{K'}{\bar{K}}e^{-t/\bar{K}}\sum_{j=10}^{\infty}\int^{t} e^{s/\bar{K}}\frac{dO'_j(s)}{ds}ds. \tag{6}$$

Next, O'_1 is taken as the previous term in the right side of this equation, that is

$$O'_1(s) = \frac{1}{\bar{K}}e^{-s/\bar{K}}, \tag{7}$$

so that

$$O'_2(s) = -\frac{K'}{\bar{K}}e^{-s/\bar{K}}\int_0^s e^{\xi/\bar{K}}\frac{dO'_1(\xi)}{d\xi}d\xi, \tag{8}$$

or

$$O'_2(s) = \frac{K'}{\bar{K}^3}se^{-s/\bar{K}}. \tag{9}$$

In general,

$$O'_j(s) = -\frac{K'}{\bar{K}}e^{-s/\bar{K}}\int_0^s e^{\xi/\bar{K}}\frac{dO'_{j-1}(\xi)}{d\xi}d\xi, \quad j = 1, 2, \cdots$$

It will be shown in section 3 that only three terms in the summation series are sufficient to obtain an approximation of O_1 with acceptable accuracy. For the details of this approximation scheme of SDE's and a discussion on the convergence, the reader is referred to Serrano (1988). It is sufficient to state here that since this is not a perturbation approximation, arbitrary large variances, only restricted by the stability of the equation, are allowed and that in general only a few terms are required to be computed.

Thus taking a maximum j as equal to 3 in Eq. (6) and solving,

$$O_1(t) = e^{-t/\bar{K}}[\frac{1}{\bar{K}} + \frac{K't}{\bar{K}^3} - \frac{(K')^2t}{\bar{K}^4} + \frac{(K')^2t^2}{2\bar{K}^5} + \frac{(K')^3t}{\bar{K}^5} - \frac{(K')^3t^2}{\bar{K}^6} - \frac{(K')^3t^3}{6\bar{K}^7}]. \tag{10}$$

We now focus our attention on the differential equation governing the outflow rate from reservoir 2, which is given by Eq. (1) when $i = 2$ and whose forcing function is the outflow rate from reservoir 1, as given by Eq. (10). Following a similar procedure to the one applied to reservoir 1, one obtains

$$\begin{aligned} O_2(t) = e^{-t/\bar{K}}[\frac{t}{\bar{K}^2} - \frac{K't}{\bar{K}^4} + \frac{K't^2}{\bar{K}^4} + \frac{(K')^2t}{\bar{K}^4} - \frac{2(K')^2t^2}{\bar{K}^5} \\ - \frac{(K')^3t}{\bar{K}^5} + \frac{(K')^2t^3}{2\bar{K}^6} + \frac{3(K')^3t^2}{2\bar{K}^6} + \frac{(K')^3t^4}{6\bar{K}^8} - \frac{9(K')^3t^3}{6\bar{K}^7}] \end{aligned} \tag{11}$$

For most practical purposes many of the terms in Eqs. (10) and (11) are negligible. In fact, terms containing powers of K' higher than 3 have been omitted in Eq. (11), since they are very small in magnitude. If the number of reservoirs in the cascade, n, is equal to 2 then O_2 represents the random IUH for the watershed. A similar procedure would be followed if the number of reservoirs is greater than two, in which case the more complex convolution integrals can be solved with a Gaussian quadrature. Thus sample functions of the IUH can be generated if the probability density function of K' is known and sample realizations of this parameter are available. Sample functions of the IUH should be at the heart of simulation models which utilize the IUH in the synthesis of streamflow.

A more objective description of the statistical properties of the random IUH is obtained after deriving expressions for the moments of O_2. For example, assuming that K' is a Gaussian random variable, the mean of the IUH can be obtained by taking expectations on both sides of Eq. (11). Using the properties of Gaussian random variables and neglecting the terms very small in magnitude, it reduces to

$$E\{O_2(t)\} = \sigma^2 e^{-t/\bar{K}}[\frac{t}{\sigma^2\bar{K}^2} + \frac{t}{\bar{K}^4} - \frac{2t^2}{\bar{K}^5} + \frac{t^3}{2\bar{K}^6}], \tag{12}$$

where $E\{\cdot\}$ denotes the expectation operator and σ^2 is the variance of K'.

Similarly the correlation function can be obtained from Eq. (11). Combining the correlation function with the mean one can obtain the variance of the IUH defined as $Var\{O_2(t)\} = E\{O_2^2(t)\} - E^2\{O_2(t)\}$:

$$Var\{O_2\} = \sigma^2 e^{-2t/\bar{K}}[\frac{t^2}{\bar{K}^6} - \frac{2t^3}{3\bar{K}^7} + \frac{t^4}{3\bar{K}^8}]. \tag{13}$$

The variance offers an objective measure of the uncertainty associated with the predictability of the IUH. If this measure is incorporated in a streamflow forecasting model it can provide some upper and lower bounds for the error in the forecast (i.e., the peak flow rate).

3 Application to the Middle Thames Watershed

The methodology presented in Section 2 was applied to the derivation of the stochastic IUH of the Middle Thames Watershed in Southern Ontario, Canada (Fig. 1). The Middle Thames is approximately rectangular in shape, it covers an area of 298 km^2 with a fairly flat topography (Serrano et al., 1985). The watershed land use is mainly agriculture and its natural drainage system has been considerably improved by the massive construction of surface and particularly tile drains for several decades.

The starting point in the present application is a study conducted by Serrano et al. (1985) in which an investigation to detect the changes in the characteristics of the IUH of the Middle Thames River which could be attributed to drainage was done. In over 30 years of streamflow records, they selected 14 pairs of effective rainfall-storm runoff hydrographs which satisfied certain homogeneity requirements. Out of each pair, a realization of the IUH was obtained after fitting a Nash cascade by using the matching moments technique (Chow et al., 1988). Table 1 reproduces the cascades derived by Serrano et al. (1985). Next an attempt was made to investigate a trend in either the values of K, n, time to peak, peak flow rate or time base of the IUH. Since no significant trend with respect to time was found, there is reason to believe that the 14 IUH's are time independent and rather represent individual realizations of a stochastic IUH. Table 1 indicates that the average value of n is 2.24 and varies between 1.16 and 3.23. In contrast K has an average value of 13.40 and varies between 6.23 and 24.33. Thus the sample data suggests that K is the parameter exhibiting the highest uncertainty. For these reasons we assumed n to be equal to 2.0, and constant, and concentrated the efforts on studying the effect of a much greater uncertainty in K on the shape of IUH. Thus K is best described as a random variable whose probability density function is to be found.

The probability density function of K was investigated graphically as well as analytically (Haan, 1977). After fitting straight lines to the plotting positions of K for Normal and Log-Normal laws it was found that either distribution would be an adequate candidate to model the frequency behaviour of K, although the Log-Normal distribution seemed to fit the data better. The Chi-squared test, the Kolmogorov-Smirnov test and the W test were conducted in order to investigate the feasibility of the same distributions. According to the Chi-squared test, the hypothesis of a Normal distribution was not rejected. According to the Kolmogorov-Smirnov test, the hypothesis of either Normal or

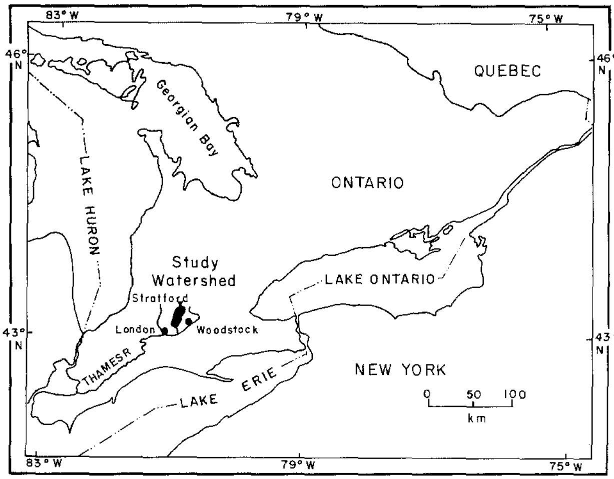

Figure 1. Location of the Middle Thames River

Table 1. Characteristics of the impulse response function derived for selected storms of the Middle Thames River

Storm No.	Storm Date	n	K(hr)	Time To Peak(hr)	Peak (m^3/s)
1	59-Apr-28	2.78	10.40	18	2.27
2	68-Apr-03	2.21	15.55	23	1.81
3	77-Apr-16	2.54	11.79	18	2.14
4	60-Apr-30	2.57	11.37	18	2.21
5	67-Apr-17	2.80	9.71	17	2.42
6	76-Mar-25	2.41	8.81	12	2.97
7	61-May-06	3.23	6.23	14	3.42
8	68-Nov-28	2.47	9.59	14	2.69
9	76-Mar-31	1.98	20.33	20	1.51
10	63-May-10	2.30	17.43	23	1.56
11	69-May-18	1.58	24.33	14	1.56
12	79-Apr-13	1.25	21.97	5	2.30
13	80-Jul-08	2.01	9.22	9	3.29
14	80-Jul-28	1.16	10.82	2	5.22
Average	1956-1980	2.24	13.40	15	2.53

Log-Normal was not rejected. Finally, according to the W test both hypotheses were rejected. The dissimilar results of the tests may be due to the insufficient number of data points available, in spite of the fact that they were carefully selected from over 30 years of streamflow records. Since two of the tests indicate that there is no reason to believe that K does not follow a Normal distribution, we assume that the density function for this parameter is indeed Normal at this stage.

Before applying the equations derived in Section 2, a check on the convergence speed of the approximation scheme of Eq. (6) and the the one implicit in Eq. (11) was done. For a realization of K' a computation of O'_j with respect to time for four iterations is illustrated in Fig. 2. Note that no more than three iterations are necessary to achieve an acceptable accuracy. Similarly, Fig. 3 illustrates O''_j, which is the approximation variable for the second reservoir, with respect to time for four iterations and again observe that no more than three iterations are necessary.

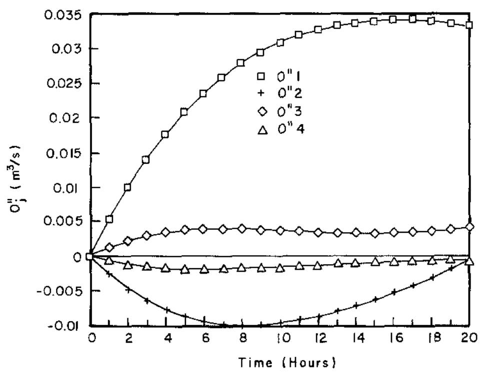

Figure 2

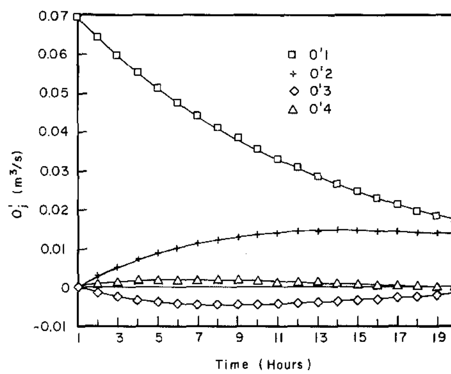

Figure 3

Figure 2. Test of convergence for the solution for reservoir 1

Figure 3. Test of convergence for the solution for reservoir 2

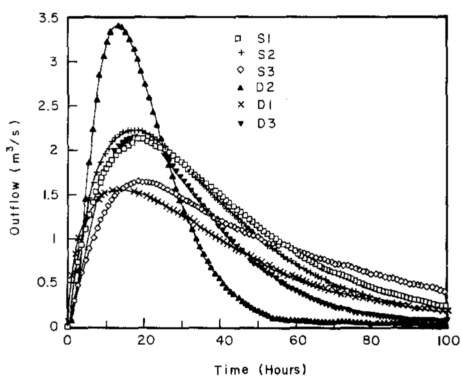

Figure 4

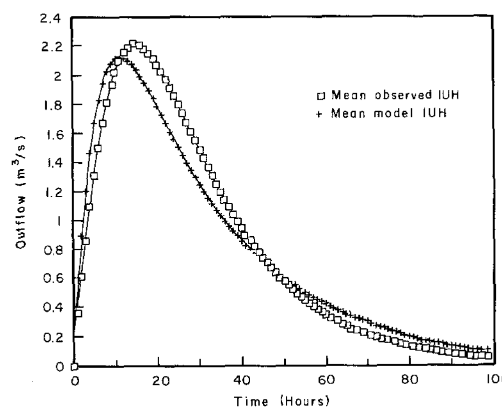

Figure 5

Figure 4. Observed samples of the IUH versus model samples of the IUH

Figure 5. Mean model IUH versus mean observed IUH

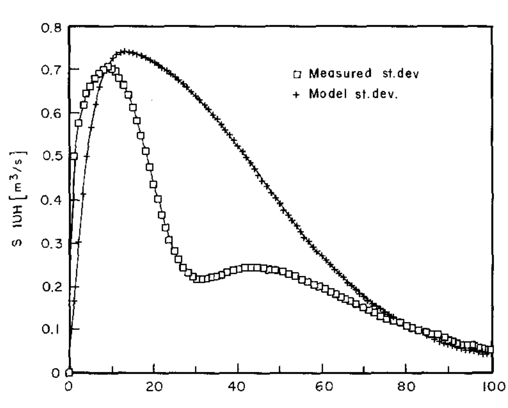

Figure 6. Model standard deviation of IUH versus observed standard deviation of IUH

A few sample functions of the IUH were computed by using a Gaussian random numbers generator in combination with Eq. (11) with the conversion factor to m^3/s of $A/3.6$, where $A = 298\ km^2$. The population mean and variance of K were assumed equal to the corresponding sample ones. Fig. 4 shows three generated sample IUH's (S1, S2, and S3) along with three fitted IUH's from observed data (D1, D2, D3). Note that, at least qualitatively, simulated IUH's are similar to observed ones.

A more objective verification of the model is achieved after comparing the observed mean and variance of the IUH with the corresponding simulated statistics. The observed mean IUH, $\hat{O}_2$ was computed after the equation (Nash, 1957)

$$\hat{O}_2(t) = \frac{1}{\overline{K}\Gamma(\overline{n})}[\frac{t}{\overline{K}}]^{\overline{n}-1}e^{-t/\overline{K}}, \tag{14}$$

where $\overline{n}$ is the average observed number of reservoirs, $\overline{K}$, is the average observed storage constant, and $\Gamma(\ )$ is the Gamma function. The simulated mean IUH was computed after Eq. (12) with $\sigma^2 = \hat{\sigma}^2 = S_K^2 = 5.36^2$, which is the observed variance of K. Fig. 5 shows the observed mean and the simulated mean IUH, which indicates a very close agreement.

The observed variance of the IUH was computed numerically at several time intervals based on the fourteen observed IUH. The simulated variance was computed by using Eq. (13). Fig. 6 illustrates the observed standard deviation along with the simulated standard deviation of the IUH with respect to time. Generally there is good agreement between observed and simulated standard deviation between 0 and 10 hr, after 65 hr and particularly at the maximum values. The discrepancy between 10 and 65 hr is perhaps due to the arbitrary procedure employed in the calculation of the observed variance, and to the unknown differences between the sample mean and sample variance of K, $\overline{K}$ and S_K respectively, with respect to the corresponding population parameters. It seems that with a higher number of sample storms one could reduce the difference between observed and simulated variance in the IUH. Since the objective of the present study was to illustrate the methodology, no attempt to calibrate the model was done. Such a procedure would involve the adjustment of the S_K such that a measure of the error of observed with respect to simulated variance in the IUH is minimum.

In general, however, the model equations are quite satisfactory since the observed and simulated statistics of the IUH are of the same order of magnitude. Thus the stochastic IUH, summarized in Eqs. (11) through (13), accounts for both climatological and model uncertainty and can be used as a more general model for the synthesis of streamflow data and flood forecasting with a measure of variability. For example, the results of the present application indicate that the peak flow rate of the IUH of the Middle Thames River has a mean of 2.21 m^3/s with a standard deviation of plus or minus 0.74 m^3/s. This uncertainty measure could be easily transmitted to an event rainfall-runoff simulation model to predict variability in the streamflow forecast. In addition, sample functions of the IUH could be easily incorporated into a streamflow simulation model to test the performance of projected structures with data which exhibits the same statistical properties of the prototype stream.

4 Summary and conclusions

The present article demonstrated the concept that climate variability affecting rainfall and runoff data and errors generated by approximations, simplification and the adoption of an imperfect conceptual model to represent the IUH of a watershed result in an uncertain estimation of the magnitude and shape of the IUH, which is best described in statistical terms.

If a conceptual model is employed in the derivation of the IUH, then the parameter uncertainty in the model continuity differential equations should be evaluated by using the theory of SDE's. The main contribution of this article has been the introduction of a methodology to derive the stochastic IUH when the above two fields, conceptual models and SDE's, are combined. The methodology was illustrated with the application to the Middle Thames River in Ontario. The main steps of the method were: (1) the selection of several pairs of effective rainfall-storm runoff hydrograph events under homogeneous conditions; (2) fitting a conceptual model to each storm event; (3) evaluation of the statistical properties of the fitted conceptual model parameters (probability density function, moments); and (4) solution of the resulting SDE's arising from the conceptual model subject to the random parameters. The resulting stochastic IUH exhibited similar statistical properties to the corresponding ones of the individual observed IUH's.

A stochastic IUH accounting for climatological and model uncertainty constitutes a more general model, which may prove most useful in any of the wide range of applications currently available for the IUH. Streamflow forecasting is the most commonly encountered, but other applications such as detection of environmental impact could find important benefits where climatological and model uncertainty could be separated from deterministic trends to be detected.

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