Pergamon

Mathl. Comput. Modelling Vol. 24, No. 4, pp. 15–25, 1996
Copyright©1996 Elsevier Science Ltd
Printed in Great Britain. All rights reserved
0895-7177/96 $15.00 + 0.00

PII: S0895-7177(96)00104-5

New Contributions to the Solution of Transport Equations in Porous Media

S. E. SERRANO
Department of Civil Engineering
University of Kentucky, Lexington, KY 40506, U.S.A.

G. ADOMIAN
General Analytics Corporation
155 Clyde Road, Athens, GA 30605, U.S.A.

(Received April 1995; accepted March 1996)

Abstract—In this article recent new developments in the method of decomposition as applied to the general solution of the three-dimensional deterministic/stochastic transport equation in heterogeneous porous media are presented with several applications to convection-dispersion models, spatial scale-dependent and time scale-dependent dispersion equations in heterogeneous porous media, simulation examples with coefficients of variability of up to 500%, and verification with exact solutions. The decomposition method offers fundamental advantages that could be used to build a more realistic theory of groundwater flow and contaminant transport than the current small perturbations theories.

Keywords—Groundwater pollution, Scale-dependent transport equations, Decomposition solutions.

1. INTRODUCTION

Practical problems involving groundwater management, groundwater pollution control, and aquifer restoration today require the use of a mathematical model capable of forecasting the time and space evolution of a contaminant in a particular aquifer. The model is usually composed of a set of partial differential equations representing solute dispersion in porous media. Solving these equations for the concentration distribution subject to the several physical uncertainties, and the difficult to measure *in situ* initial conditions and uncertain chemical reactions and environmental conditions, is one of the main difficulties in environmental engineering today. In addition, dispersion parameters appear to increase with the scale of observation, or the time after the spill. For these reasons, equations subject to random dispersion parameters or randomly distributed nonpoint sources are now considered as appropriate tools to account for the random variability and uncertainty in the physical parameters and the input data. The resulting deterministic or stochastic, linear or nonlinear, partial differential equations require a general analytical procedure capable of approximating the solution in an accurate and systematic way.

An extensive hydrologic literature has already addressed some of the above problems by using several methods to solve the differential equations. The major advantage of existing solution methods of stochastic transport equations is that they have permitted the development of some fundamental understanding of the phenomenon of mass transport in aquifers. The major disadvantage is that most of them have been built based on excessive restrictions and assumptions

Typeset by $\mathcal{A}_{\mathcal{M}}\mathcal{S}$-TEX

created for the purpose of making the mathematics of the problem tractable and the solution of the equations possible, and not necessarily to reflect physical conditions. Several physical assumptions combined with classical small perturbation solutions have produced solutions to differential equations which may not be a proper representation of the physical problem to model. For a summary and a critical review of the existing stochastic theories of transport, the reader is referred to [1–3].

Alternative theories are beginning to appear in the literature. Serrano and Unny [4] introduced the method of decomposition [5,6] as a fundamental tool to approximate the transient groundwater flow equation subject to stochastic transmissivity and recharge with a favorable comparison with the exact solution. Serrano [7,8] presented a general solution of the random advective dispersive equation in porous media, where the random terms could appear anywhere in the differential equation. Subsequently, semianalytical solutions of transport equations in groundwater were introduced with the possibility to consider irregular domains, and a theorem with proof establishing the bounds in the simulation time step to assure uniform convergence in the decomposition series [9]. Examples with normal, and sometimes large variability were given and compared with exact solutions. The method of decomposition has also been used to solve the stochastic Richard's equation of infiltration subject to hysteretic soil-water functional relationships [10,11], opening the way to address highly nonlinear phenomena in hydrology.

The purpose of this article is to present some recent refinements in the decomposition theory [5,6,12] with new applications to deterministic/stochastic transport equations in porous media. In Section 2, a new decomposition solution of the general stochastic transport equation in porous media is presented. Section 3 illustrates applications to convection dispersion models with comparison examples with exact solutions. Emphasis is given to the illustration of practical simulation techniques and the convergence of individual terms in the series. Section 4 shows new application solutions to scale dependent transport equations in aquifers.

2. GENERAL SOLUTION OF TRANSPORT EQUATIONS IN POROUS MEDIA

Consider the general three-dimensional stochastic transport equation in porous media given by [7,8]

$$\frac{\partial u}{\partial t}(x,t,\omega) + A(x,t,\omega)u = g(x,t,\omega) \qquad (x,t,\omega) \in G \times [0,T] \times \Omega, \tag{1}$$

where u is the system output representing (i.e., hydraulic head, contaminant concentration), g is a second-order stochastic process, G is subset of $\Re^3$ with boundary ∂G on $(0 < T < \infty)$, x represents three-dimensional space, A is an m^{th} order random partial differential operator. We write (1) as

$$Lu = g - Au. \tag{2}$$

Using decomposition [5], we define L^{-1} as the definite integral from zero to t, with decomposition of u into $\sum_{n=0}^{\infty} u_n$ and identification of u_0 as $u(0)$. We have

$$\begin{aligned} L^{-1}Lu &= L^{-1}g - L^{-1}Au, \\ u - u(t=0) &= L^{-1}g - L^{-1}Au, \\ u &= u(t=0) + L^{-1}g - L^{-1}Au. \end{aligned} \tag{3}$$

Define $u_0 = u(t=0) + L^{-1}g$ and $u = \sum_{n=0}^{\infty} u_n$,

$$u = u_0 - L^{-1}A\sum_{n=0}^{\infty} u_n, \tag{4}$$

$$
\begin{aligned}
u_1 &= -L^{-1}Au_0, \\
u_2 &= -L^{-1}Au_1 = (-L^{-1}A)^2u_0, \\
&\vdots \\
u_n &= -L^{-1}Au_{n-1}.
\end{aligned}
\tag{4, cont.}
$$

We can then write the approximant

$$
\phi_m[u] = u_0 - \sum_{n=0}^{m-2} L^{-1}Au_n = \sum_{n=0}^{m-1} \left(-L^{-1}A\right)^n u_0. \tag{5}
$$

EXAMPLE.

$$
\frac{\partial u}{\partial t} = g - Au, \tag{6}
$$

where

$$
\begin{aligned}
A &= -D\frac{\partial^2}{\partial x^2} + v\frac{\partial}{\partial x}, \\
u &= u(t=0) + L^{-1}g, \\
u_1 &= -L^{-1}\left[-D\frac{\partial^2}{\partial x^2} + v\frac{\partial}{\partial x}\right]u_0 = L^{-1}D\frac{\partial^2}{\partial x^2}u_0 - L^{-1}v\frac{\partial}{\partial x}u_0, \\
&\vdots
\end{aligned}
\tag{7}
$$

with D, v constant parameters. $\langle u\rangle$, where $\langle\ \rangle$ denotes the expectation operator, is found from $\langle\phi_m[u]\rangle$:

$$
\langle u_0\rangle = \langle u(t=0)\rangle + L^{-1}\langle g\rangle \qquad \text{or} \qquad u(t=0) + L^{-1}\langle g\rangle, \tag{8}
$$

depending on the given initial conditions,

$$
\langle u_1\rangle = L^{-1}D\frac{\partial^2}{\partial x^2}\langle u_0\rangle - L^{-1}\langle v\rangle\frac{\partial}{\partial x}\langle u_0\rangle, \tag{9}
$$

or even

$$
\begin{aligned}
\langle u_1\rangle &= L^{-1}\langle D\rangle\frac{\partial^2}{\partial x^2}\langle u_0\rangle - L^{-1}\langle v\rangle\frac{\partial}{\partial x}\langle u_0\rangle \\
&\vdots
\end{aligned}
\tag{10}
$$

if D, v are stochastic. g could be a general stochastic process. First and second-order statistics can be found easily. Because of the fast convergence of the series it is usually accurate to obtain $\phi_4[t]$ and $\phi_4[t']$ and average to derive the correlation function (the errors are discussed in [4,6]). The results are easily generalized to three dimensions and, by assuming Gaussian behavior, to higher moments. Finally, by using the Adomian polynomials [5], the results can be extended to nonlinear equations as well. For example, if equation (1) also has a term $f(u)$, the approximate A_n polynomials for $f(u)$ are $f(u) = \sum_{n=0}^{\infty} A_n$ and proceeding exactly as with the u_n. The series $\sum_{n=0}^{\infty} A_n$ forms a generalized Taylor series about the function $u_0(x)$. For boundary value problems which are nonlinear, evaluating the constants of integration must be done at each level of ϕ_m [5].

A rigorous framework for the convergence of decomposition series has been developed by Gabet [13–15] by connecting the method to well-known formulations where classical theorems (fixed point theorem, substituted series, etc.) could be used. Other rigorous work on the convergence was published by Abbauoui and Cherruault [16–18].

3. APPLICATIONS TO CONVECTION DISPERSION MODELS

In an effort to illustrate the simplicity and the convergence of the decomposition method in cases of large variances, we first reconsider the groundwater flow problem conceived by the classical small perturbation solution:

$$\begin{aligned} \frac{d}{dx}\left(K(x)\frac{dh}{dx}\right) &= 0, \qquad 0 \le x \le l_x, \\ h(0) &= H_1, \qquad h(l_x) = H_2, \end{aligned} \tag{11}$$

where h represents the hydraulic head (m); x is horizontal distance (m); l_x is the aquifer length (m); and $K(x)$ is the hydraulic conductivity (m/month). We have added a set of boundary conditions. Define $L_x = \frac{\partial^2}{\partial x^2}$, and write (11) as

$$L_x h = -\frac{dLnK(x)}{dx}\frac{dh}{dx} \tag{12}$$

or

$$h = -L_x^{-1}\frac{dLnK(x)}{dx}\frac{dh}{dx}. \tag{13}$$

The decomposition series yields $h = h_0 + h_1 + h_2 + \cdots$, where

$$\begin{aligned} h_0 &= H_1 + ax, \qquad a = \frac{H_2 - H_1}{l_x}, \\ h_1 &= -L_x^{-1}\frac{dLnK(x)}{dx}\frac{dh_0}{dx} = -a\int LnK(x)\,dx, \\ h_2 &= -L_x^{-1}\frac{dLnK(x)}{dx}\frac{dh_1}{dx} = aL_x^{-1}\frac{dLnK(x)}{dx}LnK(x), \\ &\ \vdots \end{aligned} \tag{14}$$

Following the classical formulation, $LnK(x) = K_l + K'(x)$, where K_l is a constant; and the process $K'(x)$ has the properties $\langle K'(x)\rangle = 0$, $\langle K'(x_1)K'(x_2)\rangle = \sigma_y^2 e^{-\rho(x_1-x_2)^2}$, σ_y^2 is the log hydraulic conductivity variance parameter, and ρ is the correlation decay parameter. Taking expectations on (14), we obtain the series for the mean hydraulic head:

$$\begin{aligned} \langle h_0\rangle &= H_1 + ax, \\ \langle h_1\rangle &= -aK_l x, \\ \langle h_2\rangle &= a\sigma_y^2 x \\ &\ \vdots \end{aligned} \tag{15}$$

As an example, consider an aquifer with large mean hydraulic conductivity of 200.0 m/month and a large variability in the log hydraulic conductivity $\sigma_y^2 = 3.0$, which for a log-normal distribution translates into $K_l = 3.8$, $H_1 = 10.0$ m, $H_2 = 11.0$ m, $l_x = 1000.0$ m. Thus the maximum value for the first term in (15) $\max(|\langle h_0\rangle|) = 11.0$ m, $\max(|\langle h_1\rangle|) = 3.8$ m, $\max(|\langle h_2\rangle|) = 3.0$ m, etc. Clearly the decomposition solution (15) exhibits uniform convergence, even in the case of very large variances. This result has been obtained without neglecting elements of the differential equation judged to be "small," and in this example the series has been truncated after a convergent series gives a solution at the desired resolution. While this result is mathematically correct, on close examination one discovers that $\sigma_y^2 = 3.0$ corresponds to a coefficient of variability $C_v =$ 1908.55% and a standard deviation in the field hydraulic conductivity of $\sigma_k = 3817.1$ m/month. This contradicts any observed hydraulic conductivity and implies the existence of physically unrealizable negative conductivities. While recent manipulations of the small perturbation models claim accuracy for $\sigma_y^2 = 7.0$, clearly the underlying conductivity field is fictitious. This leads to the conclusion that coefficients of variability of the order of 50% in the field hydraulic conductivity are indeed large and that values beyond 100% represent academic examples.

On the other hand, one may attempt to solve (11) by properly constructing a decomposition series without the logarithmic transformation. It is easy to show that upon taking expectations, all the terms in the series vanish except the first, h_0 in (15), thus giving the correct physical result: in the absence of recharge, the mean steady hydraulic head is independent of the hydraulic conductivity and its variance, and must equal the deterministic solution (a straight line between the two boundaries.) Therefore, while the logarithmic transformation conveniently adjusts the variances for small pertubation solutions, it probably yields an incorrect hydrologic model [2].

The next example is the convection dispersion equation (CDE), which still is the heart of many dispersion models in porous media, at least at the laboratory scale. The CDE also constitutes the first approximation in some scale dependent equations, and is part of the kernel in some integral equation solutions. Therefore, the ability to accurately, and efficiently, estimate solutions to the CDE renders a tool useful in applications.

Consider the case of a short term chemical spill in an infinite one-dimensional aquifer governed by the deterministic CDE given by

$$\begin{gathered}\frac{\partial C}{\partial t} - D\frac{\partial^2 C}{\partial x^2} + u\frac{\partial C}{\partial x} = 0, \qquad -\infty < x < \infty, \quad 0 < t, \\ C(\pm\infty, t) = 0, \qquad C(x,0) = f(x),\end{gathered} \tag{16}$$

where $C(x,t)$ is the chemical concentration in the solution (mg/l); D is the dispersion coefficient (m^2/month); u is the aquifer pore velocity (m^2/month); x represents aquifer longitudinal distance (m); t represents time after the spill (month); and $f(x)$ is a smooth function representing the initial chemical spatial distribution in the aquifer.

Now, set $L_t C = \frac{\partial C}{\partial t}$ and rewrite the CDE as

$$L_t C = D\frac{\partial^2 C}{\partial x^2} - u\frac{\partial C}{\partial x}, \tag{17}$$

or

$$C = DL_t^{-1}\frac{\partial^2 C}{\partial x^2} - uL_t^{-1}\frac{\partial C}{\partial x}. \tag{18}$$

The decomposition series are $C = C_0 + C_1 + C_2 + \cdots$, where the first term satisfies $LC_0 = 0$, $C_0(0) = f(x)$, and

$$\begin{aligned} C_0 &= f(x), \\ C_1 &= Dtf''(x) - utf'(x), \\ &\vdots \\ C_n &= DL_t^{-1}\frac{\partial^2 C_{n-1}}{\partial x^2} - uL_t^{-1}\frac{\partial C_{n-1}}{\partial x}. \end{aligned} \tag{19}$$

Note that one may easily adjust the simulation time step to assure convergence, which would require that $Dt/2 < 1$, or $ut/2 < 1$. See [9] for a theorem with proof on the uniform convergence of the above series.

Without loss of generality, and as an example, let us assume an initial condition of the form

$$f(x) = C_0 = \frac{C_i}{\sqrt{4\pi}}e^{-x^2/4}, \tag{20}$$

which is a Gaussian distribution with plume variance $\sigma^2 = 2.0\,\mathrm{m}$. As $\sigma^2 \to 0$, $f(x) \to C_i\delta(x)$, a Dirac's delta function of strength C_i, theoretically an instantaneous spill. From the above equations, the first terms in the series are

$$C_1 = \frac{tf(x)}{2}\left(D\left(\frac{x^2}{2}-1\right)+ux\right),$$
$$C_2 = \frac{t^2 f(x)}{2}\left(D^2\left(\frac{3}{4}-\frac{3}{4}x^2-\frac{3}{8}x^3+\frac{x^5}{8}\right)-2uD\left(\frac{3}{4}x-\frac{x^3}{4}\right)+u^2\left(\frac{x^2}{4}-\frac{1}{2}\right)\right) \tag{21}$$
$$\vdots$$

The exact solution of the CDE is, in this case,

$$C=\frac{1}{\sqrt{4\pi Dt}}\int_{-\infty}^{\infty} e^{-(x-ut-x')^2/4Dt}.\frac{C_i}{\sqrt{4\pi}}e^{-x'^2/4}\,dx' = \frac{C_i}{\sqrt{4\pi(1+Dt)}}e^{-(x-ut)^2/4(1+Dt)} \tag{22}$$

Let us assume that $C_i = 100.0\,\text{mg/l}$, $D = 0.01\,\text{m}^2/\text{month}$, and $u = 1.0$ m/month. Figure 1 illustrates a comparison between the exact solution of the CDE six months after the spill, and the decomposition solution as approximated by $\phi_3[C]$ with the first three terms in the series. The accuracy of the decomposition with three terms is remarkable, and in this case only two terms would produce an acceptable approximation. Figure 2 is a graph of the individual terms, C_0, C_1, and C_2 as a function of distance for the same simulation time, which demonstrates numerically the fast and uniform convergence of the decomposition series.

4. APPLICATIONS TO SCALE DEPENDENT TRANSPORT MODELS

Differential equations that account for the increase in the dispersion parameters with the scale of observation are beginning to appear in the hydrologic literature. These equations are difficult to solve with the traditional analytic techniques, even in the case of homogeneous one-dimensional aquifers. In this section we illustrate new decomposition solutions. First consider the space variable dispersion equation (VDE) in a two-dimensional heterogeneous phreatic aquifer with Dupuit assumptions subject to recharge from rainfall as given by Serrano [19].

$$\frac{\partial \bar{C}}{\partial t} - h_1(x)\frac{\partial^2 \bar{C}}{\partial x^2} + h_2(x)\frac{\partial \bar{C}}{\partial x} + h_3\bar{C} - h_4(x)\frac{\partial^2 \bar{C}}{\partial y^2} = 0, \qquad -\infty < x < \infty, \quad -\infty < y < \infty, \quad 0 < t \tag{23}$$

subject to the same boundary and initial conditions and

$$h_1(x) = \frac{\alpha_x}{nh_0}\left(Irx - h_0'\bar{T}\right), \quad h_2(x) = \frac{1}{nh_0}\left(Irx - h_0'\bar{T} - \alpha_x rI\right), \quad h_3 = \frac{Ir}{nh_0} \tag{24}$$
$$h_4(x) = \frac{\alpha_t}{nh_0}\left(Irx - h_0'\bar{T}\right), \quad r = 1 - \left(\frac{\sigma_T}{\bar{T}}\right)^2, \tag{25}$$

where $\bar{C}(x,y,t)$ is the mean solute concentration (mg/l); I is the mean monthly recharge from rainfall (m/month) responsible for the creation of nonuniform flow in the aquifer; n is the aquifer porosity; α_x is the small scale longitudinal dispersivity (m); α_y is the small scale transverse dispersivity (m); h_0' is the hydraulic gradient at the origin; h_0 is the mean saturated thickness (m); $\bar{T}$ is the mean aquifer transmissivity (m^2/month); σ_T is the transmissivity standard deviation (m^2/month); y represents plan distance (m) perpendicular to x; and the rest of the terms as before.

For the purposes of illustration, let us consider the one-dimensional version of the VDE in a heterogeneous aquifer (for the homogeneous case see [20]). Proceeding as in the previous sections we write the VDE as

$$C = h_1(x)L_t^{-1}\frac{\partial^2 C}{\partial x^2} - h_2(x)L_t^{-1}\frac{\partial C}{\partial x} - h_3 L_t^{-1}C, \tag{26}$$

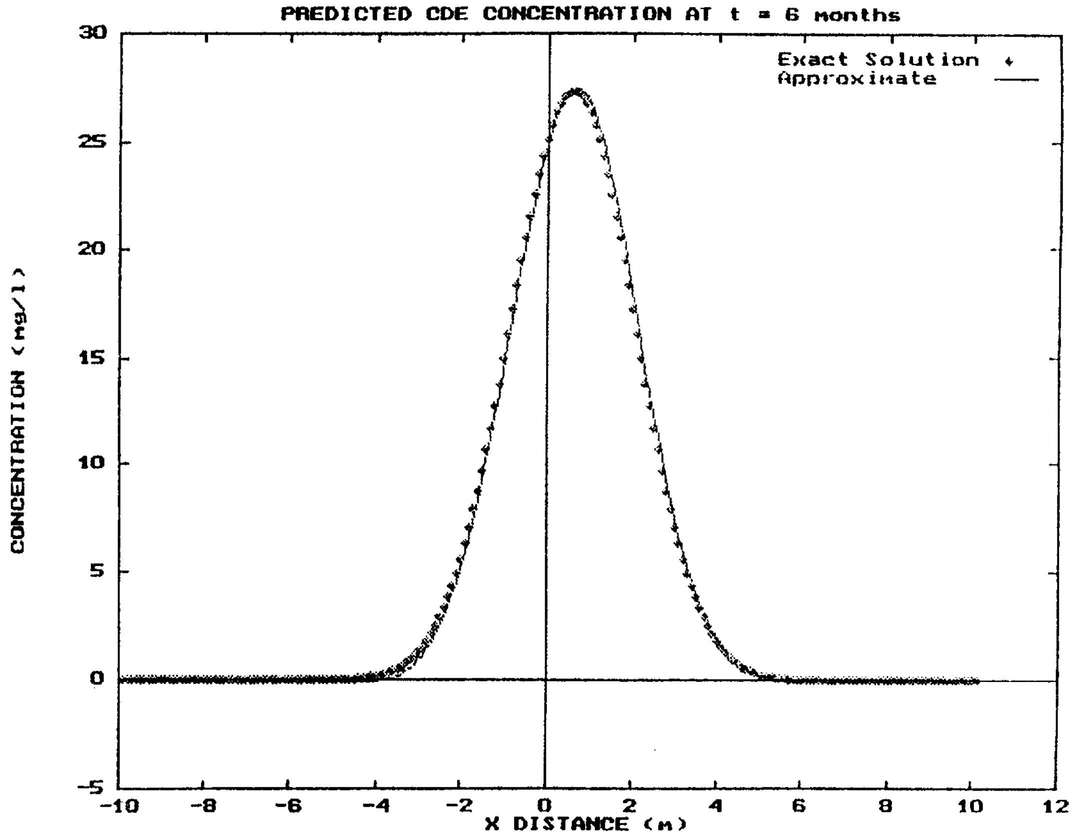

Figure 1. Predicted CDE concentration at $t = 6$ months.

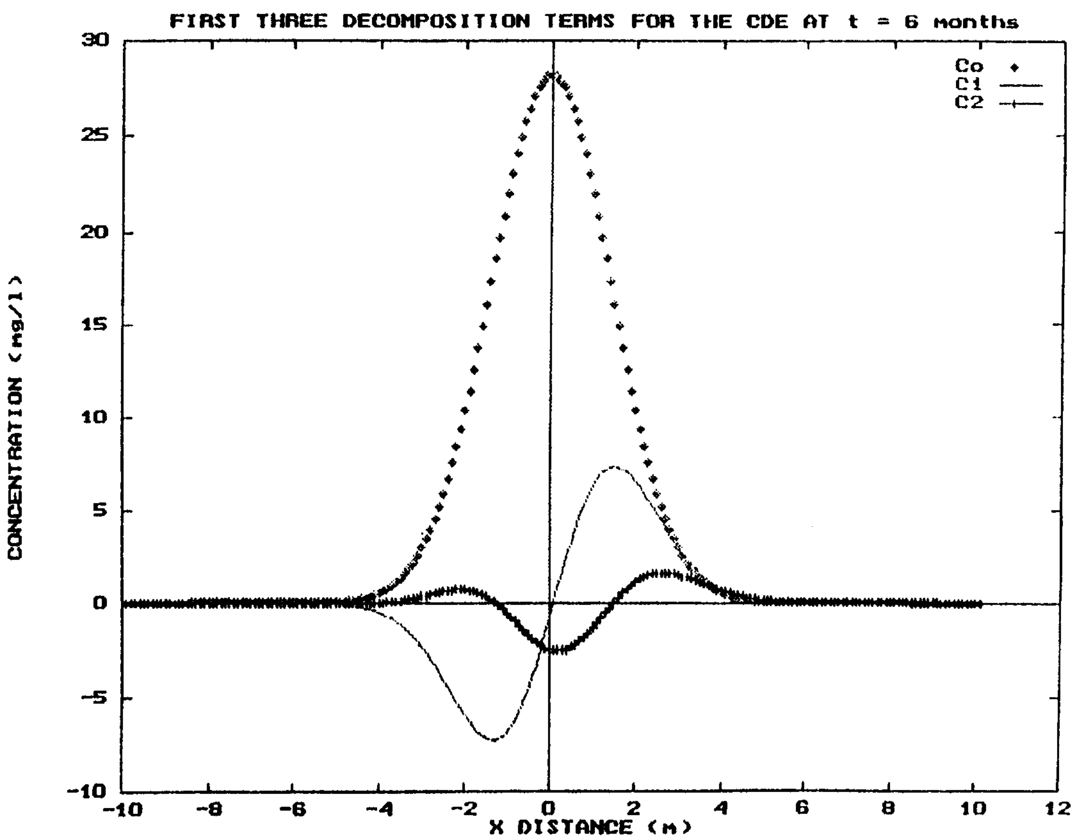

Figure 2. First three decomposition terms for the CDE at $t = 6$ months.

and the individual terms in the decomposition series are given by

$$
\begin{aligned}
C_0 &= f(x),\\
C_1 &= h_1(x)L_t^{-1}\frac{\partial^2 C_0}{\partial x^2} - h_2(x)L_t^{-1}\frac{\partial C_0}{\partial x} - h_3 L_t^{-1}C_0 = th_1 f'' - th_2 f' - th_3 f,\\
C_2 &= h_1(x)L_t^{-1}\frac{\partial^2 C_1}{\partial x^2} - h_2(x)L_t^{-1}\frac{\partial C_1}{\partial x} - h_3 L_t^{-1}C_1,\\
C_2 &= \frac{t^2}{2}\left[h_1\left(h_1 f^{iv} + 2h_1' f''' - h_2 f''' - 2h_2' f'' - h_3 f''\right)\right.\\
&\quad \left. -h_2\left(h_1 f''' + h_1' f'' - h_2 f'' - h_2' f' - h_3 f'\right) - h_3\left(h_1 f'' - h_2 f' - h_3 f\right)\right]\\
&\ \vdots
\end{aligned}
\tag{27}
$$

As an example, assume the following parameter values: $\alpha = 0.1\,\mathrm{m}$, $I = 0.02\,\mathrm{m/month}$, $h_0' = -0.001$, $\bar{T} = 100.0\,\mathrm{m^2/month}$, $h_0 = 10.0\,\mathrm{m}$, a large transmissivity standard deviation $\sigma_T = 100.0\,\mathrm{m^2/month}$, equivalent to a coefficient of variability $C_v = 100\%$, and the rest of the parameters as in Section 3. Figure 3 is a graph of the first three terms of the VDE solution, C_0, C_1, and C_2 as a function of distance six months after the spill, which again demonstrate numerically the fast and uniform convergence of the decomposition series even in cases of large variances.

In the next application we focus our attention on the time VDE in a two-dimensional heterogeneous aquifer [19]

$$
\begin{gathered}
\frac{\partial \bar{C}}{\partial t} - \bar{D}_x(t)\frac{\partial^2 \bar{C}}{\partial x^2} + \bar{u}_x\frac{\partial \bar{C}}{\partial x} - \bar{D}_y(t)\frac{\partial^2 \bar{C}}{\partial y^2} = 0,\\
-\infty < x < \infty, \quad -\infty < y < \infty, \quad 0 < t,
\end{gathered}
\tag{28}
$$

subject to the same boundary and initial conditions. The time dependent dispersion coefficients are, respectively,

$$
\bar{D}_x(t) = D_x + 2r_1\rho\bar{u}_x t, \quad \bar{D}_y = D_y + \frac{r_1}{2}\rho^2 D_y t, \qquad r_1 = \left(\frac{h_0'\sigma_T}{nh_0}\right)^2, \tag{29}
$$

where D_x and D_y are the longitudinal and transverse small scale dispersion coefficients, respectively; ρ is the transmissivity correlation decay parameter (m^{-1}), i.e., $\langle T(x_1)T(x_2)\rangle = \sigma_T^2 e^{-\rho|x_1-x_2|}$; and $\bar{u}_x = -h_0'\bar{T}/nh_0$ is the mean groundwater pore velocity (m/month). Considering the one-dimensional version of this equation and proceeding as before, the first terms in the decomposition series are:

$$
\begin{aligned}
C_0 &= f(x),\\
C_1 &= L_t^{-1}\bar{D}_x(t)\frac{\partial^2 C_0}{\partial x^2} - \bar{u}_x L_t^{-1}\frac{\partial C_0}{\partial x} = \left(D_x t + r_1\rho\bar{u}_x t^2\right) f''(x) - \bar{u}_x t f'(x) + \bar{u}_x\frac{t^2}{2} f''(x)\\
C_2 &= L_t^{-1}\bar{D}_x(t)\frac{\partial^2 C_1}{\partial x^2} - \bar{u}_x L_t^{-1}\frac{\partial C_1}{\partial x},\\
C_2 &= \left(D_x^2 t^2 + r_1\rho D_x\bar{u}_x t^3 + r_1^2\rho^2\bar{u}_x^2\frac{t^4}{2}\right) f^{iv}(x)\\
&\quad - \left(D_x\bar{u}_x\frac{t^2}{2} + \left(2r_1\rho\bar{u}_x + \frac{D_x}{2}\right)\bar{u}_x\frac{t^3}{3} + r_1\rho\bar{u}_x^2\frac{t^4}{12}\right) f'''(x) + \bar{u}_x^2\frac{t^2}{2} f''(x)\\
&\ \vdots
\end{aligned}
\tag{30}
$$

The exact solution to the time VDE is

$$
\bar{C}(x,t) = \frac{C_i}{\sqrt{4\pi(1+\Psi(t))}} e^{-(x-\bar{u}_x t)^2/4(1+\Psi(t))}, \qquad \Psi(t) = \int_0^t \bar{D}_x(t')\,dt'. \tag{31}
$$

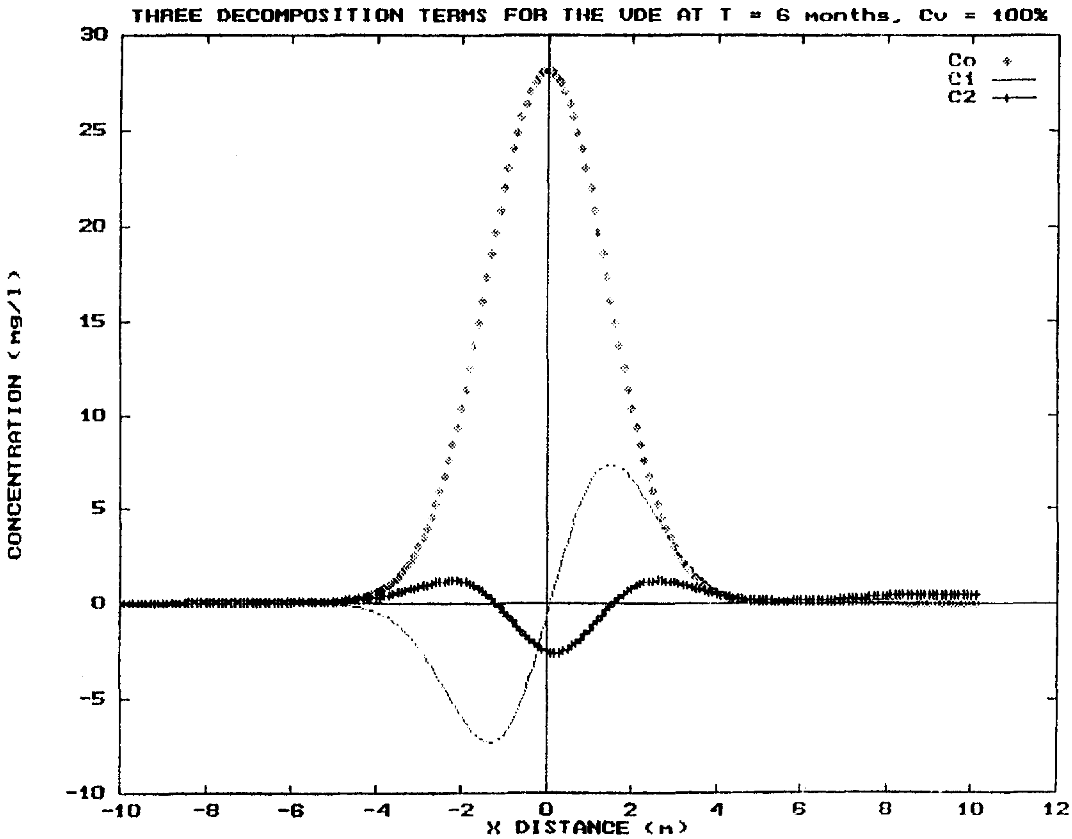

Figure 3. Three decomposition terms for the VDE at $t = 6$ months.

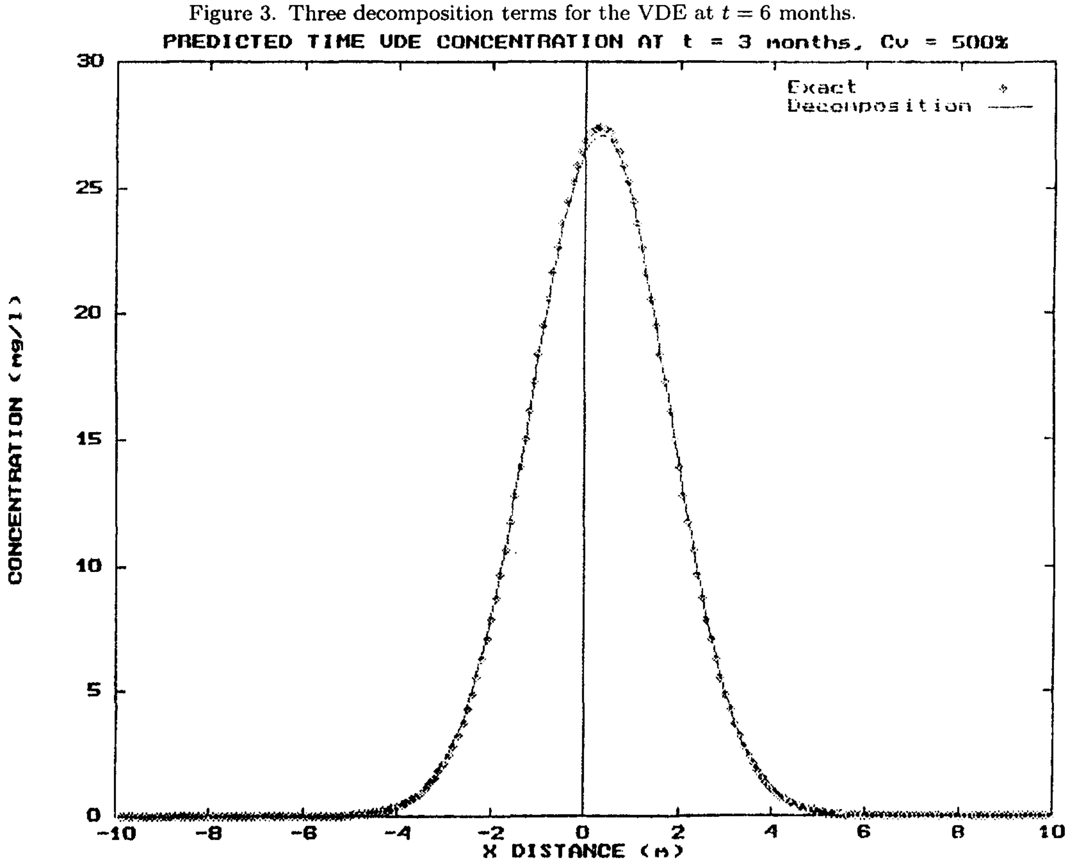

Figure 4. Predicted time VDE concentration at $t = 6$ months.

As an illustration we adopt $D_x = D = 0.01\,\mathrm{m}^2/\mathrm{month}$, $\rho = 0.1\ \mathrm{m}^{-1}$, $\bar{T} = T = 100.0\,\mathrm{m}^2/\mathrm{month}$, a very large transmissivity standard deviation $\sigma_T = 500.0\,\mathrm{m}^2/\mathrm{month}$, equivalent to a coefficient of variability $C_v = 500\%$, and the rest of the parameters as before. Figure 4 illustrates a comparison between the exact solution and the time VDE and the decomposition solution as approximated by the summation of the first three terms of the series. Once again, note the accuracy of the decomposition solution even in cases of unusually large variances. Depending on the particular differential equation and the values of the parameters, the convergence of the series may be assured by adjusting the simulation time step.

5. CONCLUSIONS AND RECOMMENDATIONS

New developments in the decomposition method with applications to the general solution of the three-dimensional deterministic/stochastic, linear or nonlinear, transport equation in heterogeneous porous media have been presented. Emphasis has been given to the illustration of the method and on practical simulation of potential applications in hydrology, such as scale-dependent equations. It is clear that the extensions to higher dimensions and to nonlinear equations are straightforward. It was shown that the decomposition method is an accurate, systematic and general analytic technique with clear advantages over methods that require many restrictive assumptions to make the mathematics tractable.

Still remaining is an investigation on the use of the decomposition method in conjunction with numerical techniques to smooth uncertain initial conditions. Since most data bases are obtained from limited punctual information derived from isolated monitoring wells, the sensitivity of the decomposition solution to error in the initial condition is a problem of practical significance. Another area to study is the development of sequential solutions for prolonged time simulations. As the variances in the random quantities increase, the maximum simulation time required for convergence decreases, and thus a combination of more terms in the approximant in concert with a cascade of successive solutions could be attempted.

REFERENCES

1. J.H. Cushman, Development of stochastic partial differential equations in subsurface hydrology, *Stochastic Hydrol. Hydraul.* **1** (4), 241–262, (1987).
2. J.H. Cushman, Comment on "Three-dimensional stochastic analysis of macrodispersion in aquifers", *Water Resour. Res.* **19**, 1641–1642, (1983).
3. G. Sposito and W.A. Jury, Fundamental problems in the stochastic convection dispersion model of solute transport in aquifers and field soils, *Water Resour. Res.* **22** (1), 77–88, (1986).
4. S.E. Serrano and T.E. Unny, Semigroup solutions to stochastic unsteady groundwater flow subject to random parameters, *Stochastic Hydrol. Hydraul.* **1** (4), 281–296, (1987).
5. G. Adomian, *Solving Frontier Problems of Physics—The Decomposition Method*, Kluwer Academic, (1994).
6. G. Adomian, *Non-Linear Stochastic Operator Equations*, Academic Press, (1986).
7. S.E. Serrano, General solution to random advective transport equation in porous media. 1. Stochasticity in the sources and in the boundaries, *Stochastic Hydrol. Hydraul.* **2** (2), 99–112, (1988).
8. S.E. Serrano, General solution to random advective-dispersive transport equation in porous media. 2. Stochasticity in the parameters, *Stochastic Hydrol. Hydraul.* **2** (2), 99–112.
9. S.E. Serrano, Semianalytical methods in stochastic groundwater transport, *Appl. Math. Modelling* **16**, 181–191, (1992).
10. S.E. Serrano, Stochastic differential equations models of erratic infiltration, *Water Resour. Res.* **26** (4), 703–711, (1990).
11. S.E. Serrano, Modeling infiltration in hysteretic soils, *Adv. Water. Resour.* **13** (1), 12–23, (1990).
12. G. Adomian, A review of the decomposition method and some recent results for nonlinear equations, *Computers Math. Applic.* **21** (5), 101–127, (1991).
13. L. Gabet, The decomposition method and distributions, *Computers Math. Applic.* **27** (3), 41–49, (1994).
14. L. Gabet, The decomposition method and linear partial differential equations, *Mathl. Comput. Modelling* **17** (6), 11–22, (1993).
15. L. Gabet, *Equisse d'une Théorie Décompositionnelle et application aux equations aux dérivées partialles*, Dissertation, Ecole Centrale de Paris, France, (1992).
16. K. Abbaoui and Y. Cherruault, Convergence of Adomian's method applied to differential equations, *Computers Math. Applic.* **28** (5), 103–109, (1994).

17. Y. Cherruault, Convergence of Adomian's method, *Kybernetes* **18** (2), 31–38, (1989).
18. Y. Cherruault, G. Saccomardi and B. Some, New results for convergence of Adomian's method applied to integral equations, *Mathl. Comput. Modelling* **16** (2), 85–93, (1992).
19. S.E. Serrano, Groundwater flow and contaminant transport in aquifiers subject to large parameter uncertainty, *Computational Stochastic Mechanics*, (Edited by A.H.D. Cheng and C.Y. Yang), pp. 475–491, Elsevier Applied Science, (1993).
20. S.E. Serrano, The form of the dispersion equation under recharge, and variable velocity and its analytical solution, *Water Resour. Res.* **28** (7), 1801–1808, (1992).