Development and Verification of an Analytical Solution for Forecasting Nonlinear Kinematic Flood Waves

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Abstract: A new approximate analytical solution to the nonlinear kinematic wave equation is proposed. The solution has been derived by combining an implicit solution obtained with the method of characteristics with analytical decomposition and successive approximation. The new solution was verified favorably with a finite-difference approximation. A field verification via a comparison with observed storm hydrographs from the Schuylkill River near Philadelphia indicated that with constant lateral flow the nonlinear model reasonably predicted the observed flow rates, except during periods of intense rainfall. An improved nonlinear model that accounts for variable lateral flow due to changing effective precipitation is proposed. Numerical measures of accuracy indicated that the new variable-rate nonlinear model performs better than the constant lateral flow and the linear kinematic wave model. The linear kinematic wave model produced poor forecasts, especially in the flow time and flood-peak time. The new solution is simple to apply and permits the efficient forecast of nonlinear kinematic flood waves without the usual stability restrictions of numerical models. The new formula also permits the analysis of the effect of nonlinear parameters in the area-discharge relationship on hydrograph characteristics. The hydrograph peak time appears to be very sensitive to the magnitude of the nonlinear exponent β. For values of $\beta < 1$, the peak flow occurs at a time earlier than that predicted by a linear hydrograph ($\beta = 1$). For values of $\beta > 1$, the peak flow occurs at a time later than that predicted by a linear hydrograph.

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Introduction

Kinematic wave modeling methods are gaining wide acceptance as a fast and accurate way of handling a wide range of water modeling problems, such as the spatial representation of watersheds, overland flow routing, and channel flow routing (Akan 1989; Wong and Chen 1997; Tisdale and Hamrick 1999; Wong 2002). The kinematic wave motion is a useful approximation of natural flood wave propagation in cases when the channel slope dominates in the momentum equation (Chow et al. 1988; Singh 1996). Under these conditions the acceleration and pressure terms in the latter may be neglected. The resulting flood wave does not get longer, or attenuate, or subside, and the flood peak stays at the maximum depth (Hromadka and De Vries 1988; Ponce 1991). It has been found that more accurate flood forecasts are obtained with the kinematic wave equation than with conventional conceptual reservoir methods in urban watersheds (Xiong and Melching 2005).

Simple analytical solutions to the linear kinematic wave equation have been proposed, as well as several solutions that result from the application of direct numerical methods, and characteristic methods (e.g., Li et al. 1975; Jayawardena and White 1977; Ross et al. 1979; Weinmann and Laurenson 1979; Singh 1996; Jaber and Mohtar 2002; Singh and Woolhiser 2002; Ajami et al. 2004). However, the kinematic wave model is, in general, represented by a nonlinear partial differential equation. It is desirable to obtain a simple solution to the nonlinear equation in order to produce fast and efficient forecasting models without the usual stability restrictions of numerical solutions.

The application of the method of characteristics to the nonlinear kinematic wave equation yields an implicit solution of difficult application. To obtain an explicit solution, the method of characteristics is combined with analytical decomposition and successive approximation. Decomposition has been shown to be a systematic method to solve nonlinear equations in subsurface hydrology (Adomian 1994; Serrano 1995, 1997, 2001, 2003a,b; Serrano and Adomian 1996). In the domain of hydraulics, a theoretical development has already been produced for the analytical solution of the Navier–Stokes flow equations of a viscous compressible fluid (Adomian 1995). Wazwaz (1995) illustrated the application of decomposition of kinematic-wave type of equations for integer nonlinearities. By combining decomposition with the method of characteristics, we include in this paper the general case of fractional power nonlinearities that usually appear in hydrologic modeling.

Nonlinear Kinematic Wave Model

The following kinematic flood routing equation combines the continuity and momentum equations, when the acceleration and the pressure terms in the latter are neglected (Chow et al. 1988; Singh 1996)

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$$\frac{\partial Q}{\partial x}+\alpha\beta Q^{B-1}\left(\frac{\partial Q}{\partial t}\right)=q$$

$$0\leqslant x<\infty$$

$$0<t$$

$$Q(0,t)=Q_0(t)$$

$$Q(x,0)=f(x) \tag{1}$$

where $Q(x,t)$=flow rate (meter^3/second); x=distance from a streamflow station with a known hydrograph $Q_0(t)$; t=time (hour); $f(x)$=initial spatial flow rate distribution along the river channel (meter^3/second), such that $f(0)=Q_0(0)$; the channel cross-sectional area is in general expressed as $A=\alpha Q^{\beta}$; α has the dimensions of (meter$^{2-3\beta}$second$^{\beta}$); β=dimensionless; and $q(x)$=lateral inflow rate per unit length of channel (meter^2/second). Eq. (1) is a nonlinear partial differential equation.

No Lateral Flow

A solution to Eq. (1) begins with a naive attempt to obtain a characteristics solution for the special case when $q=0$. The characteristics equation of Eq. (1) is determined by (Jeffrey 2003)

$$\frac{dt}{dx}=\alpha\beta Q^{\beta-1} \tag{2}$$

and the compatibility condition of Eq. (1) is

$$\frac{dQ}{dx}=0 \tag{3}$$

Integrating the compatibility condition Eq. (3) along a characteristic that passes through the point $(0,\xi)$ yields

$$Q(x,\xi)=Q_0(\xi) \tag{4}$$

where the Cauchy condition (initial condition) has been used. Using this result in the characteristic Eq. (2) gives

$$t=\alpha\beta Q_0(\xi)^{\beta-1}x+\xi \tag{5}$$

Simultaneous solving and eliminating ξ from Eqs. (4) and (5) gives an implicit solution to Eq. (1)

$$Q=Q_0(t-\alpha\beta Q^{\beta-1}x) \tag{6}$$

In order to obtain an explicit solution, a procedure that combines a decomposition series of Eq. (6) (Adomian 1994; Serrano 2003b) with a successive approximation of the dependent variable (2003a, Serrano 2004) is introduced. Let us write Eq. (6) as

$$Q=Q_0[t-\alpha\beta N(Q)x]$$
$$N(Q)=Q^{\beta-1} \tag{7}$$

where the nonlinear term, $N(Q)$, is expanded with the Adomian polynomials, A_n. Eq. (7) becomes

$$Q=Q_0\left(t-\alpha\beta\sum_{n=0}^{\infty}A_n x\right)$$
$$N(Q)=Q^{\beta-1} \tag{8}$$

The first term in the Adomian polynomials is calculated as

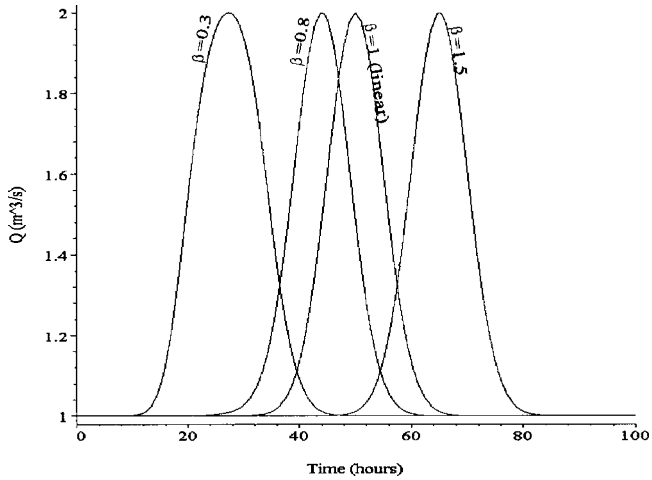

Fig. 1. Effect of nonlinear parameter, β, on flood hydrograph, $q=0$

$$A_0=N(Q_0)=Q_0^{\beta-1} \tag{9}$$

from which the second approximation to Q is obtained from Eq. (8) as

$$Q_1=Q_0(t-\alpha\beta A_0 x) \tag{10}$$

This process is continued recursively as follows

$$A_1=Q_1\frac{dN(Q_0)}{dQ_0}$$

$$Q_2=Q_0[t-\alpha\beta(A_0+A_1)x]$$

$$A_2=Q_2\frac{dN(Q_0)}{dQ_0}+\frac{Q_1^2}{2!}\frac{dN^2(Q_0)}{dQ_0^2}$$

$$Q_3=Q_0[t-\alpha\beta(A_0+A_1+A_2)x]$$

$$\vdots$$

$$Q_n=Q_0\left(t-\alpha\beta\sum_{j=1}^{n}A_{n-1}x\right) \tag{11}$$

If the series converges fast, and it is truncated at $n=N$, an approximate analytical solution to Eq. (1) is

$$Q\approx Q_0(t-\alpha\beta Q_N x) \tag{12}$$

In the verification section, it is shown that Eq. (12) is reasonably accurate when $N=1$ for typical values of the parameters α and β. The simplicity of Eq. (12) allows for a flexible and fast forecast of the flow rate without the usual stability complications of numerical solutions. The nonlinear solution also allows a study of the effect of the nonlinear parameters on the characteristics of the hydrograph. Fig. 1 shows a simulation of a flood hydrograph at a fixed location as predicted by Eq. (12) with $\alpha=3$ m$^{2-3\beta}$ s$^{\beta}$ and various values of β. The peak flow time appears to be significantly sensitive to the magnitude of the nonlinear parameter β. For values of $\beta<1$, the peak flow occurs at a time earlier than that predicted by the linear hydrograph ($\beta=1$). For values of $\beta>1$, the peak flow occurs at a time later than that predicted by the linear hydrograph. This suggests that, depending on the stream parameters, the simulation of flood propagation may not be appropriately described by a linear model.

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Constant Lateral Flow

For the case when $q \neq 0$, set $L_x = \partial/\partial x$ and write Eq. (1) as

$$Q = Q_0 + qx - \alpha\beta L_x^{-1} \sum_{n=0}^{\infty} N(Q) \frac{\partial Q}{\partial t} \tag{13}$$

where L_x^{-1}=integral from 0 to x; and q has been assumed constant, although the general case of $q(x)$ is a straightforward extension. Eq. (13) represents one of many possible decomposition schemes. As before by expanding the nonlinear term, $N(Q)$, with the Adomian polynomials then Eq. (13) becomes

$$Q = Q_0 + qx - \alpha\beta L_x^{-1} \sum_{n=0}^{\infty} A_n \frac{\partial Q}{\partial t} \tag{14}$$

Once again, the decomposition expansion is combined with successive approximation and the characteristic solution. Thus, if $N(Q) \sim A_0$, Eq. (14) would reduce to

$$Q = Q_0 + qx - \alpha\beta L_x^{-1} A_0 \frac{\partial Q}{\partial t} \tag{15}$$

Using Eq. (9), Eq. (15) satisfies the differential equation given by

$$\frac{\partial Q}{\partial x} + \alpha\beta Q_0^{\beta-1}\left(\frac{\partial Q}{\partial t}\right) = q \tag{16}$$

The characteristic equation of Eq. (16) is

$$\frac{dt}{dx} = \alpha\beta Q_0^{\beta-1} \tag{17}$$

and the compatibility condition is

$$\frac{dQ}{dx} = q \tag{18}$$

Integrating Eq. (18)

$$Q(x,t) = Q_0(\xi) + qx \tag{19}$$

Integrating the characteristic Eq. (17) gives

$$t = \alpha\beta Q_0^{\beta-1}(\xi)x + \xi \tag{20}$$

Using Eq. (19) to eliminate ξ, Eq. (20) yields an implicit solution to Eq. (1)

$$Q \approx Q_0[t - \alpha\beta(Q - qx)^{\beta-1}x] + qx \tag{21}$$

Using the concept of double decomposition (Adomian 1994) and a procedure similar to that used in Eqs. (7) and (12) when q was zero, an approximate solution to Eq. (1) follows

$$Q \approx Q_0\{t - \alpha\beta[Q_0(t) - qx]^{\beta-1}x\} + qx \tag{22}$$

The above result was obtained when $N(Q) \approx A_0$ in Eq. (14). Eq. (22) may now be used to calculate more terms in the Adomian polynomials and thus obtain an improved solution, as was done when the lateral inflow was set to zero. However, it appears that Eq. (22) is a simple and sufficiently accurate equation as will be shown in the "Verification of Nonlinear Models" section. The nonlinear solution also allows a study of the effect of the nonlinear parameters on the hydrograph features. Fig. 2 shows a simulation of a flood hydrograph at x=10 m as predicted by Eq. (22) with α=1 $m^{2-3\beta}$ s^{β} and various values of β. Once again, the occurrence of the peak flow appears to be significantly sensitive to the magnitude of the nonlinear parameter β. For values of $\beta < 1$, the peak flow occurs at a time earlier than that predicted by the linear hydrograph (β=1). For values of $\beta > 1$, the peak flow occurs at a time later than that predicted by the linear hydrograph. This suggests that, depending on the stream parameters, the simulation of flood propagation may not be appropriately described by a linear model.

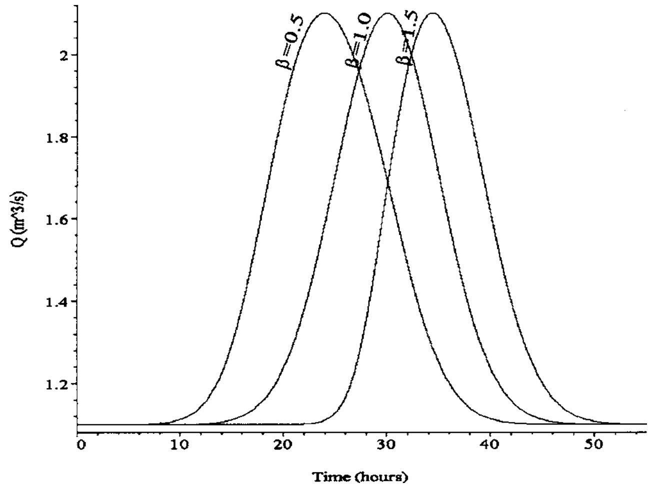

Fig. 2. Effect of nonlinear parameter, β, on flood hydrograph, $q \neq 0$

Variable Lateral Flow

To account for variable lateral flow, while maintaining a simple, yet nonlinear, model, the lateral flow representation could be modified so as to account for the time variability of effective precipitation during high-intensity storms. For small subwatersheds the lateral inflow may be given as

$$q(x,t) = q_0 + \frac{cA_s P_e(t)}{L} \tag{23}$$

where q_0=constant lateral flow contribution from groundwater flow (meter^2/second) estimated from the difference between average baseflow values at the downstream and the upstream stations, respectively; A_s=watershed area between the monitoring stations (meter^2); $P_e(t)$=spatially averaged effective precipitation rate in the subwatershed obtained from hourly rainfall rate estimates after infiltration rate has been subtracted (millimeter/hour); L=distance along the stream between the upstream and downstream stations (meter); and $c = 0.2778 \times 10^{-6}$ m mm^{-1} h s^{-1} is a units conversion factor. In Eq. (23), consideration was not given to overland flow storage or surface routing effects.

Using Eq. (23) the nonlinear kinematic wave Eq. (13) now becomes

$$Q = Q_0 + \left[q_0 + \frac{cA_s P_e(t)}{L}\right]x - \alpha\beta L_x^{-1} \sum_{n=0}^{\infty} N(Q) \frac{\partial Q}{\partial t} \tag{24}$$

Following Wazwaz's (2000) algorithm for the general calculation of decomposition series, take $Q_0 + q_0 x$ as the basis for the calculation of the second term in the series, then an approximation to Eq. (15) would be

$$Q \approx Q_0[t - \alpha\beta(Q_0(t) - q_0 x)^{\beta-1}x] + q_0 x + \frac{cA_s P_e(t)}{L}x \tag{25}$$

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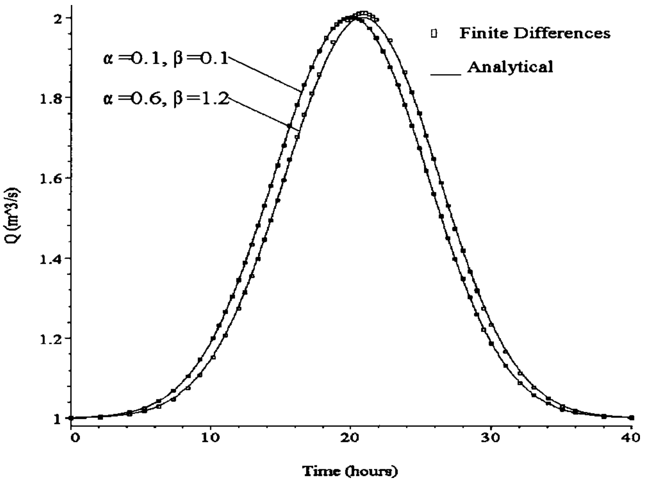

Fig. 3. Comparison between analytical and numerical solutions, $q=0$

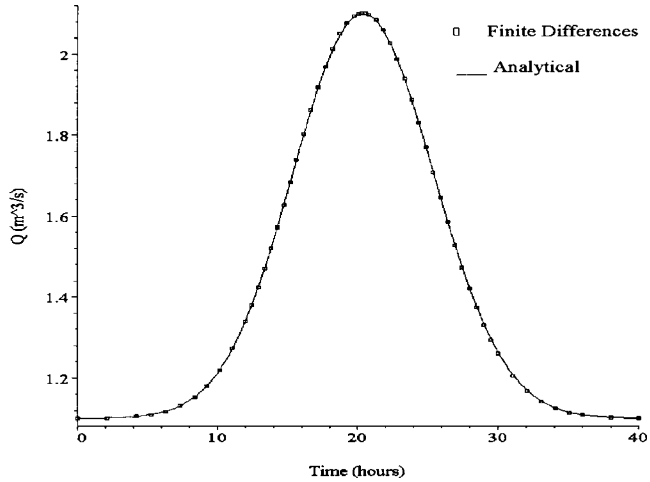

Fig. 4. Comparison between analytical and numerical solutions, $q \neq 0$

Verification of Nonlinear Models

Verification with Independent Numerical Solutions

The nonlinear models derived in the previous sections rely on the convergence of decomposition series. The convergence of decomposition series has been extensively treated in the mathematical and hydrologic literature. It depends on the values of the parameters and the domain dimensions. For domain sizes that satisfy the Lipschitz condition (Oden 1977), it can be seen that the series converges fast. For a rigorous mathematical discussion on the convergence problem of decomposition series, the reader is referred to Abbaoui and Cherruault (1999), Cherruault (1989), and Cherruault et al. (1992). It is also important to mention the rigorous mathematical framework for the convergence of decomposition series developed by Gabet (1992, 1993, 1994). He connected the method of decomposition to well-known formulations where classical theorems (e.g., fixed-point theorem, substituted series, etc.) could be used. For a discussion on the convergence of decomposition series of hydrologic convection-diffusion equations, including theorems with proofs, see Serrano (2003a, b). Serrano and Adomian (1996) provide additional comparisons between exact and truncated decomposition solutions.

To further corroborate the accuracy of the series when only a few decomposition terms are used, the new solutions were compared with corresponding numerical solutions to Eq. (1). First, Eq. (12) was computed for several values of N and compared to corresponding finite difference solutions of Eq. (1). The simulations indicated that Eq. (12) is a reasonably accurate approximation when $N=1$ for the typical values of the parameters α and β. Fig. 3 shows a comparison of flood hydrographs as described by Eq. (12) and by a finite difference solution to Eq. (1) for an arbitrary initial condition $Q_0(t)$ at $x=0$. The two solutions are in good agreement. Next, Eq. (22) was computed for several values of N and compared to corresponding finite difference solutions of Eq. (1). The results indicated that Eq. (22) is a reasonably accurate approximation when $N=1$ for the typical values of the parameters α and β. Fig. 4 shows a typical comparison of hydrographs at a fixed location after an arbitrary initial condition, as simulated by Eq. (22) and a finite difference solution to Eq. (1) when $\alpha=0.1\ \mathrm{m}^{2-3\beta}\,\mathrm{s}^{\beta}$, $\beta=0.5$. The two solutions are in good agreement.

Field Verification in Schuylkill River in Southeast Pennsylvania

The new approximate analytical solution to the nonlinear kinematic wave equation was tested using data from the Schuylkill River in Southeast Pennsylvania. Major towns in the watershed include Pottsville, Reading, Pottstown, Norristown, Conshohocken, and Philadelphia, Pa. The river travels approximately 210 km from its headwaters at Tuscarora Springs in Schuykill County to its mouth at the Delaware River in Philadelphia, Pa. The Schuylkill River is the largest tributary of the Delaware River and is a major contributor to the Delaware Estuary. Major tributaries of the Schuylkill, in downstream order, are Mill Creek, the West Creek, Perkiomen Creek, Wissahikon Creek, French Creek, and Tulpehocken Creek. The watershed encompasses an area of approximately 5,200 km^2. The Schuylkill River has been an important source of drinking water in the region for over 2 centuries. Approximately 1.5 million people receive their drinking water from the Schuylkill River and its tributaries.

For the purpose of this application, flow between the station at Norristown, with a watershed drainage area of 4,558 km^2, and the station at Philadelphia, Pa. with a drainage area of 4,903 km^2 are used. In this section, the river flows through a predominantly urban environment that includes residential, industrial, and commercial development. Discharge rate data at these stations are provided online by the U.S. Geological Survey, as described in the bibliography (U.S. Geological Survey 2005). The same source provided individual discharge versus cross-sectional area data for the above stations. Using the information at these stations, a rating-curve relationship of the form $A=\alpha Q^{\beta}$, with A the channel wetted area (meter^2), was fitted with average parameter values $\alpha=4.6\ \mathrm{m}^{2-3\beta}\,\mathrm{s}^{\beta}$, and $\beta=0.594$. For the hourly simulations a series of recent intense flooding events between July 10 and August 10, 2004 were selected. A cumulative precipitation of about 208 nm was recorded at Norristown during this period. Based on an average base flow at the two stations under consideration during this time, and a distance of about 21 km between the two stations, it was estimated that the parameter $q=0.0008\ \mathrm{m}^2/\mathrm{s}$. $Q_0(t)$ was taken as the flow rate recorded at Norristown, which is provided by the U.S. Geological Survey every 15 min. Eq. (22) was used to forecast the flow rate at Philadelphia, Pa. Eq. (22) is nothing more than a simple formula for the temporal shifting of the upstream hydrograph, $Q_0(t)$, with an added lateral flow contribution.

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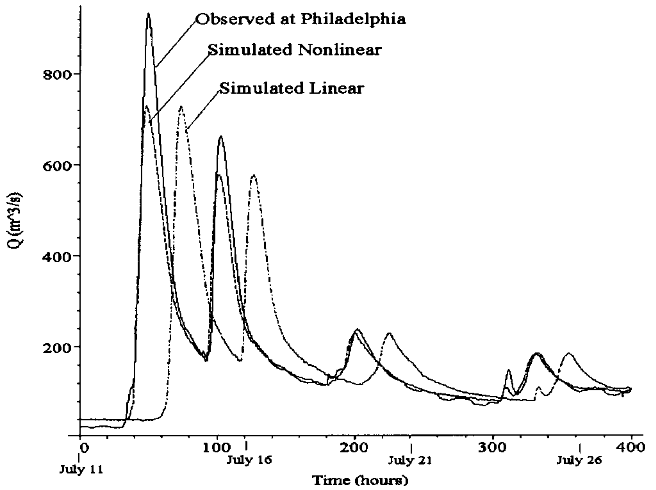

Fig. 5. Observed and predicted hourly hydrographs, Schuylkill River, Philadelphia, Pa., July 2004

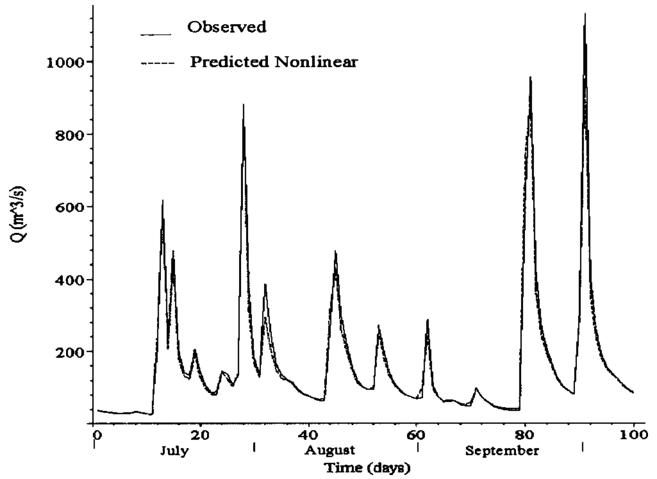

Fig. 6. Observed and predicted daily flow rates for variable q, July–September, 2004

Fig. 5 shows a comparison between the hourly observed and the predicted hydrographs according to Eq. (22) at Philadelphia for the chosen period of analysis. The linear hydrograph was produced by setting $\beta=1$. Generally, it is seen that the nonlinear model Eq. (22) coincides reasonably well with the measured hydrograph, except for the two initial flood peaks. Assuming that the observed peak flow rates were accurately recorded, the discrepancy between the observed and simulated flow rates during very high flood peaks may be due to surcharge overland flow or variable lateral flow, q, that causes the regime to depart from the usual kinematic flow conditions. This suggests that a model that includes effective precipitation, such as Eq. (26), might improve the estimates of peak flow rates. Fig. 5 also indicates that the linear hydrograph does not accurately predict the time to peak and appears to be an inadequate model.

To further test the ability of the models to predict observed values, the period of analysis was extended to include a complete hydrologic year, beginning July 1, 2004, and the inclusion of estimates of daily effective precipitation in Eq. (25). The subwatershed drainage area between Norristown and Philadelphia gages was estimated as $A_s=3.45\times10^8\ \text{m}^2$. The constant lateral flow contribution from groundwater flow estimated from the difference between average base flow values at the downstream and the upstream stations, respectively, was $q_0=0.001\ \text{m}^2/\text{s}$. Daily precipitation from rainfall was provided online by NOAA's National Climatic Data Center as described in the bibliography (NOAA 2005).

The first 100 days after July 1, 2004, that is from July through September 2004, were used as the calibration period for the daily effective precipitation loss. A constant loss rate of 75 mm/day was subtracted from the total daily rainfall values. All other parameter values estimated during the hourly simulations (Fig. 5) were used. Fig. 6 illustrates a comparison between observed and predicted daily flow rates for variable q, according to Eq. (25). Eq. (25) better predicted the flow forecast than Eq. (22). The inclusion of estimates of effective precipitation significantly improved the accuracy of the model with respect to the observed flow rates, especially during peak times. To quantitatively assess the accuracy of the model-predicted values with respect to corresponding observed ones an estimate of fitness, γ (meter^3/second), was defined as

$$\gamma=\sqrt{\frac{\sum_{i=1}^{N_p}(Q_i-\hat{Q}_i)^2}{N_p}} \tag{26}$$

where Q_t=observed flow rate at a given time (meter^3/second); $\hat{Q}_t$=simulated flow rate at the same time (meter^3/second); and N_p=number of points. Eq. (26) is the standard error of estimate and a measure of the accuracy of the predicted flows, with large values indicating inaccurate estimates. The standard error for the July–September predictions of Fig. 6 was $\gamma=31.12\ \text{m}^3/\text{s}$.

The rest of the hydrologic year, from September 2004 to May 2005, was used as a validation period for the model Eq. (25). In other words, all parameters from the calibration period were used unchanged and applied. Fig. 7 shows a comparison between observed and predicted daily flow rates for variable q, according to Eq. (25), for this period. The goodness of fit yielded $\gamma=13.21\ \text{m}^3/\text{s}$. Thus, for the validation period the model performed even better. In comparison the linear model, not shown in Fig. 7, incorrectly predicted the timing of the peaks and of the entire hydrograph in general. Fig. 8 shows a detail of the simulations with the linear, the nonlinear and the observed values. For the nonlinear model $\gamma=18.94\ \text{m}^3/\text{s}$ as compared to

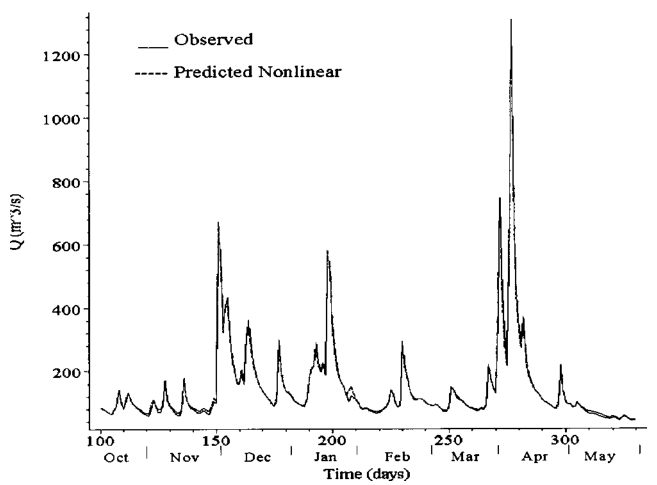

Fig. 7. Observed and predicted daily flow rates for variable q, September 2004–May 2005

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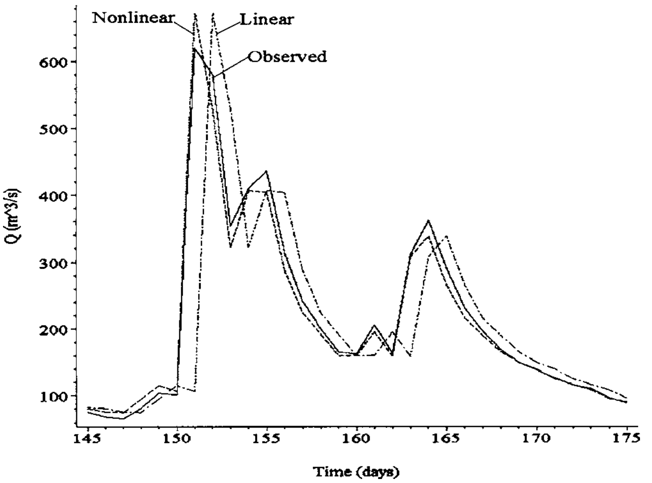

Fig. 8. Detail of Fig. 7 illustrating comparison between linear and nonlinear models

$\gamma = 109.18\ \mathrm{m^3/s}$ for the linear model. Clearly, the nonlinear model is significantly more accurate than the linear one.

Summary and Conclusions

A new approximate analytical solution to the nonlinear kinematic wave equation has been derived by combining an implicit solution obtained with the method of characteristics with an analytical decomposition and successive approximation. Three cases were considered: no lateral inflow, constant lateral inflow, and variable lateral inflow responding to changing effective precipitation. The new solution was verified and compared favorably with a finite-difference approximation. Field verification via a comparison with observed storm hydrographs in the Schuylkill River was conducted for hourly flow rates during and intense flooding period of 1 month during July 2004, and for daily flow rates during the hydrologic year of 2004–2005. The hourly simulations indicated that with a constant lateral inflow, estimated from the difference between average base flow values at the downstream and the upstream stations, the nonlinear solution reasonably predicts the correct flow rates, except during periods of extreme rainfall. During those times, the results suggest that a variable lateral inflow responding to changing effective precipitation significantly improves the accuracy of the nonlinear model. A simple model that uses the concept of constant loss rate of precipitation was proposed to that effect. Additional field verification in the Schuylkill River included daily simulations during the months of July through September, 2004. Using the same parameter values, additional validation of the nonlinear model was conducted during the rest of the hydrologic year from September 2004 to May 2005. Numerical measures of goodness of fit between observed and predicted models indicated that the variable-rate nonlinear model predicted the observed flows better than the constant lateral flow and the linear kinematic wave model. The linear kinematic wave model, on the other hand, appeared to significantly underestimate the flood-peak time. Numerical measures of goodness of fit between observed and predicted models indicated that the linear model produced a poor fit to the observed data. The new solution is easy to apply and permits the efficient forecast of nonlinear kinematic flood waves without the usual stability restrictions of numerical models. The new formula also permits the analysis of the effect of nonlinear parameters in the area-discharge relationship on hydrograph characteristics. The occurrence of the hydrograph peak flow appears to be significantly sensitive to the magnitude of the nonlinear exponent β. For values of $\beta < 1$, the peak flow occurs at a time earlier than that predicted by the linear hydrograph ($\beta = 1$). For values of $\beta > 1$, the peak flow occurs at a time later than that predicted by the linear hydrograph. This result is explained by the well known effects of lesser, or greater, river storage corresponding to various values of β.

Application of Eq. (25) in a river setting requires an inflow hydrograph at an upstream station, the drainage area between the upstream and down stream station, the distance between stations along the main river channel, data to estimate the parameters α and β of a rating-curve relationship of the form $A = \alpha Q^{\beta}$ at a few cross sections in the river, estimates of the base flow at the upstream and downstream stations, measures of average precipitation in the contributing area, and an estimate of infiltration or loss rate to subtract from precipitation. The application of Eq. (25) is nothing more than a simple formula for the temporal shifting of the upstream hydrograph, $Q_0(t)$, at hourly or daily times t, with an added lateral flow contribution that depends on effective rainfall. The nonlinear model appears to produce accurate results when the kinematic wave assumptions are applicable, that is when downstream hydrograph results from a temporal shifting of the upstream one, with a correction for added lateral flow. In circumstances when significant forward and backward hydrograph dispersion occurs, this model should not be applied in place of the full dynamic wave equations. The lateral flow correction in Eq. (25) is an adjustment for added flow in between stations. When the inflow hydrograph is significantly different in shape and magnitude from the resulting downstream one the correction will not be sufficient and this model should not be applied.

Acknowledgments

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