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A simple approach to groundwater modelling with decomposition

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Abstract Mathematical models are the means to characterize variables quantitatively in many groundwater problems. Recent advances in applied mathematics have perfected what is now called Adomian's decomposition method (ADM), a simple modelling procedure for practical applications. Decomposition exhibits the benefits of analytical solutions (i.e. stability, analytic derivation of heads, gradients, fluxes and simple programming). It also offers the advantages of traditional numerical methods (i.e. consideration of heterogeneity, irregular domain shapes and multiple dimensions). In addition, decomposition is one of the few systematic procedures for solving nonlinear equations. By far its greatest advantage is its simplicity of application. It may produce simple results for preliminary simulations, or in cases with scarce information. The method is described with simple applications to regional groundwater flow. Many applications in groundwater flow and contaminant transport are available in the literature.

Key words mathematical modelling; Adomian's decomposition method

Une approche simple de modélisation des eaux souterraines par décomposition

Résumé Les modèles mathématiques sont un moyen de caractériser quantitativement les variables dans de nombreux problèmes d'eaux souterraines. Les avancées récentes en mathématiques appliquées ont mis au point ce qu'on appelle aujourd'hui la méthode de décomposition d'Adomian, une procédure simple de modélisation pour des applications pratiques. Cette décomposition présente les avantages des solutions analytiques (stabilité, expression analytique des charges, gradients et flux, et programmation simple). Elle offre également les avantages des méthodes numériques traditionnelles (prise en compte de l'hétérogénéité, formes irrégulières de domaines, et dimensions multiples). En outre, la décomposition est l'une des quelques procédures systématiques permettant de résoudre des équations non linéaires. Son plus grand avantage est de loin sa simplicité d'application. Elle peut produire des résultats simples pour les simulations préliminaires, ou en cas d'informations rares. La méthode est décrite avec des applications simples à l'écoulement régional des eaux souterraines. De nombreuses applications aux écoulements souterrains et au transport de polluants sont disponibles dans la littérature.

Mots clefs modélisation mathématique; méthode de décomposition d'Adomian

INTRODUCTION

Many problems in groundwater require a quantitative characterization and forecasting of variables, such as hydraulic heads, hydraulic gradients, fluxes and pore velocities. The characterization of aquifer variables is usually accomplished via the solution of a differential equation subject to a set of boundary conditions. Traditional methods of solution are broadly divided into numerical and analytical solutions. Traditional numerical solutions involve a discretization of the spatio-temporal aquifer domain and the solution of the resulting matrix equations. Discretization schemes permit a simplification of the model equations and the consideration of irregular domain shapes. However, they may generate instability, they constitute a numerical linearization of nonlinear problems, and the programming and computing execution times may be significant. On the other hand, traditional analytical solutions are continuous in space and time, and render a more stable solution, but they are usually applicable to linear problems and require regular domain shapes.

Recent advances in applied mathematics have perfected what is now called Adomian's Decomposition Method (Adomian 1994, Wazwaz 2000, Rach 2008, Duan and Rach 2011). It consists of deriving an infinite series that, in many cases, converges to an exact solution. For a simple introduction to the method with applications in groundwater, engineering analysis and stochastic methods, the reader is referred to Serrano (2010, 2011). For nonlinear equations in particular, decomposition is one of the few systematic solution procedures available. For instance, Serrano and Workman (1998) and Serrano *et al*. (2007) presented new solutions of the nonlinear Boussinesq equation. Moutsopoulos (2007, 2009) used decomposition to derive the most characteristic case of nonlinear, non-Darcian, unconfined flows in groundwater. With the concepts of partial decomposition and of double decomposition, the process of obtaining an approximate solution in several dimensions is simplified. As long as the initial term in a decomposition series (e.g. the forcing function or the initial/boundary conditions) is described in analytic form, a partial decomposition procedure may offer a simplified approximate solution to many modelling problems. If the shape of domain boundaries can be specified in analytic form (e.g. fitting a curve to a few surveyed points), decomposition yields a solution to problems in aquifers with irregular shapes. If aquifer parameters such as transmissivity can be specified in analytic form (e.g. by fitting a smooth surface to point observations), the method can consider heterogeneity. Pumping wells and transient solutions for various governing equations may be easily derived. The propagation of contaminants subject to nonlinear reactions or nonlinear decay (Serrano 2003) are among the examples of application of decomposition. The main feature of the method is simplicity; it is a useful tool for preliminary simulations or in cases with little data. For certain nonlinear problems, the calculation of the Adomian polynomials may require some effort. Many authors have applied the variational iteration method (Abdou and Soliman 2005, He and Wu 2006) to overcome the difficulty arising in calculating Adomian polynomials. Another approach to solving nonlinear equations is the homotopy perturbation method (He 2000, 2006, Liao 2004), which does not require the assumption of "smallness" in the random parameters, and therefore it overcomes the shortcomings of the classical perturbation method.

In this article decomposition is illustrated with a simple example involving regional groundwater flow subject to mixed boundary conditions. Then, an application of the concept of partial decomposition to a practical scenario under irregular boundaries is shown. Since the boundary conditions are specified for an axis only, a creative combination of partial solutions may be useful in obtaining simple approximations.

ADOMIAN'S DECOMPOSITION METHOD

The method is described with a simple example of regional groundwater flow in an unconfined aquifer with mixed boundary conditions (Fig. 1). The aquifer is limited by the main stream on one side, two tributaries on two sides, and a consolidated geological unit on one side. Assume that the planar dimensions of the aquifer are approximately rectangular in shape; locate the origin of the coordinates system at the confluence between one tributary and the main river; the heads along the tributaries and the main stream are given according to previously surveyed functions. Assuming that Dupuit assumptions are valid and that the aquifer is homogeneous and isotropic, the governing flow equation is given by:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{R_g}{T} \qquad 0 \le x \le l_x, 0 \le x \le l_y \tag{1}$$

where $h(x,y)$ is the hydraulic head function of x and y; $l_x = 860$ m; $l_y = 2000$ m; the aquifer transmissivity, $T = 700$ m^2/month; and the mean recharge from rainfall, $R_g = 10$ mm/month. For boundary conditions, we have a specified head at the rivers,

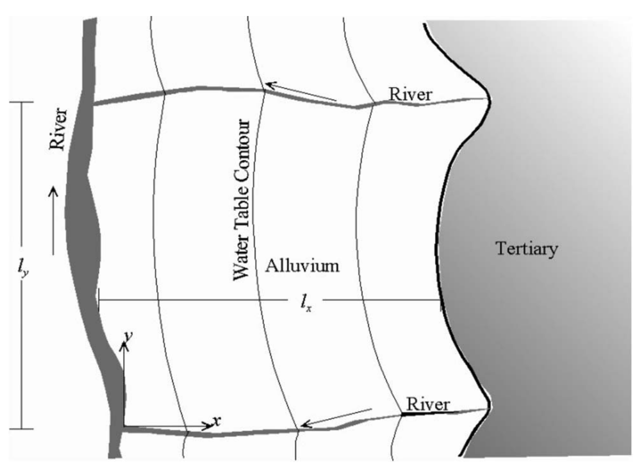

Fig. 1 Regional groundwater flow with mixed boundary conditions.

and a no-flow boundary condition at the intersection of the aquifer with the Tertiary formation. Thus the boundary conditions imposed on (1) are:

$$\begin{aligned} &h(0,y) = f_1(y) \\ &\frac{\partial h}{\partial x}(l_x, y) = 0 \\ &h(x,0) = f_2(x) \\ &h(x,l_y) = f_3(x) \end{aligned} \tag{2}$$

Assume that the head at the main river and its tributaries has been obtained from field measurements of river stage and fitted to the following functions: $f_1(y) = 241 - 0.001y$, $f_2(x) = Cx^2 + Ax + f_1(0)$, $f_3(x) = Ex^2 + Bx + f_1(l_y)$, where $C = -R_g/(2T)$, $A = -2Cl_x$, $B = -2El_x$ and $E = -R_g/T$.

Now define the operators $L_x = \partial^2/\partial x^2$ and $L_y = \partial^2/\partial y^2$. The inverse operators L_y^{-1} and L_x^{-1} are the corresponding two-fold indefinite integrals with respect to x and y, respectively. Equation (1) becomes:

$$L_x h + L_y h = -\frac{R_g}{T} \tag{3}$$

There are two partial decomposition expansions to equation (3): the x-partial solution and the y-partial solution. The x-partial solution, h_x, results from operating with L_x^{-1} in equation (3) and re-arranging. Thus equation (3) becomes:

$$h_x = -L_x^{-1}\frac{R_g}{T} - L_x^{-1}L_y h_x \tag{4}$$

Expanding h_x on the right-hand side as an infinite series $h_x = h_{x0} + h_{x1} + h_{x2} + \ldots$, equation (4) becomes:

$$h_x = -L_x^{-1}\frac{R_g}{T} - L_x^{-1}L_y\left(h_{x0} + h_{x1} + h_{x2} + \ldots\right) \tag{5}$$

The choice of h_{x0} often determines the level of difficulty in calculating subsequent decomposition terms and the rate of convergence (Adomian 1994, Wazwaz 2000). A simple choice is to set h_{x0} as equal to the first three terms on the right-hand side of equation (5). Thus, the first approximation to the solutions is:

$$\begin{aligned} h_{x0} &= k_1(y) + k_2(y)x - L_x^{-1}\frac{R_g}{T} \\ &= k_1(y) + k_2(y)x - \frac{R_g x^2}{2T} \end{aligned} \tag{6}$$

where the integration "constants" k_1 and k_2 must be found from the x boundary conditions in equation (2):

$$\begin{aligned} &h(0,y) = f_1(y) = h_{x0}(y) = k_1(y) \\ &\frac{\partial h}{\partial x}(l_x,y) = 0 \\ &\qquad = \frac{\partial h_{x0}}{\partial x}(l_x,y) = k_2(y) - \frac{R_g l_x}{T} \\ &\qquad \Rightarrow k_2(y) = \frac{R_g l_x}{T} \end{aligned}$$

Equation (6) becomes:

$$h_{x0} = f_1(y) + \frac{R_g l_x x}{T} - \frac{R_g x^2}{2T} \tag{7}$$

Equation (7) satisfies the governing equation (1) and the x boundary conditions in equation (2), but not necessarily those in the y direction. Now, to obtain the y-partial solution to equation (1), h_y, operate with L_y^{-1} on equation (1) and rearrange:

$$h_y = -L_y^{-1}\frac{R_g}{T} - L_y^{-1}L_x h_y \tag{8}$$

Expanding h_y on the right-hand side as an infinite series $h_y = h_{y0} + h_{y1} + h_{y2} + \ldots$, equation (8) becomes:

$$h_y = -L_y^{-1}\frac{R_g}{T} - L_y^{-1}L_x\left(h_{y0} + h_{y1} + h_{y2} + \ldots\right) \tag{9}$$

Again, if we take h_{y0} as the first three terms on the right-hand side of equation (9), we obtain the first approximation, that is:

$$\begin{aligned} h_{y0} &= k_3(x) + k_4(x)y - L_y^{-1}\frac{R_g}{T} \\ &= k_3(x) + k_4(x)y - \frac{R_g y^2}{2T} \end{aligned} \tag{10}$$

where the integration "constants" k_3 and k_4 are found from the y boundary conditions (equation (2)):

$$h_{y0} = f_2(x) + \left(\frac{f_3(x) - f_2(x)}{l_y} + \frac{R_g l_y}{2T}\right)y - \frac{R_g y^2}{2T} \tag{11}$$

The y-partial solution satisfies the differential equation (1) and the y boundary conditions in (2), but not necessarily those in the x direction. We now have two partial solutions to equation (1): the x-partial solution (equation (7)) and the y-partial solution (equation (11)). Since both are solutions to h, an algebraic reduction of the two partial solutions yields:

$$h_0(x,y) = \left(\frac{h_{x0}(x,y) + h_{y0}(x,y)}{2}\right) \tag{12}$$

To obtain the second term in the combined series, h_1, we need to re-derive a new x-partial solution, a new y-partial solution and combine them as above. Thus, the second term in the x-partial solution, h_{x1}, may be derived from the x-partial solution expansion (5):

$$h_{x1} = k_5(y) + k_6(y)x - L_x^{-1}L_y h_0$$

where h_0 is given by equation (12) and k_5 and k_6 are such that equation (13) satisfies homogeneous (i.e. zero) x boundary conditions in equations (12). Hence:

$$h_{x1} = -\frac{R_g}{2T}\left(l_x x - \frac{x^2}{2}\right) \tag{13}$$

Similarly, the second term in the y-partial solution, h_{y1}, may be derived from y-partial solution expansion (9):

$$h_{y1} = k_7(x) + k_8(x)y - L_y^{-1}L_x h_0$$

where h_0 is given by equation (12), and k_7 and k_8 are such that equation (14) satisfies homogeneous (i.e. zero) y boundary conditions in (2). Hence:

$$h_{y_1} = -\frac{R_g}{2T}\left(\frac{7l_y y}{6} - y^2 - \frac{y^3}{6l_y}\right) \tag{14}$$

Subsequently, h_1 is obtained by combining equations (13) and (14):

$$h_1(x,y) = \left(\frac{h_{x1}(x,y) + h_{y1}(x,y)}{2}\right) \tag{15}$$

Higher-order terms are derived similarly. The ith-order terms in the x-partial solution, h_{xi}, may be derived from equation (5):

$$h_{xi} = k_{4i+1}(y) + k_{4i+2}(y)x - L_x^{-1}L_y h_{i-1}$$

where h_{i-1} is the previous combined term in the decomposition series, and k_{4i+1} and k_{4i+2} are such that homogeneous (i.e. zero) x boundary conditions in (2) are satisfied. Similarly, the ith-order term in the y-partial solution, h_{yi}, may be derived from equation (9):

$$h_{yi} = k_{4i+3}(x) + k_{4i+4}(x)y - L_y^{-1}L_x h_{i-1}$$

where $h_{i\text{-}1}$ is the previous combined term in the decomposition series, and k_{4i+3} and k_{4i+4} are such that homogeneous (i.e. zero) y boundary conditions in (2) are satisfied. Similarly to (15), the ith combined term is given by:

$$h_i(x,y) = \left(\frac{h_{xi}(x,y) + h_{yi}(x,y)}{2}\right) \tag{16}$$

Lastly, we approximate the final solution with N terms, $h \approx h_0 + h_1 + \ldots + h_N$, where each term in the series is a combination of two partial solutions, one in x and one in y. Due to the high rate of convergence of decomposition solutions, the hydrologist often finds that one or two terms in the above iteration might be reasonably accurate in many practical applications. In general, for higher-dimensional problems, a simple partial decomposition may be constructed for each coordinate. This facilitates the building of a multi-dimensional problem with simpler mathematical derivations than traditional analytical methods, and simpler computer programming than traditional numerical methods. Recent improvements in standard mathematics software, such as Maple, require minimal efforts in the mathematical derivations, the calculations of head gradients and fluxes, and the graphing of the regional water table and directional flux vector. In many practical situations, simple groundwater models may be constructed without specialized groundwater software. Figure 2 shows the distribution of the regional water table for the current example with four decomposition terms.

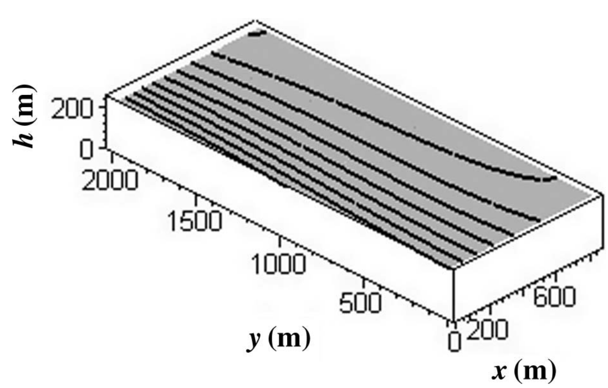

Fig. 2 Regional water table distribution.

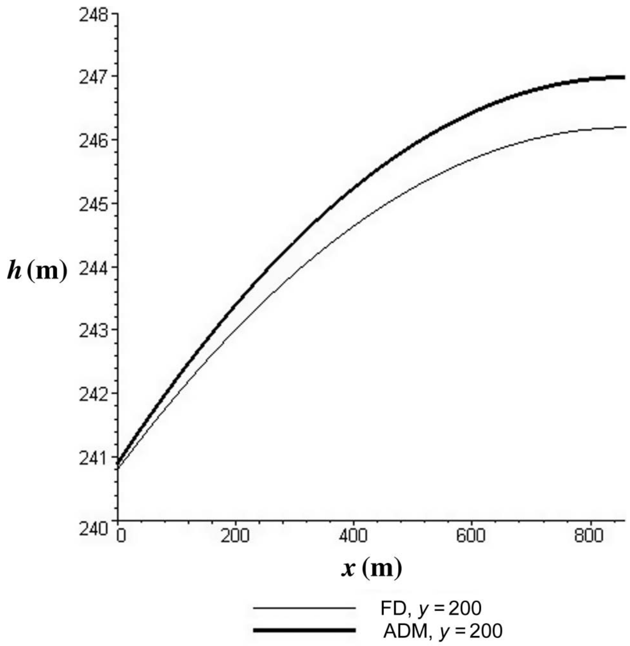

Fig. 3 Comparison between decomposition and finite difference, $y = 200$ m.

Figure 3 illustrates a comparison of groundwater heads *versus* x at $y = 200$, according to decomposition and a finite-difference solution obtained via a Gauss-Seidel iteration in conjunction with successive over-relaxation. Figure 4 illustrates a comparison of groundwater heads *versus* x at $y = 1000$, according to decomposition and a finite-difference solution. A maximum relative difference between the two methods of about 0.32% occurs in the middle of the no-flow boundary, that is on $x = l_x$, $y = l_y/2$. To observe the convergence rate, we compare a decomposition approximation with respect to the exact head at the southeast boundary, $f_2(l_x)$. Figure 5 shows the relative error after approximating head with n terms, $h(l_x,0) \approx h_n(l_x,0)$, where n changes from 1 to 9. The convergence is fast for the first four terms, after which it stabilizes. For this problem, a minimum error is obtained with four decomposition terms, but reasonable accuracy is obtained with only one or two terms. An exact solution of (1) is also possible by traditional analytical methods, in particular via a combination of separation of variables and Fourier series (Powers 1979). Among the advantages of traditional analytical solutions are stability, analytic derivation of heads, gradients, fluxes and simple computer programming. However, traditional analytical methods restrict their application to homogeneous aquifers, regular domain shapes and linear equations. Problems in multiple dimensions are also difficult with traditional analytical methods. In contrast, traditional numerical methods, such as finite differences, permit the consideration of heterogeneity, irregular domains and nonlinear equations. However, numerical methods restrict the solution to discrete locations or nodes. Numerical solutions increase the complexity of computer programming,

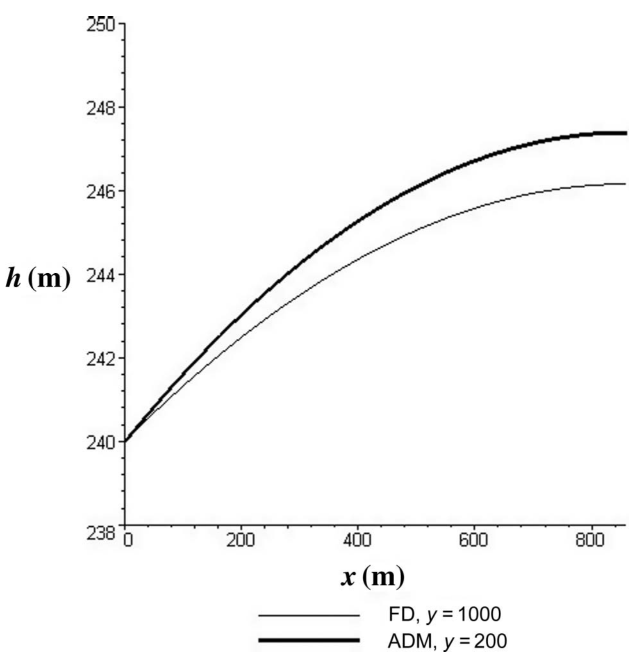

Fig. 4 Comparison between decomposition and finite difference, $y = 1000$ m.

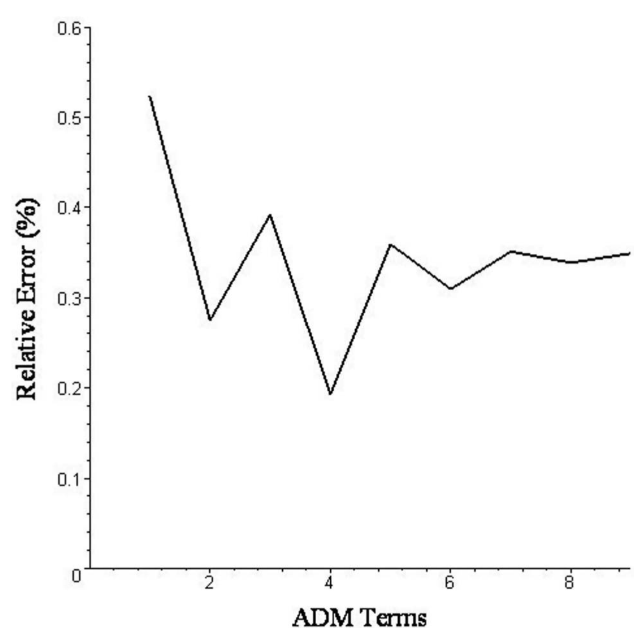

Fig. 5 Head relative error with respect to the number of ADM terms used at $(l_x, 0)$.

computer execution time, numerical round-off errors and numerical instability. Decomposition seems to offer an alternative that exhibits the benefits of traditional analytical methods as well as those of classical numerical methods.

APPLICATIONS TO MODELLING GROUNDWATER FLOW IN IRREGULAR DOMAINS

We now illustrate the use of partial decomposition solutions to the simulation of groundwater flow in practical scenarios with mixed boundaries. Since the first few terms in a partial decomposition expansion contain most of the information in a decomposition series, and since a partial solution is a weak mathematical solution with specified boundaries in one axis and unspecified in the other ones, a creative combination of partial solutions often yields simple models in practical situations. Figure 6 (not drawn to scale) shows a plan view of an aquifer bounded by rivers with a deep excavation inside where the head is maintained at $h = H_0 = 50$ m, $a \leq x \leq b$, $c \leq y \leq d$. Assuming (1) is valid, the boundary conditions are now:

$$\begin{aligned} &h(0,y) = f_1(y),\ h(l_x,y) = f_2(y),\\ &h(x,0) = f_3(x),\ h(x,l_y) = f_4(x) \end{aligned} \tag{17}$$

where $l_x = 600$ m, $l_y = 2000$ m, $T = 500$ m^2/month, $R_g = 10$ mm/month, $f_1(y) = 100 - 0.0005y$, $f_2(y) = 103 - 0.001y$, $f_3(x) = 100 + 0.008x - 0.000005x$, $f_4(x) = 99 + 0.009333x - 0.0000x^2$, $H_0 = 50$ m, $a = 3000$ m, $b = 400$ m, $c = 800$ m and $d = 1200$ m.

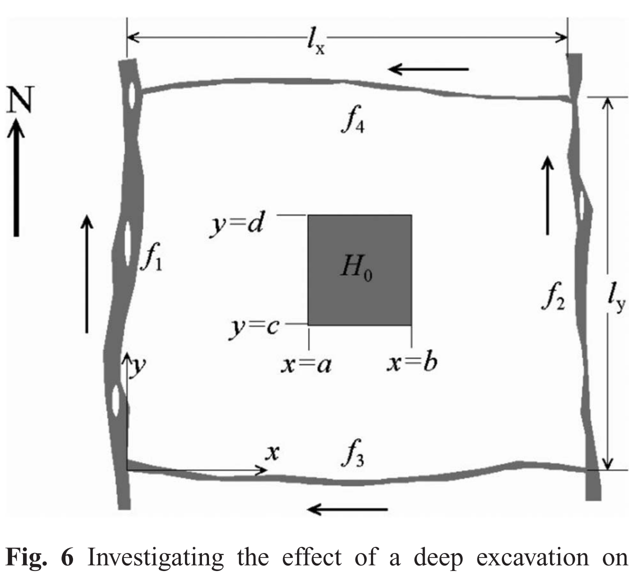

Fig. 6 Investigating the effect of a deep excavation on regional groundwater.

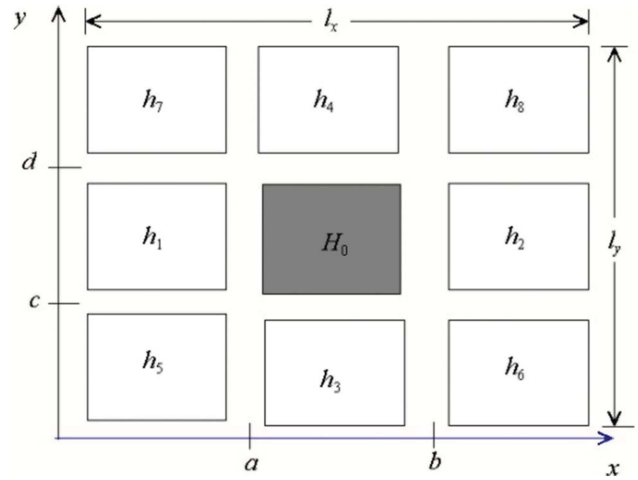

Fig. 7 Division of the aquifer into eight partial decomposition solutions.

We now subdivide the aquifer into eight regions surrounding the excavation, h_1 to h_8, such that each constitutes a partial decomposition solution with clearly-defined boundary conditions in one axis (Fig. 7). For Region 1 west of the excavation, the first term in the x-partial solution, h_{1x}, is given by equation (6) as:

$$\begin{aligned} &h_{1x} = k_{1x}(y) + m_{1x}(y)x - L_x^{-1}R_g/T\\ &h_{1x}(0,y) = f_1(y)\\ &h_{1x}(a,y) = H_0\\ &0 \leq x \leq a,\ c \leq y \leq d \end{aligned} \tag{18}$$

where k_{1x} and m_{1x} are found from the x boundary conditions at $x = 0$ and $x = a$, respectively. For Region 2 east of the excavation, the first term in the x-partial solution, h_{2x}, is given by equation (6) as:

$$\begin{aligned} &h_{2x} = k_{2x}(y) + m_{2x}(y)x - L_x^{-1}R_g/T\\ &h_{2x}(b,y) = H_0\\ &h_{2x}(l_x,y) = f_2(y)\\ &b \leq x \leq l_x,\ c \leq y \leq d \end{aligned} \tag{19}$$

where k_{2x} and m_{2x} are found from the x boundary conditions at $x = b$ and $x = l_x$, respectively. For Region 3 south of the excavation, the first term in the y-partial solution, h_{3y}, is given by equation (10) as:

$$
\begin{aligned}
& h_{3y} = k_{3y}(x) + m_{3y}(x)y - L_y^{-1}R_g/T \\
& h_{3y}(x,0) = f_3(x) \\
& h_{3y}(x,c) = H_0 \\
& a \le x \le b,\ 0 \le y \le c
\end{aligned}
\tag{20}
$$

where k_{3y} and m_{3y} are found from the y boundary conditions at $y = 0$ and $y = c$, respectively. For Region 4 north of the excavation, the first term in the y-partial solution, h_{4y}, is given by equation (10) as:

$$
\begin{aligned}
& h_{4y} = k_{4y}(x) + m_{4y}(x)y - L_y^{-1}R_g/T \\
& h_{4y}(x,d) = H_0 \\
& h_{4y}(x,l_y) = f_4(x) \\
& a \le x \le b,\ d \le y \le l_y
\end{aligned}
\tag{21}
$$

where k_{4y} and m_{4y} are found from the y boundary conditions at $y = d$ and $y = l_y$, respectively. For Region 5 southwest of the excavation, the first term in the x-partial solution, h_{5x}, is given by equation (6) as:

$$
\begin{aligned}
& h_{5x} = k_{5x}(y) + m_{5x}(y)x - L_x^{-1}R_g/T \\
& h_{5x}(0,y) = f_1(y) \\
& h_{5x}(a,y) = h_{3y}(a,y) \\
& 0 \le x \le a,\ 0 \le y \le c
\end{aligned}
\tag{22}
$$

where k_{5x} and m_{5x} are found from the x boundary conditions at $x = 0$ and $x = a$, respectively. Notice that at $x = a$ we use $h_{3y}(a,y)$ from equation (20). For Region 6 southeast of the excavation, the first term in the x-partial solution, h_{6x}, is given by equation (6) as:

$$
\begin{aligned}
& h_{6x} = k_{6x}(y) + m_{6x}(y)x - L_x^{-1}R_g/T \\
& h_{6x}(b,y) = h_{3y}(b,y) \\
& h_{6x}(l_x,y) = f_2(y) \\
& b \le x \le l_x,\ 0 \le y \le c
\end{aligned}
\tag{23}
$$

where k_{6x} and m_{6x} are found from the x boundary conditions at $x = b$ and $x = l_x$, respectively. At $x = b$ we use $h_{3y}(b,y)$ from equation (20). For Region 7 northwest of the excavation, the first term in the x-partial solution, h_{7x}, is given by equation (6) as:

$$
\begin{aligned}
& h_{7x} = k_{7x}(y) + m_{7x}(y)x - L_x^{-1}R_g/T \\
& h_{7x}(0,y) = f_1(y) \\
& h_{7x}(a,y) = h_{4y}(a,y) \\
& 0 \le x \le a,\ d \le y \le l_y
\end{aligned}
\tag{24}
$$

where k_{7x} and m_{7x} are found from the x boundary conditions at $x = 0$ and $x = a$, respectively. At $x = a$ we use $h_{4y}(a,y)$ from equation (21). Lastly, for Region 8 northeast of the excavation, the first term in the x-partial solution, h_{8x}, is given by equation (6) as:

$$
\begin{aligned}
& h_{8x} = k_{8x}(y) + m_{8x}(y)x - L_x^{-1}R_g/T \\
& h_{8x}(b,y) = h_{4y}(b,y) \\
& h_{8x}(l_x,y) = f_2(y) \\
& b \le x \le l_x,\ d \le y \le l_y
\end{aligned}
\tag{25}
$$

where k_{8x} and m_{8x} are found from the x boundary conditions at $x = b$ and $x = l_x$, respectively. At $x = b$ we use $h_{4y}(b,y)$ from equation (21). Hence, the first solution to equation (1) subject to the boundary conditions specified in equation (17) is built by a piecewise contribution from each region:

$$
h(x,y) = \left\{
\begin{array}{l}
h_{1x}(x,y),\ 0 \le x \le a,\ c \le y \le d \\
h_{2x}(x,y),\ b \le x \le l_x,\ c \le y \le d \\
h_{3x}(x,y),\ a \le x \le b,\ 0 \le y \le c \\
h_{4x}(x,y),\ a \le x \le b,\ d \le y \le l_y \\
h_{5x}(x,y),\ 0 \le x \le a,\ 0 \le y \le c \\
h_{6x}(x,y),\ b \le x \le l_x,\ 0 \le y \le c \\
h_{7x}(x,y),\ 0 \le x \le a,\ d \le y \le l_y \\
h_{8x}(x,y),\ c \le x \le l_x,\ d \le y \le l_y
\end{array}
\right\}
\tag{26}
$$

The calculation of equation (26) is easily done with the function *piecewise*( ) available in most standard

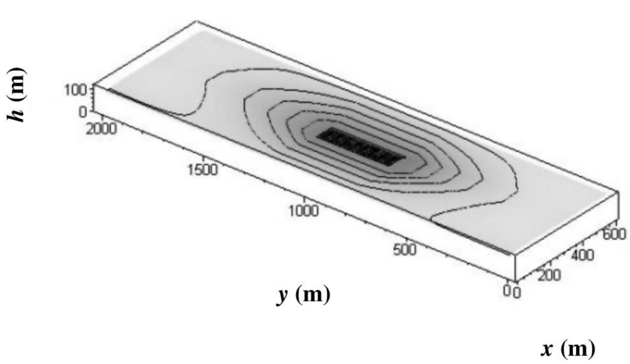

Fig. 8 Groundwater head contours according to equation (26) (m).

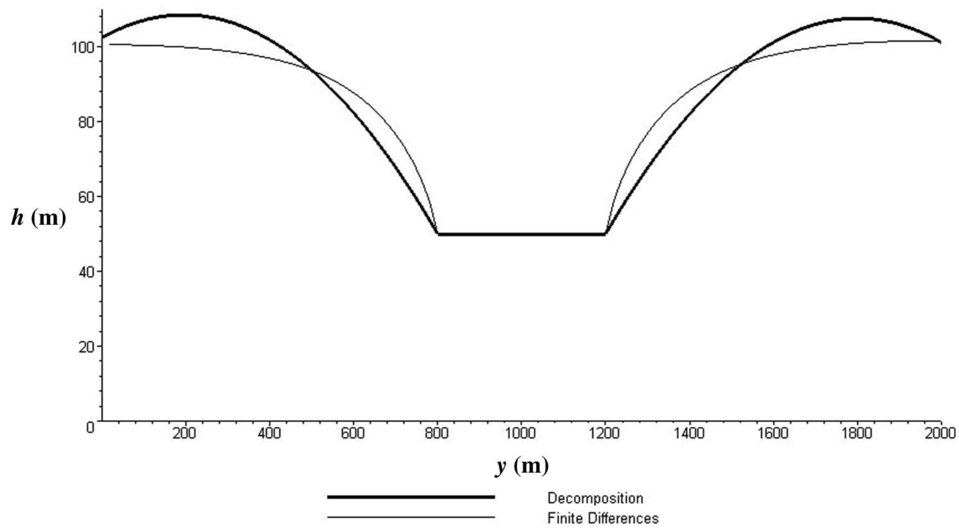

Fig. 9 Head comparison, $h(400,y)$, between decomposition and finite differences.

mathematics software. Notice that equations (18)–(25) constitute simple equations requiring integral calculus only. Thus, a few lines of computer code will suffice in the calculation of heads, gradients and fluxes, and in the production of contour plots and directional flow vectors. Figure 8 shows the groundwater head contour distribution according to equation (26). The simplicity of the calculation is remarkable. Smoothness and accuracy may be improved by calculating additional decomposition terms from the x-partial expansion (equation (5)) or the y-partial expansion (equation (9)). Having an analytical solution permits the calculation of gradients and fluxes analytically by using the *diff*( ) function in mathematics software in conjunction with Darcy's law. This obviates the need of a numerical approximation of gradients at nodes. Figure 9 illustrates a comparison of groundwater heads against y at $x = 400$ according to decomposition and a finite-difference solution obtained via a Gauss-Seidel iteration in conjunction with successive over-relaxation. The programming of the numerical solution was considerably more involved and the execution time was significantly greater than those of the decomposition solution.

SUMMARY AND CONCLUSION

Adomian's (1994) method of decomposition has been presented as a simplified mathematical modelling technique for groundwater characterization, forecasting and resource evaluation purposes. The method exhibits many of the advantages of classical analytical methods, as well as the advantages of traditional numerical procedures. The most important advantage is its simplicity of implementation. It could prove useful when insufficient information is available, or for approximate preliminary calculations. The method was illustrated with two simple applications to regional groundwater flow modelling and compared with traditional finite differences.

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