Groundwater

Regional Groundwater Flow in the Louisville Aquifer

by Syafrin Tiaif[1] and Sergio E. Serrano[2]

Abstract

The unconfined alluvial aquifer at Louisville, Kentucky, is an important source of water for domestic and industrial uses. It has been the object of several modeling studies in the past, particularly via the application of classical analytical solutions, and numerical solutions (finite differences and finite elements). A new modeling procedure of the Louisville aquifer is presented based on a modification of Adomian's Decomposition Method (ADM) to handle irregularly shaped boundaries. The new approach offers the simplicity, stability, and spatial continuity of analytical solutions, in addition to the ability to handle irregular boundaries typical of numerical solutions. It reduces to the application of a simple set of algebraic equations to various segments of the aquifer. The calculated head contours appear in reasonably agreement with those of previous studies, as well as with those from measured head values from the U.S. Geological Survey field measurement program. A statistical comparison of the error standard deviation is within the same range as that reported in previous studies that used complex numerical solutions. The present methodology could be easily implemented in other aquifers when preliminary results are needed, or when scarce hydrogeologic information is available. Advantages include a simple approach for preliminary groundwater modeling; an analytic description of hydraulic heads, gradients, fluxes, and flow rates; state variables are described continuously over the spatial domain; complications from stability and numerical roundoff are minimized; there is no need for a numerical grid or the handling of large sparse matrices; there is no need to use specialized groundwater software, because all calculations may be done with standard mathematics or spreadsheet programs. Nonlinearity, the effect of higher order terms, and transient simulations could be included if desired.

Introduction

The unconfined alluvial aquifer at Louisville, Kentucky (Figure 1), lies in a valley filled with sand and gravel deposits (Hamilton 1944; Rorabaugh 1956; Palmquist and Hall 1960; Gallaher and Price 1966). The direction of groundwater flow is from the adjacent limestone and shale valley wall (Hamilton 1944; Rorabaugh 1956; Palmquist and Hall 1960; Gallaher and Price 1966) toward the Ohio River and major pumping centers in the city of Louisville (Figures 1 and 3). The alluvium consists of unconsolidated glacial outwash sand and gravel deposits approximately 30 m in thickness, and forms a highly productive aquifer that is hydraulically connected to the Ohio River. The Louisville Aquifer has been an important source of water for domestic and industrial uses in the area. Groundwater pumpage peaked in the early 1940s when an estimated 6.8×10^6 m^3/month was used (Lyverse et al. 1996). Reduced economic growth and a sewer-usage tax levied on groundwater discharges in 1947 were a few of the reasons groundwater withdrawals decreased. Pumpage and water levels were relatively stable until the mid 1960s when water levels began to rise. Groundwater levels continued to rise until 1979 when they again stabilized (Lyverse et al. 1996). In 1980, groundwater levels reached an apparent equilibrium level. Over the last 30 years, groundwater levels have fluctuated around this equilibrium due to seasonal variations in recharge from precipitation and pumping. Figure 2 shows historical field-measured water levels at the Library Well number 381441085452701 (Kentucky Water Science Center 2013) in meters above the sea level. The northeast portion of the alluvium is an especially prolific water-bearing formation (Rorabaugh 1956), where the Louisville Water Company has been using riverbank filtration wells to draw water from the Ohio River through the aquifer at their B.E. Payne Water Treatment Plant near Prospect, Kentucky. The Louisville aquifer has been extensively studied in the past. For an annotated bibliography see Starn and Mull (1994).

Several modeling studies of regional groundwater flow in the Louisville aquifer have been conducted in the past. These studies employed analytical solutions (Tiaif 1993), finite difference schemes (Tiaif 1993; Schafer 2000), and finite element approaches (Lyverse et al.

[1]University of Bengkulu, Jl. Raya Kandang Limun, Bengkulu, Indonesia.

[2]Corresponding author: HydroScience Inc., 1217 Charter Lane, Ambler, PA 19002; hydroscience@earthlink.net

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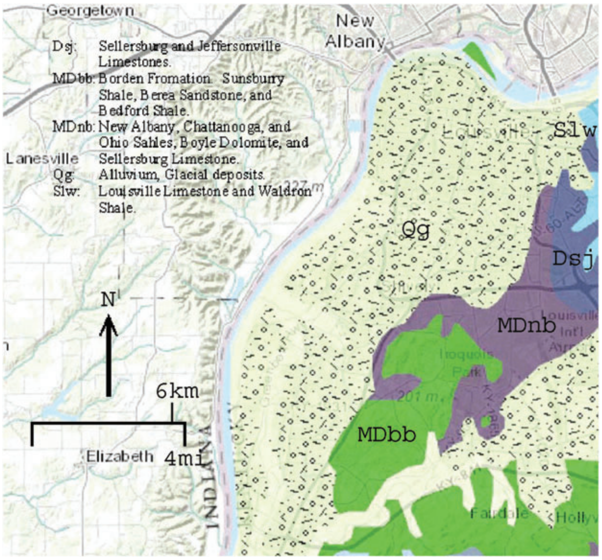

Figure 1. The Louisville Aquifer, Kentucky (courtesy of the Kentucky Geologic Map Information Service, kgs.uky.edu).

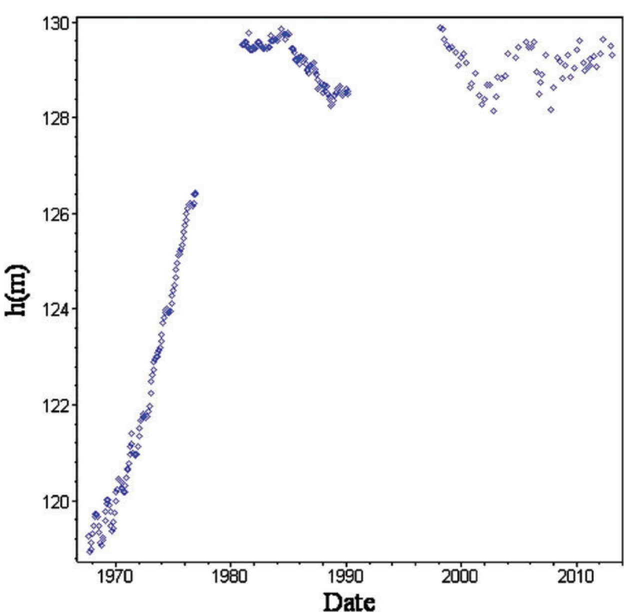

Figure 2. Effect of pumping on heads at the Louisville Library well.

1996). Classical analytical solutions (e.g., Fourier series, Laplace transform) are simple to program, are usually stable, and offer a spatially continuous description of state variables. However, classical analytical solutions require regular domain shapes (e.g., simple one dimensional or rectangular geometries), and cannot handle nonlinearities. On the other hand, numerical solutions (e.g., finite differences, finite elements) reduce the differential equations to a simpler system of algebraic equations, can handle irregular domain shapes, and may handle numerical approximations of nonlinearities. However, numerical solutions yield the state variables at discrete nodes only (i.e., they require a grid), are difficult to program (i.e., they require specialized computer software), and often have difficulties with instabilities and roundoff errors. An alternative procedure, Adomian's Decomposition Method (ADM), offers the simplicity, stability, and spatial continuity of analytical solutions, in addition to the ability to handle system nonlinearities of numerical solutions (Adomian 1994; Wazwaz 2000; Rach 2008; Duan and Rach 2011; Rach et al. 2013). ADM consists in deriving an infinite series that, in many cases, converges to the exact solution.

In the Appendix S1 of this article, we present a general description of ADM in regional steady groundwater equations. We consider the problem of the effect of recharge on head equations, and show the accuracy of ADM with respect to well-known exact solutions. Next, we briefly introduce the concept of modeling irregularly shaped domains as a prelude to the general development in the main section of this paper. Subsequently, we illustrate the problem of modeling the effect of pumping wells on regional heads, modeling of heterogeneous aquifers and nonlinear flow, which is one of the most important features of ADM. In the Appendix S2 of this article, we present a discussion on the convergence of ADM, and emphasize the principle that ADM series converge to the exact linear or nonlinear solution of the differential equation. We present two examples, one linear and one nonlinear, of decomposition series of partial differential equations in groundwater. We suggest additional references containing rigorous mathematical theory confirming ADM convergence for a wide class of equations in science and engineering. Next, we compare a truncated decomposition series with a finite difference solution of regional flow with mixed boundaries. Lastly, we include a numerical example that illustrates the principle that a few terms in a series is a reasonable approximation for many practical applications. Further details on groundwater applications of ADM, derivations, computer program listings in Maple, modeling transient flow, unsaturated flow, contaminant transport, and practical details on simulations are presented in Serrano (2010). Applications on statistical and stochastic systems are discussed in Serrano (2011). For nonlinear equations, decomposition is one of the few systematic solution procedures available. For instance, Serrano et al. (2007) and Serrano and Workman (1998) presented new solutions of the nonlinear Boussinesq equation to study the problem of stream-aquifer interaction. Moutsopoulos (2009, 2007) used decomposition to derive the most characteristic case of nonlinear, non-Darcian, unconfined flows in groundwater. With the concepts of partial decomposition and of double decomposition, the process of obtaining an approximate solution in several dimensions is simplified. As long as the initial term in a decomposition series (e.g., the forcing function or the initial/boundary conditions) is described in analytic form, a partial decomposition procedure may offer a simplified approximate solution to many modeling problems. If aquifer parameters such as transmissivity may be specified in analytic form (e.g., by fitting a smooth surface to point observations), the method may consider heterogeneity. Transient solutions for various

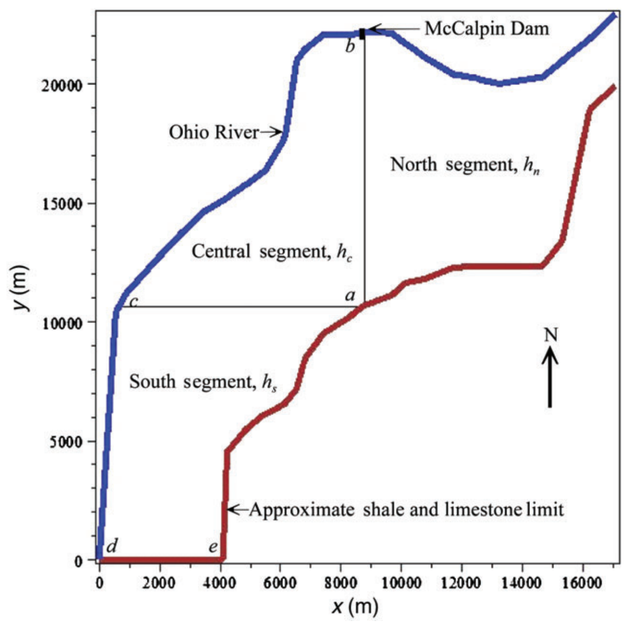

Figure 3. Division of aquifer domain into three segments.

governing equations have been derived (Serrano 2012). The propagation of contaminants subject to nonlinear reactions or nonlinear decay (Serrano 2003) are among the examples of application of decomposition.

The purpose of the present paper is to introduce a new modification of ADM to allow the modeling of regional groundwater flow in an irregularly shaped domain, such as the Louisville aquifer. By specifying the domain boundaries in analytic form (e.g., fitting a curve to a few surveyed points), using the concept of partial decomposition, and a one-term ADM expansion for separate segments of the aquifer, we obtain a simple representation of the effect of recharge and pumping. This approach consists of the application of a set of algebraic equations to three sections of the aquifer. Thus, the first step is to define the aquifer boundaries by piecewise continuous functions derived from individual points read from a map of the area. To investigate the effect of recharge from rainfall, we then divide the Louisville aquifer into three segments: The North Segment, the Central Segment, and the South segment. The criterion for the definition of segments is arbitrary and it follows an understanding of the hydrology of the region. Thus, the North Segment is assumed to be controlled by the higher water levels in the Ohio River upstream the McCalpin dam (point b in Figure 3), as well as the no-flow boundary condition on the south. The Central Segment is controlled by the lower water levels in the river downstream the dam and the head at the common boundary with the North Segment (line ab in Figure 3). The South Segment is also controlled by the levels in the river, the common boundary with the Central Segment (line ac in Figure 3), and the abrupt change in direction and shape of the no-flow boundary. As in previous studies of the area, we assume the two-dimensional planar unconfined flow equation with Dupuit assumptions as valid. Next, we alternatively apply partial decomposition solutions of the governing equation to each segment: A y-partial solution in the North Segment, an x-partial solution in the Central Segment, and a y-partial solution in the South Segment. A partial solution in one direction is obtained by integrating the differential equation in that direction. The alternation of x- with y-partial solutions assures that a given segment has specified boundary conditions given by the partial solutions at adjacent segments. An x- or a y-partial solution in a segment is given by a simple algebraic equation. The effect of pumping is added via a partial decomposition solution for the entire aquifer. We verify the new model by simulating the steady heads using average parameter values reported in previous hydrogeologic studies. The calculated head contours appear in reasonably agreement with those of previous studies, as well as with those from measured head values from the U.S. Geological Survey field measurement program. A statistical comparison of the error standard deviation is within the same range as those reported in previous studies that used complex numerical solutions.

The present methodology could be easily implemented in other aquifers when preliminary results are needed, or when scarce hydrogeologic information is available. Important advantages of the present method include a simple approach for preliminary groundwater modeling; an analytic description of hydraulic heads, gradients, fluxes, and flow rates; state variables are described continuously over the spatial domain; complications from stability and numerical roundoff are minimized; there is no need for a numerical grid or the handling of large sparse matrices; there is no need to use specialized groundwater software, because all calculations may be done with standard mathematics or spreadsheet programs. Nonlinearity, the effect of higher order terms, and transient simulations could be included if desired.

Modeling Regional Flow in the Louisville Aquifer

We adopt a Cartesian coordinate system with the x axis parallel to an east-west direction and the y axis parallel to the north-south direction. In this study, all planar distances (x, y) were measured with respect to an arbitrary origin at the southwest corner of the aquifer (point d in Figure 3). The origin has State of Kentucky plane coordinates $(x_d = 355, 250.590$ m, $y_d = 66, 044.511$ m). The next step in the analysis is the fitting of the aquifer irregular boundaries to analytic functions. The irregular shape of the Ohio river (i.e., constant head) boundary may be defined by a piecewise continuous function derived from individual points read from a map of the area (Figure 3). The McCalpin dam is located at point b in Figure 3. The stage of the Ohio river upstream the dam (i.e., east of point b) is about an average of 10.46 m higher than that downstream the dam (i.e., southwest of point b). Hence, there is a jump discontinuity in the boundary head at point b. Figure 3 also shows the approximate south boundary of the alluvial aquifer, defined by the shale and limestone, which was characterized as a no flow boundary by previous hydrogeological studies. An additional assumption here is the extension of line *de* in Figure 3, which we adopt as a no flow boundary in the y direction due to a predominantly west direction of regional flow along this line. The shape of the known-head and no-flow aquifer boundaries were described by piecewise continuous functions derived from individual points read from a map of the area. These points may be fitted to either functions of x or functions of y, depending on whether one defines x or y as the independent variable. The x east and west limits of the aquifer are defined as the functions $X_e(y)$ and $X_w(y)$, respectively (Figure 3). The y north and south limits of the aquifer are defined as the functions $Y_n(x)$ and $Y_s(x)$, respectively. The simplest approach is to fit the surveyed points via a linear interpolation, but higher-order polynomials are possible. These functions are an integral part of the solution to the differential equations.

The Effect of Recharge

To characterize the effect of natural recharge in the aquifer, we subdivide the aquifer alluvium domain into three segments, North, Central, and the South (Figure 3). The North Segment is bounded on the north by the Ohio River, on the south and the east by the no-flow boundary, and on the west by the north-south line *ab* drawn from the McCalpin dam to the no-flow boundary. The Central segment of the aquifer is limited on the north and the west by the Ohio River, on the south by the east-west line *ac*, and on the east by the North Segment. The South Segment of the aquifer is limited on the north by the central segment, on the south and the east by the no flow boundary, and on the west by the Ohio River. The criterion for the definition of segments is arbitrary and it follows an understanding of the hydrology of the region. Thus, the North Segment is assumed to be controlled by the higher water levels in the Ohio River upstream the McCalpin dam, as well as the no-flow boundary condition on the south. This suggests a y-partial decomposition solution of the governing equations, which would take advantage of the north and south boundary conditions. The Central Segment is controlled by the lower water levels in the river downstream the dam, and the head at the common boundary with the North Segment (line *ab*). This suggests the use of an x-partial decomposition solution that would take advantage of the east and the west boundary conditions. The South Segment is also controlled by the levels in the river, the common boundary with the Central Segment (line *ac*), and the abrupt change in direction and shape of the no-flow boundary. For these reasons, a y-partial decomposition solution may be suitable. Alternating x- and y-partial solutions assures a continuity of heads at the segments' interface.

Consistent with previous studies of the aquifer (Tiaif 1993; Lyverse et al. 1996), we consider Dupuit assumptions of regional flow and a homogeneous transmissivity as reasonable. The North Segment is subject to a constant-head boundary condition on the north, and a no-flow boundary condition on the south. To calculate the hydraulic head on the North Segment, we apply a first-order y-partial decomposition expansion of the Boussinesq equation subject to an irregularly shaped Dirichlet boundary condition on the north and an irregularly shaped Neuman boundary condition on the south. The governing differential equation with Dupuit assumptions is given by

$$\frac{\partial^2 h_n}{\partial x^2} + \frac{\partial^2 h_n}{\partial y^2} = \frac{R_g}{T}, \quad Y_g(x) \le y \le Y_n(x)$$
$$h_n(x, Y_n(x)) = f_u, \quad \frac{\partial h_n}{\partial \eta}(x, Y_s(x)) = 0 \qquad (1)$$

where x and y are horizontal planar coordinates with respect to the lower left limit of the aquifer (point d in Figure 3) (m); $h_n(x,y)$ is the hydraulic head in the North Segment (m); R_g is the mean aquifer recharge from rainfall (m/month); T is the mean aquifer transmissivity (m^2/month); $Y_s(x)$ is the y coordinate of the no-flow boundary on the south (m); $Y_n(x)$ is the y coordinate of the constant-head boundary at the Ohio river (m) on the north; f_u is the mean surface water elevation in the Ohio River upstream the McCalpin dam (m); and $\partial h_n/\partial\eta = 0$ represents a zero gradient along a line η perpendicular to Y_s (i.e., no flow condition). We define the operators $L_x = \partial^2/\partial x^2$ and $L_y = \partial^2/\partial y^2$. The inverse operators $L_y{}^{-1}$ and $L_x{}^{-1}$ are the corresponding twofold indefinite integrals with respect to x and y, respectively. Equation 1 reduces to

$$L_x h_n + L_y h_n = -\frac{R_g}{T} \qquad (2)$$

The y-partial solution results from operating with $L_y{}^{-1}$ on Equation 2 and rearranging:

$$h_n = -L_y^{-1}\frac{R_g}{T} - L_y^{-1} L_x h_n \qquad (3)$$

Expand h_y in the right side as the series $h_n = h_{n0} + h_{n1} + h_{n2} + \cdots$ to obtain

$$h_n = -L_y^{-1}\frac{R_g}{T} - L_y^{-1} L_x (h_{n0} + h_{n1} + h_{n2} + \cdots) \qquad (4)$$

The first term is given by

$$h_n \approx h_{n0} = k_n(x) + m_n(x)\, y - L_y^{-1}\frac{R_g}{T}$$
$$= k_n(x) + m_n(x)\, y - \frac{R_g y^2}{2T} \qquad (5)$$

where the functions $k_n(x)$ and $m_n(x)$ assure that the y conditions at the irregularly shaped boundaries are satisfied. Substituting the head at the Ohio river, Equation 1, into Equation 5, we obtain

$$k_n(x) = f_u - m_n(x)\, Y_n(x) + \frac{R_g}{2T} Y_n^2(x) \qquad (6)$$

For the no-flow condition in Equation 1, a head contour near the boundary is parallel to the boundary itself. Thus,

$$0 = m_n(x) - \frac{R_g}{2T}\left(2Y_s(x) + \frac{1}{\cos\theta}\right) \tag{7}$$

where θ is the angle of the boundary with respect to the x axis. Using a simple identity and rearranging,

$$m_n(x) = \frac{R_g}{2T}\left(2Y_s(x) + \sqrt{1 + Y_s'(x)^2}\right) \tag{8}$$

where $Y_s'(x)$ is the first-order derivative of Y_s with respect to x. As discussed in the Appendices S1 and S2, the fast rate of convergence of ADM series often results in reasonably accurate one-term approximations. Assuming $h_n \approx h_{n0}$, the hydraulic head in the North Segment is given by Equations 5, 6, and 8.

Because the Central Segment limits with the North Segment on the east (on line ab), we use $h_n(x_a, y)$, where x_a is the x coordinate of line ab, as the east boundary condition for the former. We apply an x-partial decomposition solution of the Boussinesq equation, subject to an irregularly shaped Dirichlet boundary condition on the west and $h_n(x_a, y)$ on the east. The governing equation is

$$\begin{aligned}&\frac{\partial^2 h_c}{\partial x^2} + \frac{\partial^2 h_c}{\partial y^2} = -\frac{R_g}{T}, \qquad X_w(y) \le x \le x_a \\ &h_c(X_w(y), y) = f_d, \qquad h_c(x_a, y) = h_n(x_a y)\end{aligned} \tag{9}$$

where $h_c(x, y)$ is the hydraulic head in the Central Segment (m); f_d is the mean surface water elevation in the Ohio River downstream the McCalpin dam (m); $X_w(y)$ is the x coordinate of the Ohio river boundary on the west (m); and the rest of the terms as before. The x-partial solution results from operating with L_x^{-1} on Equation 9 and rearranging:

$$h_c = -L_x^{-1}\frac{R_g}{T} - L_x^{-1}L_y h_c \tag{10}$$

Expand h_c in the right side as the series $h_c = h_{c0} + h_{c1} + h_{c2} + \cdots$ to obtain

$$\begin{aligned}h_c = {}& k_c(y) + m_c(y)\,x - L_x^{-1}\frac{R_g}{T} \\ & - L_x^{-1}L_y\left(h_{c0} + h_{c1} + h_{c2} + \cdots\right)\end{aligned} \tag{11}$$

The first term is given by

$$\begin{aligned}h_c \approx h_{c0} &= k_c(y) + m_c(y)\,x - L_x^{-1}\frac{R_g}{T} \\ &= k_c(y) + m_c(y)\,x - \frac{R_g x^2}{2T}\end{aligned} \tag{12}$$

where functions $k_x(y)$ and $m_x(y)$ assure that the conditions at the irregularly shaped boundaries are satisfied.

Substituting the boundary conditions in Equation 9 into Equation 12 we obtain

$$\begin{aligned}k_c(y) &= f_d - m_c(y)\,X_w(y) + \frac{R_g}{2T}X_w^2(y) \\ m_c(y) &= \left(\frac{h_n(x_a, y) - f_d}{x_a - X_w(y)}\right) + \frac{R_g}{2T}\left(x_a + X_w(y)\right)\end{aligned} \tag{13}$$

Assuming $h_c \approx h_{c0}$, the hydraulic head in the Central Segment is given by Equations 12 and 13.

Because the South Segment limits with the Central Segment on the north (line ac), as well as the Ohio river, we use $h_s = h_c(x, y_a)$, $\quad x > x_c$, and $h_s = f_d$, $x \le x_c$ as the north boundary condition for this segment. Where x_c is the x coordinate of point c in Figure 3. This suggests a y-partial decomposition solution of the Boussinesq equation, subject to an irregularly shaped no-flow boundary condition on the south and a piecewise function, $h_c(x, y_c)$ or f_d, on the north. Following a similar derivation as for the North Segment, the hydraulic head in the South Segment is given by

$$\begin{aligned}h_s(x, y) &= k_s(x) + m_s(x)\,y - \frac{R_g}{2T}y^2 \\ k_s(x) &= f_d U(x_c - x) + h_c(x, y_c)\,U(x - x_c) \\ &\quad - m_s(x)\min(y_c, Y_n(x)) + \frac{R_g}{2T}\min(y_c, Y_n)^2(x) \\ m_s(x) &= \frac{R_g}{2T}\left(2Y_s(x) + \sqrt{1 + Y_s'(x)^2}\right)\end{aligned} \tag{14}$$

where $h_s(x, y)$ is the head in the South Segment (m); the coordinates of point c are (x_c, y_c); $U(x_c - x) = 1$, $x \le x_c$, $U(x_c - x) = 0$, $x \ge x_c$ is the Heaviside unit step function; $\min(y_c, Y_n) = y_c$, $y_c < Y_n$, and $\min(y_c, Y_n) = Y_n$, $y_c > Y_n$ is the minimum function (m). The functions $k_s(x)$ and $m_s(x)$ assure that the conditions at the irregularly shaped boundaries are satisfied. Combining the partial solutions from the three segments, Equations 5, 12, and 14, the effect of recharge from rainfall, $h_r(x, y)$, is given by the piecewise function

$$h_r(x, y) \approx \left\{\begin{array}{lll} h_n(x, y), & x_a \le x, & Y_s(x) \le y \le Y_n(x) \\ h_c(x, y), & x_c \le x \le x_a, & y_a \le y \le Y_n(x) \\ h_s(x, y), & x_d \le x, & Y_s(x) \le y \le y_a \end{array}\right\} \tag{15}$$

where x_d is the x coordinate of point d.

The Effect of Pumping

To model the effect of pumping from various wells on the regional hydraulic head, we use a y-partial decomposition solution of the Boussinesq equation for the entire aquifer. A detailed description of the application of partial decomposition solutions to simulate the effect of pumping wells on regional flow is given in Appendix S1 and in Serrano (2010). Applying this approach to the Louisville aquifer as described in the previous section, the hydraulic head due to pumping, h_p, is given by

$$h_p(x, y) = k_p(x) + m_p(x)\, y + F(x, y)$$

$$F(x, y) = \sum_{i=1}^{N_w} \frac{Q_i}{2\pi A_i T} \log\left[(x_i - x)^2 + (y_i - y)^2\right]$$

$$k_p(x) = -\, m_p(x)\, Y_n(x) - F(x, Y_n(x))$$

$$m_p(x) = \frac{F(x, Y_s(x)) - F\left(x, Y_s(x) + \sqrt{1 + Y_s'(x)^2}\right)}{\sqrt{1 + Y_s'(x)^2}} \qquad (16)$$

where (x_i, y_i) are the *(x,y)* coordinates of well i (m); A_i is cross-section area of well i (m^2); and Q_i is the pumping rate of well i (m^3/month). Because the field boundary conditions are satisfied by h_r, the functions $k_p(x)$ and $m_p(x)$ assure that homogeneous conditions at the irregularly shaped boundaries are met.

From Equations 15 and 16, the complete solution is given by the superposition of the effects of recharge and those due to pumping:

$$h(x, y) = h_r(x, y) + h_p(x, y) \qquad (17)$$

Verification with Field Measurements

The calculation of hydraulic heads is easily accomplished by applying Equation 17. We adopt steady-state parameter values reported in the literature for the Louisville aquifer. After extensive investigation and comparison of various methods, Tiaif (1993) suggested an average value of recharge is $R_g = 0.01108$ m/month. This is in reasonable agreement with previous studies (Rorabaugh 1949; Walker 1961). An average transmissivity value $T = 7,776$ m^2/month (Tiaif 1993) was adopted. This corresponds to the median values derived from pumping tests in the aquifer (Gallaher and Price 1966). The mean surface water elevation in the Ohio River upstream the McCalpin dam is $f_u = 128$ m and the mean surface water elevation in the Ohio River downstream the McCalpin dam is $f_d = 117.54$ m (Lyverse et al. 1996). As stated before, the functions $X_w(y)$, $Y_n(x)$, and $Y_s(x)$ were obtained by reading on a topographic map coordinates of arbitrary points (x_j, y_j) located on the aquifer boundaries and fitting them to piecewise continuous functions of x or y as needed. The Louisville aquifer is subject to substantial pumping, although in the last few decades the rate of pumping has been significantly reduced (Unthank et al. 1995; Lyverse et al. 1996). The rate of pumping reported in past studies gives an overall extraction for the entire aquifer, but not the wells that are subject to pumping or the specific amounts for each well (Lyverse et al. 1996). Using well data records (Kentucky Water Science Center 2013), we determined the location of about

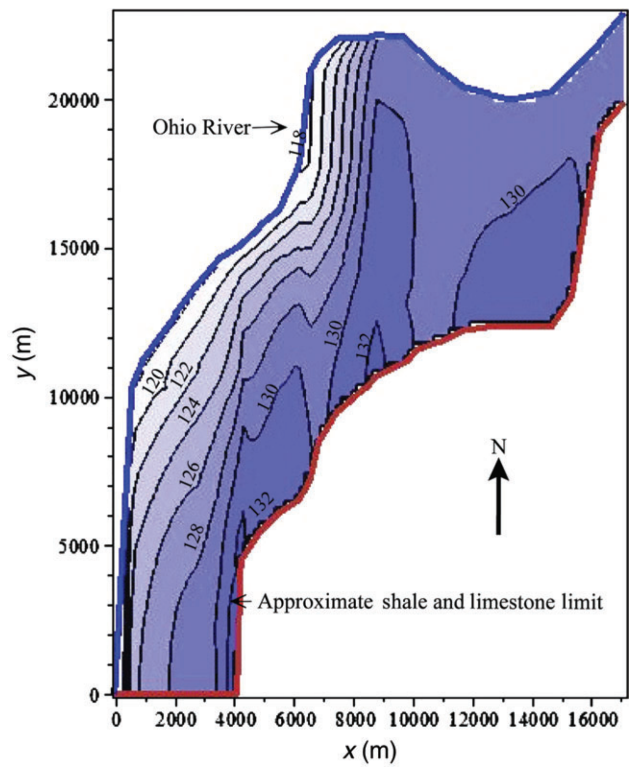

Figure 4. Steady regional hydraulic head in the Louisville aquifer.

15 wells subject to pumping and their coordinates with respect to our arbitrary origin at point d (Figure 3). Not knowing the pumping rates of individual wells, we divided the overall pumping rate of the aquifer equally among those wells under the influence of pumping. Thus, a value of $Q_i = 20,870$ m^3/month was applied to each pumping well. An average well cross-sectional area $A_i = 0.1$ m^2 was used.

The application Equation 17 is easily implemented via a few programming lines in any standard mathematics software, such as Maple. Figure 4 shows the steady head-contour map for the aquifer, which is in reasonable agreement with regional steady flow maps from previous studies (Faust and Lyverse 1987; Tiaif 1993; Lyverse et al. 1996). Water level contours show a slight deviation from the normal pattern parallel to the river in areas where groundwater withdrawals affect the natural water table gradient, especially in the industrial area in west Louisville where groundwater withdrawals have lowered the water table and caused the water level contours to curve landward. For comparison purposes, Figure 5 shows the location of 91 wells in the Louisville Aquifer under the U. S. Geological Survey field measurement program. Using the standard contour plotting routines in Maple, preliminary water table contours were drawn based on the reported well water levels for October 1982, considered a steady-state condition by previous studies (Faust and Lyverse 1987; Tiaif 1993).

An estimate of the error between simulated head and measured head at well i, e_i, is defined as

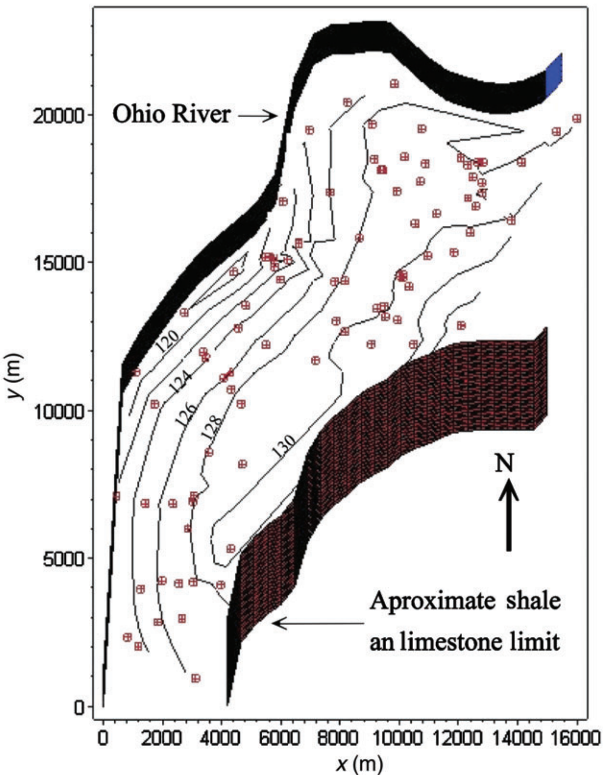

Figure 5. Location of wells under the USGS Field Measurement Program and approximate contour levels.

$e_i = (h(x_i, y_i) - h_m(x_i, y_i))$, where (x_i, y_i) are the (x, y) coordinates of well i (m); $h(x_i, y_i)$ is the simulated steady state head from Equation 17 (m); and $h_m(x_i, y_i)$ is the measured head at the same location. The mean absolute error between model and field data is given by $\mu_e \sum_{i=1}^{91} e_i/91 = 0.0001$ m, which suggests an overall low error, and thus a good agreement between model and field data. On the other hand, the standard deviation of the absolute error is given by $\sigma_e = \left(\sum_{i=1}^{91} (e_i - \mu_e)^2 /91\right)^{1/2} = 1.710$ m. A similar result was found by Lyverse et al. (1996) with a standard deviation of the absolute error (equivalent to the mean root square error) of 1.786 m in 1962 and 1.268 m in 1983 (Lyverse et al. 1996, p. 35 and figure 13a through 13e). Lyverse et al. (1996) used a Galerkin finite element solution procedure for the differential equations and implemented with the Modular Finite Element Model, MODFE (Cooley 1992; Torak 1992, 1993a, 1993b). The present methodology is considerably simpler to apply and it may prove useful to model aquifers where information is scarse. We remark that the above results were obtained with one term in an ADM expansion only. Comparison of exact analytical solutions to one-term ADM approximations have shown to be reasonably accurate for field-scale groundwater flow equations (see Appendix S1, Appendix S2, and Serrano [2010]). However, the inclusion of additional terms and of system nonlinearity may be easily added if desired.

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Supporting Information

Additional Supporting Information may be found in the online version of this article:

Appendix S1. Decomposition solution of groundwater equations. An introduction to ADM in regional groundwater. The effect of recharge, irregular domain geometry, and pumping wells on regional heads. Modeling of heterogeneity and nonlinearity.

Appendix S2. Convergence of decomposition solutions. Convergence of ADM series to exact linear or nonlinear solutions. References to rigorous mathematical theory on convergence of ADM. Comparison between truncated ADM series and finite differences. Accuracy of one- or two-term approximate solutions.

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