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# Stochastic modeling of transient stream–aquifer interaction with the nonlinear Boussinesq equation

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Received 21 January 2005; received in revised form 21 December 2005; accepted 27 December 2005

## KEYWORDS

Unconfined aquifer;  
Nonlinear equation;  
Stream–aquifer  
interaction;  
Stochastic model;  
Analytical solution

**Summary** In this article the effect of highly fluctuating stream stage on the adjacent alluvial valley aquifer is studied with a new analytical solution to the nonlinear transient groundwater flow equation subject to stochastic conductivity and time varying boundary conditions. A random conductivity field with known correlation structure represents uncertain heterogeneity. The resulting nonlinear stochastic Boussinesq equation is solved with the decomposition method. New expressions for the mean of the hydraulic head and its variance distribution are given. The procedure allows for the calculation of the mean head and error bounds in real situations when a limited sample allows the estimation of the conductivity mean and correlation structure only. Under these circumstances, the usual assumptions of a specific conductivity probability distribution, logarithmic transformation, small perturbation, discretization, or Monte Carlo simulations are not possible. The solution is verified via an application to the Scioto River aquifer in Ohio, which suffers from periodic large fluctuations in river stage from seasonal flooding. Predicted head statistics are compared with observed heads at different monitoring wells across the aquifer. Results show that the observed transient water table elevation in the observation well lies in the predicted mean plus or minus one standard deviation bounds. The magnitude of uncertainty in predicted head depends on the statistical properties of the conductivity field, as described by its coefficient of variability and its correlation length scale.  
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## Introduction

The study of stream–aquifer hydraulics is of great interest as several flow and contaminant problems can be modeled, understood and quantified. The quantification of the hydraulics of the stream–aquifer in an alluvial valley require a good knowledge of the controlling input hydrogeo-

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logical parameters, such as hydraulic conductivity, specific yield, recharge, as well as boundary effects that are associated with the stream. Small changes in the stream elevation can cause a large variation in the groundwater elevation in the aquifer. The spread of contaminants in stream–aquifer systems from the river to the aquifer or from the aquifer to the river is also of concern. The hydraulics of such systems has been studied in the literature from both a deterministic point of view as well a stochastic point of view.

Stream–aquifer systems can be quantitatively studied using Laplace's equation subject to nonlinear free surface boundary conditions and time dependent river boundary conditions. Strack (1989) has shown that when Dupuit assumptions of negligible vertical flow, as compared to horizontal, are valid the nonlinear Boussinesq equation is a viable alternative to Laplace's equation. For the Boussinesq equation the vertical coordinate does not exist and the free surface boundary condition is not needed, and hence time dependent boundary conditions can be easily incorporated into the analysis. The hydraulics of stream–aquifer systems have been studied by several researchers when the input controlling parameters are known with certainty using both analytical and numerical methods (Polubarinova-Kochina, 1962; Kirkham, 1966). The solution to Laplace's equation with free surface boundary conditions was compared with the solution of the linearized Boussinesq equation for the case of sudden drawdown in the river levels and for uniform and nonuniform rainfall (Van De Giesen et al., 1994). Tabidian et al., 1992 studied groundwater level fluctuations for changes in the stage levels in the associated streams. Hussein and Schwartz (2003) have studied the coupled groundwater–surface water problems and quantified the expected transport in both the aquifer and the stream. The coupled canal flow–groundwater flow equation has been solved analytically and the results are compared with the numerical model, MODFLOW, for small changes in the water level disturbances (Lal, 2000). Moench and Barlow (2000) have solved analytically the one-dimensional flow equation in confined and leaky aquifers and the two-dimensional flow equation in a plane perpendicular to the stream in phreatic aquifers where the stream is assumed to penetrate the full thickness of the aquifer.

Most stream–aquifer systems can be viewed as heterogeneous geological media with the governing flow equations being described as stochastic equations. When the aquifer parameters and the boundary conditions are not known with certainty, the preferred way to model the system has been stochastically. Using the stochastic approach, stream–aquifer models have been solved both analytically as well as numerically. Weissmann and Fogg (1999) and Weissmann et al. (1999) have studied the alluvial fan system from both deterministic as well as stochastic approaches, by modeling the large scale features deterministically and the intermediate heterogeneity using transition probability geostatistics. Hantush and MariZo (1997, 2002) have also studied the flow equations from a stochastic point of view.

The stream–aquifer interaction problem with time varying boundaries has been solved by Workman et al., 1997 using the deterministic linearized Boussinesq equation. In a later study, Serrano and Workman (1998) solved the same problem using the nonlinear Boussinesq equation and Adomian's method of decomposition. Decomposition is now

being used to solve deterministic, stochastic, linear or nonlinear equations in various branches of science and engineering (Adomian, 1991, 1994; Srivastava and Singh, 1999; Biazar et al., 2003; Wazwaz, 2000; Wazwaz and Gorguis, 2004; Srivastava, 2005). In groundwater flow problems, this method has been extensively used (Serrano, 1992, 1995a,b, 2003; Serrano and Unny, 1987; Serrano and Adomian, 1996; Adomian and Serrano, 1998), to obtain analytical, and sometimes closed-form, solutions to linear, nonlinear and stochastic problems. The method has been shown to be systematic, robust, and sometimes capable of handling large variances in the controlling hydrogeological parameters.

In this paper the work of Serrano and Workman (1998) has been extended to incorporate heterogeneity in the hydraulic conductivity represented stochastically. The nonlinear transient groundwater flow equation subject to stochastic conductivity and time varying boundary conditions is solved using decomposition. A random conductivity field with known correlation structure represents uncertain heterogeneity. Since decomposition does not require the assumptions of normality or smallness, we consider the practical situation when only the mean and correlation of the hydraulic conductivity are given based on a limited set of field samples. This is a common scenario in hydrologic practice. From the practical point of view, a modeling procedure that limits its application to Gaussian or any other conductivity fields is restrictive, since there is usually not enough information to ascertain the underlying probability distribution. A more common scenario in applications is one when the modeler possesses a limited sample from which only the first two moments can be estimated. Thus, a modeling procedure that allows the inclusion of these two measures, regardless of the underlying density function, appears practical. In this paper the solution and its second-order statistics are verified via an application to the Scioto River aquifer in Ohio. Predicted head statistics are compared with observed heads at different monitoring wells across the aquifer.

### Analytical solution to the stochastic transient groundwater flow equation in unconfined aquifers

The groundwater flow-equation in a horizontal unconfined aquifer of length  $l_x$  with Dupuit assumptions given by Bear (1979) as

$$\frac{\partial h}{\partial t} - \frac{1}{S} \frac{\partial}{\partial x} \left( Kh \frac{\partial h}{\partial x} \right) = \frac{R}{S} \quad 0 \leq x \leq l_x \text{ and } 0 < t \quad (1)$$

where  $h(x, t)$  is the hydraulic head (m),  $K$  is the hydraulic conductivity (m/day),  $R$  is the recharge (m/day),  $l_x$  is the length of the aquifer (m),  $x$  is the spatial coordinate (m), and  $t$  is the time coordinate (day).

The boundary conditions imposed on (1) are

$$\begin{aligned} h(0, t) &= H_1(t) \\ h(l_x, t) &= H_2(t) \\ h(x, 0) &= H_0(x) \end{aligned} \quad (2)$$

where  $H_1(t)$  and  $H_2(t)$  are time fluctuating heads at the left and the right boundaries (m).  $H_0(x)$  is the initial head across the aquifer (m).

The spatial variability in the hydraulic conductivity is represented by

$$K(x) = \bar{K} + K'(x) \quad (3)$$

where  $\bar{K}$  is the mean and  $K'(x)$  is the fluctuation part.

Common representations for the fluctuating part in the hydraulic conductivity assume the log conductivity as a normally distributed random variable. In the present work, we do not adopt a specific distribution and instead assume the random conductivity has known first and second-order statistics. From the practical hydrologic point of view, limited field data bases often offer sufficient information to infer the first two moments only. For the present exercise we consider the common case of an exponential two-point correlation function. Thus, the random component of the hydraulic conductivity is represented by

$$\begin{aligned} E\{K'(x)\} &= 0 \\ E\{K'(x_1)K'(x_2)\} &= \sigma_K^2 e^{-\rho|x_1-x_2|} \end{aligned} \quad (4)$$

where  $E\{\}$  denotes the expectation operator,  $\sigma_K^2$  is the variance in the hydraulic conductivity  $(\text{m/day})^2$ ,  $\rho$  is the correlation decay parameter  $(1/\text{m})$ , or  $(1/\rho)$  is the correlation length scale  $(\text{m})$ .

Substituting (3) into (1) we get the stochastic flow equation as

$$\frac{\partial h}{\partial t} = \frac{R}{S} + \frac{1}{S} \frac{\partial}{\partial x} \left( (\bar{K} + K')h \frac{\partial h}{\partial x} \right) \quad 0 \leq x \leq l_x \text{ and } 0 < t \quad (5)$$

Following the procedure given by Serrano and Workman (1998), the flow system (5) with associated boundary conditions (2) is solved and the  $t$ -decomposition solution to the problem is obtained. For an introduction to the method of decomposition in hydrology the reader is referred to Serrano (2001, 1997).

Let us define the operator  $L_t = \frac{\partial}{\partial t}$ . Applying the inverse operator  $L_t^{-1}$  to (5) and rearranging, we obtain

$$h = H_0(x) + \frac{Rt}{S} + L_t^{-1}N(h) \quad (6)$$

where the nonlinear operator is

$$N(h) = \frac{1}{S} \left\{ (\bar{K} + K')h \frac{\partial^2 h}{\partial x^2} + (\bar{K} + K') \left( \frac{\partial h}{\partial x} \right)^2 + h \frac{\partial K'}{\partial x} \frac{\partial h}{\partial x} \right\}$$

Using decomposition (Adomian, 1991, 1994), the solution to  $h$  may be written as the series

$$h = h_0 + h_1 + h_2 + \dots \quad (7)$$

The first term in the series  $h_0$  satisfies the first term in the right-hand side of (5) and the solution is written as

$$h_0 = H_0(x) + \frac{Rt}{S} \quad (8)$$

where  $H_0(x)$  is the initial condition. The initial condition is crucial in obtaining the solution to the problem. The initial condition can be expressed as a second order polynomial in  $x$  as

$$H_0(x, t) = A(t)x^2 + B(t)x + C(t) \quad (9)$$

where the constants  $A(t)$ ,  $B(t)$ , and  $C(t)$  should be obtained from well data across the aquifer, as is done in the present study. In cases where sufficient well data are not available,

the solution to the homogeneous groundwater flow equation with Dupuit assumptions and a constant transmissivity would be an appropriate approximation to an initial condition. This is a second order polynomial in  $x$  with the constants given by (Serrano, 1997)

$$\begin{aligned} A &= -\frac{R}{2T} \\ B &= \left( \frac{H_2(0) - H_1(0)}{l_x} + \frac{Rl_x}{2T} \right) \\ C &= H_1(0) \end{aligned} \quad (10)$$

Subsequent terms in the decomposition series (7) can be expressed as

$$\begin{aligned} h_1 &= L_t^{-1}A_0 \\ h_2 &= L_t^{-1}A_1 \\ &\vdots \\ h_{n+1} &= L_t^{-1}A_n \end{aligned} \quad (11)$$

The Adomian polynomials are given by

$$\begin{aligned} A_0 &= N(h_0) \\ A_1 &= h_1 \frac{dN(h_0)}{dh_0} \\ A_2 &= h_2 \frac{dN(h_0)}{dh_0} + \frac{h_1^2}{2!} \frac{d^2N(h_0)}{dh_0^2} \\ A_3 &= h_3 \frac{dN(h_0)}{dh_0} + h_1 h_2 \frac{d^2N(h_0)}{dh_0^2} + \frac{h_1^3}{3!} \frac{d^3N(h_0)}{dh_0^3} \\ &\vdots \end{aligned} \quad (12)$$

Thus, from (6) and (12) the second term in the series is expressed as

$$\begin{aligned} h_1 &= L_t^{-1} \frac{1}{S} \\ &\quad \times \left\{ 2\bar{K}A(t)(A(t)x^2 + B(t)x + C(t)) + \bar{K}(2A(t)x + B(t)) \right\} \\ &\quad + L_t^{-1} \frac{2\bar{K}A(t)Rt}{S^2} + L_t^{-1} \frac{1}{S} \\ &\quad \times \left\{ K' \left\{ (2A(t)x + B(t))^2 + 2A(t)h_0 \right\} + (2A(t)x + B(t))h_0 \frac{\partial K'}{\partial x} \right\} \end{aligned} \quad (13)$$

With two terms in the series, the solution to the nonlinear problem (5) is

$$\begin{aligned} h(x, t) &= A(t)x^2 + B(t)x + C(t) + \frac{Rt}{S} + L_t^{-1} \frac{1}{S} \\ &\quad \times \left\{ 2\bar{K}A(t)(A(t)x^2 + B(t)x + C(t)) + \bar{K}(2A(t)x + B(t)) \right\} \\ &\quad + L_t^{-1} \frac{2\bar{K}A(t)Rt}{S^2} + L_t^{-1} \frac{1}{S} \\ &\quad \times \left\{ K' \left\{ (2A(t)x + B(t))^2 + 2A(t)h_0 \right\} + (2A(t)x + B(t))h_0 \frac{\partial K'}{\partial x} \right\} \end{aligned} \quad (14)$$

Gabet (1993, 1994) and Abbaoui and Cherruault (1994) have shown that when decomposition series converge, they do so very rapidly and only a few terms in the series are required for an accurate solution. Serrano (1998) has shown the rapid convergence of the decomposition series for groundwater flow problems. The inclusion of additional terms in the

solution would require the availability of higher-order moments in the hydraulic conductivity.

Taking expectation on both sides of (14) we obtain the mean head as

$$E\{h(x, t)\} = Ax^2 + Bx + C + \frac{Rt}{S} + L_t^{-1} \frac{1}{S} \times \{2\bar{K}A(t)(A(t)x^2 + B(t)x + C(t)) + \bar{K}(2A(t)x + B(t))\} + L_t^{-1} \frac{2\bar{K}A(t)Rt}{S^2} \quad (15)$$

The fluctuating part in Eq. (14) is

$$\chi(x, t) = c_1(t)K'(x) + c_2(t) \frac{\partial K'(x)}{\partial x} \quad (16)$$

where

$$c_1(t) = \left( \frac{t}{S} \{ (2A(t)x + B(t))^2 + 2A(t)(A(t)x^2 + B(t)x + C(t)) \} + \left( \frac{A(t)Rt^2}{S^2} \right) \right)$$

$$c_2(t) = \left( \frac{t}{S} (2A(t)x + B(t))(A(t)x^2 + B(t)x + C(t)) + \left( \frac{Rt^2}{2S^2} (2A(t)x + B(t)) \right) \right)$$

The two-point covariance function is given by

$$E\{\chi_1(x_1, t_1)\chi_2(x_2, t_2)\} = E\left\{ \left( c_1(t_1)K'(x_1) + c_2(t_1) \frac{\partial K'(x_1)}{\partial x_1} \right) \times \left( c_1(t_2)K'(x_2) + c_2(t_2) \frac{\partial K'(x_2)}{\partial x_2} \right) \right\} \quad (17)$$

By setting  $x_1 = x_2 = x$  and  $t_1 = t_2 = t$  the variance is obtained as

$$\sigma^2(x, t) = C_k^2 \bar{K}^2 (c_1^2 + 2\rho c_1 c_2 + \rho^2 c_2^2) \quad (18)$$

## Model verification

The methodology developed above has been applied to evaluate the effect of a highly fluctuating river stage on the adjacent aquifer flow. The study area is an alluvial valley aquifer in South Central Ohio, OMSEA (Ohio Management System Evaluation Area) (Fig. 1). The Scioto River drains much of central Ohio and forms the western boundary of the OMSEA

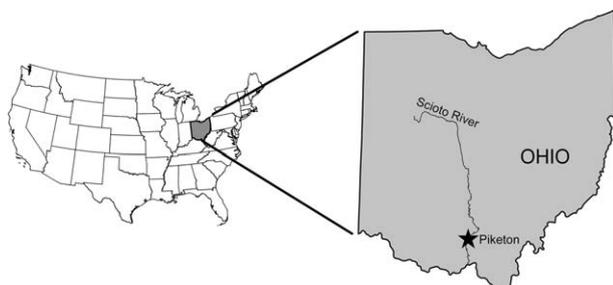


Figure 1 Location of the Scioto River Aquifer in Ohio.

site and a stream gage at Higby, Ohio (approximately 21 km upstream from the OMSEA) has monitored flow in the 13,290 km<sup>2</sup> watershed for 60 years (Nortz et al., 1994). The change in river stage between maximum and minimum flows was 7.4 m. The National Weather Service (NWS) operates a wire-weight gage at Piketon, Ohio. Jagucki et al. (1995) used the Scioto River elevations adjacent to the OMSEA site to develop a gradient correction factor of 1.52 m between the NWS gage site and the OMSEA site.

The alluvial valley aquifer is approximately 18–20 m thick and about 2 km wide (Fig. 2). The aquifer is composed of various types of sand and gravel with high transmissivity values. The US Geological Survey (USGS) conducted 13 pump tests in the aquifer over a 25-year period to determine hydrologic properties and water resource potential for a Department of Energy project (Norris and Fidler, 1969; Norris, 1983a,b; Nortz et al., 1994; Jagucki et al., 1995). A time-drawdown analysis of the data indicated the hydraulic conductivity of the regional aquifer to vary between 122 and 170 m/day with a mean value of 142 m/day. The specific yield has been estimated to be around 0.2 m<sup>3</sup>/m<sup>3</sup>.

Eleven water-table wells were constructed over a 260 ha area surrounding the OMSEA site. The wells were constructed with 152 mm diameter PVC casing. A 6.1-m long, 2.54-mm slotted, PVC screen was positioned to bracket the highest and lowest expected water-table elevations in each well. All of the water-table wells were instrumented with shaft encoders and electronic data loggers that recorded hourly water levels.

Transient stage at the Scioto River constituted the left boundary condition for the model, and well R1 the right one (Fig. 2). The three wells (R5, R4, and S10) shown in Fig. 2 lie on the flow path from the eastern edge of the OMSEA site to the Scioto River. These three monitoring wells are located R5-215 m, R4-975 m, and S10-1525 m, respectively, from the Scioto River where measurements were made. Big Beaver Creek, located at the right boundary of the flow domain, drains an area composed predominantly of the poorly permeable bedrock uplands from which runoff was rapid and bank storage was minimal. These conditions produce rapid stage fluctuations in the creek during storm events but only intermittent flow during the dry summer months. The transient nature of the boundary conditions in the river and the well is depicted in Fig. 3, which shows that the fluctuations with respect to time at the right boundary are very small as compared to those of the Scioto River on the left side. Fig. 3 shows the total change in stage recorded during the period of October 1991 to September 1992 was approximately 4.9 m. The seasonal fluctuations in the water level of the Scioto River were important and appeared to directly affect the ground-water levels in areas near the river.

The assumptions specified for the development of the mathematical model were met with the aquifer underlying the OMSEA site. Researchers have observed that the regional groundwater flow is predominantly in the horizontal direction, the water table elevations exhibited mild slopes, and the aquifer thickness is small compared to the horizontal length. These aquifer and flow characteristics justify the Dupuit assumptions. When the Dupuit assumptions are valid, the linearized Boussinesq equation appears to be a viable alternative to Laplace's equation. With the Boussinesq

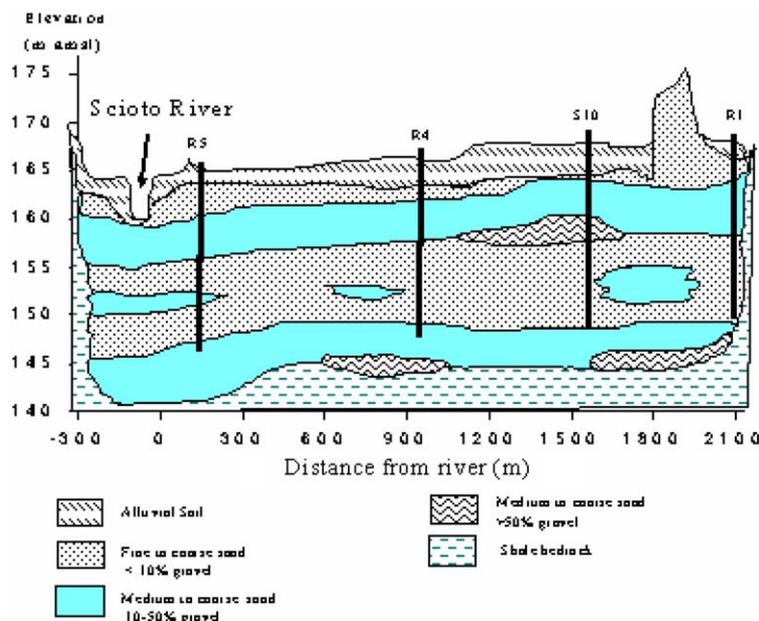


Figure 2 Cross section of the Scioto River alluvial valley aquifer.

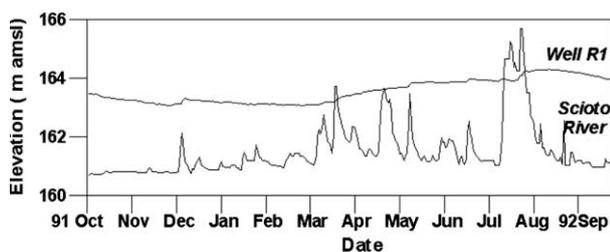


Figure 3 Daily elevation of the Scioto River at the right boundary and well R1 at the left boundary of the OMSEA aquifer from October 1991 to September 1992.

equation, the vertical coordinate is eliminated, and the free-surface boundary condition is not needed. The result is a simplified model where the effect of time-dependent river boundary conditions can be easily incorporated in the analysis.

Earlier investigations in the region have shown that the performance of the nonlinear model in predicting stream–aquifer interaction is similar to that of the linearized model. Workman et al. (1997) have used these data and modeled the water table elevations across the aquifer using the linearized Boussinesq equation. In their study they simulated the stream–aquifer interaction for linear models. They have obtained the solution by combining the solution of the steady state component and the transient component. In a later study, Serrano and Workman (1998) derived a nonlinear solution to the Boussinesq equation with transient boundary conditions using the decomposition method. They have shown that the results of the nonlinear model and the linearized model are very similar in predicting the stream–aquifer interaction at the OMSEA research site. This is basically because the aquifer thickness was large compared to the changes in the river elevation. When the saturated thickness is not large compared to the boundary conditions Serrano and Workman (1998) have shown that the nonlinear

model predicts the water table response more accurately. Also the nonlinear model better represents the dynamic response of the water table elevation when a large change in the transmissivity field occurs. In the same study, no recharge was simulated in the aquifer for the purposes of determining the direct influence of the river on groundwater levels.

Workman and Serrano (1999) studied the effects of recharge on the aquifer to determine the separate influences of river stage and recharge over a 5-year period. When the river stage reaches a height of 166 m above mean sea level, the floodplain becomes inundated and water is recharged directly to the aquifer. Over 2 m of recharge was computed during the 5-year period with most recharge events occurring during these flood events. The period of record chosen for this study (1991–1992) does not include any dates of overbank flow and allows the study of the river influence on hydraulic heads. Appendix A in the present paper consists of a sensitivity analysis of the effect of recharge relative to that of a highly fluctuating left boundary condition on the magnitude of the aquifer head. It is seen that the effect of recharge in this particular scenario is so small that the assumption of no recharge is reasonable. We remark that the methodology presented could include recharge estimates if it is found to be more important or as important as the effect of the boundaries.

The distance of influence of the river flood wave into the aquifer is dependent on the transmissivity. Since the aquifer exhibits a large variation in the hydraulic conductivity parameter a stochastic model has been developed to quantify the errors in the head variation due to errors in the hydraulic conductivity.

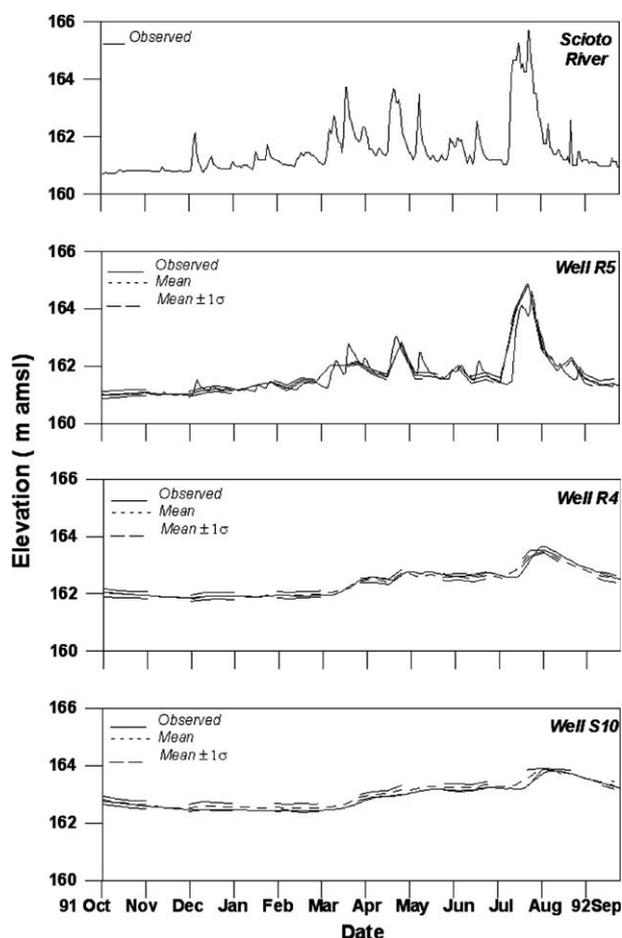
## Results and discussion

The proposed methodology has been tested on these wells that lie along the flow path and the theoretical values have

**Table 1** Controlling hydrogeological parameters for the alluvial valley aquifer

Aquifer length	2000 m
Mean hydraulic conductivity	142 m/day
Specific yield	0.2 m <sup>3</sup> /m <sup>3</sup>
Coefficient of variability in hydraulic conductivity ( $C_K$ )	0.4
Correlation length scale	500 m

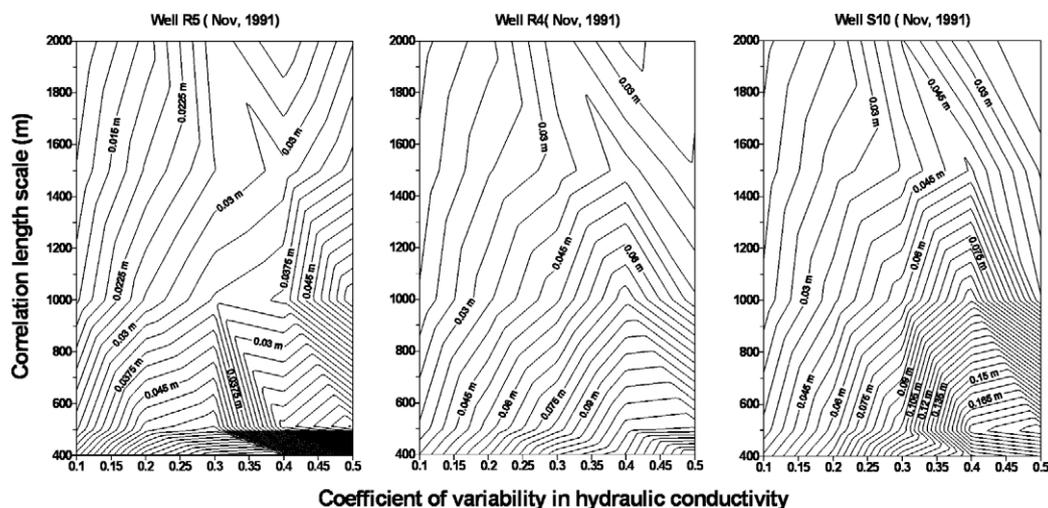
been computed with its error bounds and compared with the observed heads. A portion of the data from October 1991 to September 1992 (Jagucki et al., 1995) has been used to quantify the water table elevations at the three wells in the study domain. The controlling aquifer parameters are tabulated in Table 1. Using these parameters the transient mean water table elevation is computed from (15), and the standard deviation from (18). Fig. 4 shows the plot of the mean and mean plus or minus one standard deviation at Well R5, approximately 215 m from the Scioto River. The well exhibits a large variation in the water level over the period of 1 year from October 1991 to September



**Figure 4** Observed, mean, and mean plus and minus one standard deviation of the water table elevation at Well R5 (215 m), at Well R4 (975 m) and Well S10 (1525 m) and the daily elevations at the Scioto River.

1992. The mean and the error bounds have been computed at the well with a coefficient of variability in the hydraulic conductivity  $C_K = 0.4$  and a correlation length scale of 500 m. The mean head is in reasonable agreement with the observed head (Fig. 4). Fig. 4, the error bounds are dependent on time as well as on distance from the river boundary. Fig. 4 also shows the mean plus minus one standard deviation, and the observed head at wells R4, located 975 m, and S10, located 1525 m, respectively, from the Scioto River. From the well hydrographs in Fig. 4, we observe that the head standard deviation is seen to slightly increase with distance. This confirms prior theoretical studies (e.g., Serrano, 1995a) that suggest that the head variance increases with distance from a known head boundary condition, where it is zero. However, the variance appears to be stationary in time. Quantification of the effect of random conductivity indicate that the observations lie in the plus minus one standard deviation limit for a given variation in the hydraulic conductivity. However, it is observed that there is a slight over prediction or under prediction of the modeled head for some months at the wells. This is because we are considering the mean behavior of the system. We remark that the observed head is in fact a sample function from the ensemble. Individual sample functions may or may not lie within the bounds of the standard deviation. In Fig. 4 the months observed levels of June and July are a bit beyond these bounds. Theoretically, an appropriate comparison between model and prototype should be done by comparing mean of prototype, based on many samples, with the corresponding mean of the model; then comparing the variance of the prototype with that of the model. However, in this case only one sample over time is available and the comparison is qualitative. A quantitative assessment of the differences between a measured hydrograph and a simulated one is only possible in a traditional deterministic analysis that assumes that the measured hydrograph is the “true,” error free, one. In the present analysis, the measured hydrograph is subject to errors too (i.e., is a sample from an unknown random process), while the statistics form the simulated hydrograph (e.g., the mean hydrograph) represents the ensemble average of all samples in the model. In a stochastic analysis sense, a uniform comparison would be possible if both moments of prototype and model were available. In other words, if for example the mean prototype head is compared to the mean model head. However, only a limited measured series is available.

Next we study the effect of uncertainty in the hydraulic conductivity, as measured by its variance and correlation length scale, on the predicted heads. The sensitivity analysis on the effect of the random components of the hydraulic conductivity on the head statistics at the wells R5, R4 and S10 has been carried out (Fig. 5). It is clear from the figure that the errors increase with distance. The head standard deviation was computed for various values of the coefficient of variability in hydraulic conductivity and its correlation length scale. In this Figure we see that the maximum standard deviation is about 0.05 m and the minimum is about 0.015 m at well R5 located at 215 m from the river, the maximum standard deviation is 0.09 m and the minimum is 0.03 m at well R4 located at 975 m from the river and the maximum standard deviation is 0.165 m and the minimum standard deviation is about 0.03 at well S10 located at



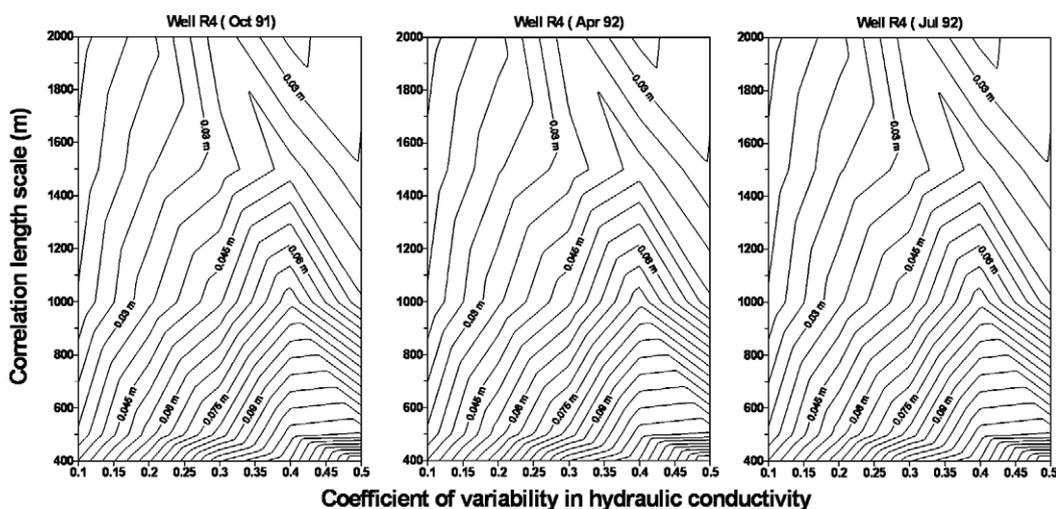
**Figure 5** Standard deviation as a function of coefficient of variability in the hydraulic conductivity and its correlation length scale at well R5, R4 and S10 for November 1991.

1525 m from the river. At well R5 we also observe that for smaller correlation length scale and larger coefficient of variability in hydraulic conductivity the errors behave in a nonlinear pattern. It is also observed that the errors are large for smaller correlation length scale than for larger lengths. Hence if we consider the aquifer to be highly heterogeneous we need to consider smaller correlation length scale. Depending on the uncertain behavior of the hydraulic conductivity the coefficient of variability in the conductivity field can be used. Ideally, the correlation length should be deduced from a fitted serial correlation coefficient if a sufficiently large sample of spatial conductivity is available. Fig. 5 further demonstrates the errors on the water table heights for a range of the controlling random components of the hydraulic conductivity. This shows that the errors are highly controlled by these parameters and an appropriate choice depending on the nature of the aquifer should be used. Ideally it should be calculated from the available information in the field. Since sufficient data on the hydro-

lic conductivity data are not available a 40% error in the hydraulic conductivity i.e  $C_K = 0.4$  has been considered. From Fig. 5 we observe that for a correlation length scale of greater than 1/4th the errors stabilize more or less. Hence in this investigation a correlation length scale of about 500 m has been considered. The error propagation with time has also been investigated and it is observed that it is stationary with respect to time (Fig. 6). For this we have considered well R4 and the error statistics for October 1991, April, 1992 and July 1992 have been computed. It is clear that there is no change in the errors in the three months at the given well location.

## Summary and conclusions

Using decomposition, the nonlinear Boussinesq equation with random hydraulic conductivity and time varying boundary conditions has been solved to simulate the stream-aquifer



**Figure 6** Standard deviation as a function of coefficient of variability in the hydraulic conductivity and its correlation length scale at well R4 for October 1991, April 1992 and July 1992.

interaction in an alluvial valley. A practical scenario has been considered when limited sampling on the hydraulic conductivity allows the estimation of its first and second-order statistics only, and thus assumptions of a specific probability distribution are not reasonable. Simple analytical expressions to the mean and standard deviation in the water table elevation have been obtained. The model has been applied to simulate the mean and standard deviation of the water table elevation in different wells across the 2 km wide aquifer, which is subject to large periodic fluctuations in river stage. Statistics of the water table elevation as a function of those of the hydraulic conductivity have been quantified. It is seen that the magnitude of uncertainty in the predicted variable depend on distance, time, coefficient of variability in the hydraulic conductivity, and on the conductivity correlation length scale. The uncertainty on the water table elevation at a particular location and time has been demonstrated for different values of the coefficient of variability and correlation length scale. It is observed that for smaller correlation length scales the errors are high and are also seen to increase with an increase in the coefficient of variability of hydraulic conductivity. For highly heterogeneous aquifers a smaller correlation length scale can be taken. The variability in hydraulic conductivity in such situations is also known to be high. Hence, the quantification of these uncertainties helps predict the error bounds in the dependent variable and in corollary variables, such as aquifer dispersion coefficient and seepage velocity.

## Appendix A. Sensitivity analysis of the effect of recharge

One of the reviewers of this paper suggested the investigation of the effect of recharge relative to that of a fluctuating boundary condition on the overall magnitude of the hydraulic head. Intuitively, one may assume that a fluctuating river boundary condition exhibiting several meters in amplitude is more important than that of recharge from rainfall, which usually amounts to much less than a meter. In this section we verify this hypothesis quantitatively by comparing the magnitude of aquifer head in an aquifer with a highly fluctuating boundary and recharge with one without recharge. For simplicity we consider the linearized equation (1) for a hypothetical aquifer subject to a periodic left boundary condition and a no flow right boundary condition:

$$\frac{\partial h}{\partial t} - \frac{T}{S} \frac{\partial^2 h}{\partial x^2} = \frac{R}{S} \quad 0 \leq x \leq l_x \text{ and } 0 < t \quad (\text{A1})$$

where  $T$  is the mean transmissivity ( $\text{m}^2/\text{day}$ ). (A1) is subject to

$$h(0, t) = H_1(t) = H_0 + A \sin\left(\frac{2\pi t}{B}\right) \quad (\text{A2})$$

$$\frac{\partial h(l_x, t)}{\partial x} = 0, \quad h(x, 0) = H_0$$

where  $A$  and  $B$  are constants. The solution of (A1) may be expressed as the summation of two components (Serrano and Unny, 1987)

$$h(x, t) = V(x, t) + W(x, t) \quad (\text{A3})$$

Substituting Eq. (A3) into Eq. (A1) we get

$$\frac{\partial W}{\partial t} - \frac{T}{S} \frac{\partial^2 W}{\partial x^2} = \frac{R}{S} - \left( \frac{\partial V}{\partial t} - \frac{T}{S} \frac{\partial^2 V}{\partial x^2} \right) \quad (\text{A4})$$

subject to

$$W(0, t) = H_1(t) - V(0, t) = W_0 \quad (\text{A5})$$

$$\frac{\partial W(l_x, t)}{\partial x} = - \frac{\partial V(l_x, t)}{\partial x} \quad (\text{A6})$$

$$W(x, 0) = H_0(x) - V(x, 0) \quad (\text{A7})$$

We choose a smooth function  $V(x, t)$  such that the boundary conditions in Eqs. (A5) and (A6) become zero:

$$V(x, t) = H_1(t) \quad (\text{A8})$$

The solution to Eq. (A4) is (Serrano and Unny, 1987)

$$W(x, t) = \int_0^t J_{t-\tau} \frac{R}{S} d\tau - \int_0^t J_{t-\tau} \frac{\partial H_1(\tau)}{\partial \tau} d\tau \quad (\text{A9})$$

where the semigroup operator associated with Eq. (A4),  $J_t$ , is given as

$$J_t f = \sum_{n=1}^{\infty} b_n(f) \phi_n(x) M_n(t) \quad (\text{A10})$$

the Fourier coefficients,  $b_n$ , are given as

$$b_n(f) = \frac{2}{l_x} \int_0^{l_x} f \sin(\lambda_n \xi) d\xi \quad (\text{A11})$$

the eigenvalues,  $\lambda_n$ , are

$$\lambda_n = \left( \frac{2n-1}{2l_x} \right) \pi \quad n = 1, 2, 3, \dots \quad (\text{A12})$$

the basis function,  $\phi_n(x)$ , is given as

$$\phi_n(x) = \sin(\lambda_n x) \quad (\text{A13})$$

and

$$M_n(t) = e^{-\lambda_n^2 T t / S} \quad (\text{A14})$$

Since the recharge and the specific yield are assumed constant, the solution of Eq. (A4) is obtained as

$$W(x, t) = \frac{R}{S} \sum_{n=1}^{\infty} b_n(1) \phi_n(x) I_n(t) - \sum_{n=1}^{\infty} b_n(1) \phi_n(x) L_n(t) \quad (\text{A15})$$

where

$$I_n(t) = \int_0^t M_n(t-\tau) d\tau = \frac{S}{\lambda_n^2 T} (1 - e^{-\lambda_n^2 T t / S}) \quad (\text{A16})$$

and

$$L_n(t) = \int_0^t M_n(t-\tau) \frac{\partial H_1(\tau)}{\partial \tau} d\tau = \frac{2\pi A S}{\lambda_n^4 T^2 B^2 + 4\pi^2 S^2} \times \left[ \lambda_n^2 T B e^{-\frac{\lambda_n^2 T t}{S}} - \lambda_n^2 T B \cos\left(\frac{2\pi t}{B}\right) - 2\pi S \sin\left(\frac{2\pi t}{B}\right) \right] \quad (\text{A17})$$

From (A3) and (A8), the solution reduces to

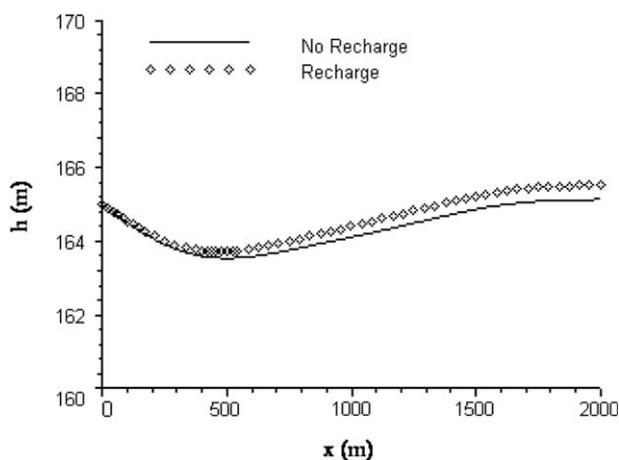
$$h_R(x, t) = H_1(t) + \frac{R}{S} \sum_{n=1}^{\infty} b_n(1) \phi_n(x) I_n(t) - \sum_{n=1}^{\infty} b_n(1) \phi_n(x) L_n(t) \quad (\text{A18})$$

where  $h_R(x, t)$  is the hydraulic head including recharge, and

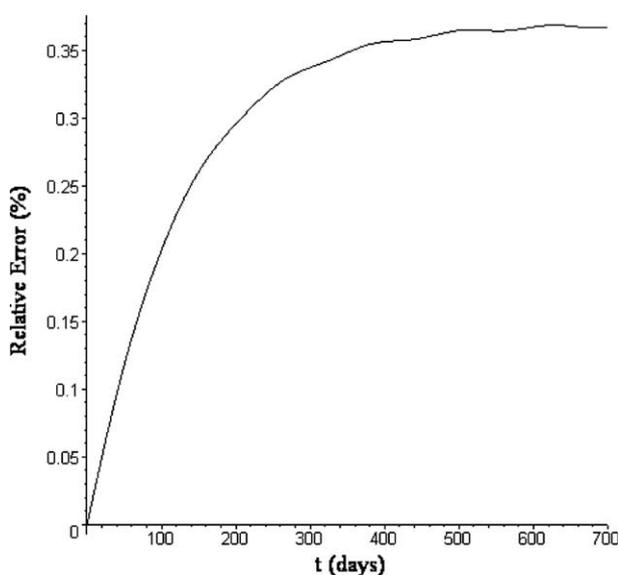
$$h(x, t) = H_1(t) - \sum_{n=1}^{\infty} b_n(1)\phi_n(x)L_n(t) \quad (\text{A19})$$

where  $h(x, t)$  is the head without recharge. Now we adopt parameter values reflecting those of the Scioto River,  $K = 142$  m/day,  $S = 0.2$ ,  $l_x = 2000$  m, a large recharge of 0.3 m/year, and average aquifer thickness  $b = 19$  m,  $A = 5$  m,  $B = 120$ , and a mean transmissivity  $T \approx Kb = 2698$  m<sup>2</sup>/day. Fig. A1 shows the typical aquifer head spatial distribution at  $t = 120$  days with and without recharge, according to Eqs. (A18) and (A19), respectively. The effect of recharge increases with distance and it is maximum at the right boundary. Recharge appears to increase a fraction of the magnitude of the hydraulic head. To assess this quantitatively we consider a measure of the relative error,  $E$ , as

$$E = \frac{(h_R(x, t) - h(x, t))}{h(x, t)} \times 100 \quad (\text{A20})$$



**Figure A1** Aquifer head spatial distribution at  $t = 120$  days with and without recharge.



**Figure A2** Maximum relative error due to recharge at the right boundary.

Numerical simulations indicated that the maximum relative error due to recharge occurs at the right boundary. This error increases with time and reaches an asymptotic value of about 0.35%. Thus, the results from this exercise suggest that the effect of a highly fluctuating river boundary condition appears to be significantly more important than recharge in determining the magnitude of the aquifer hydraulic head. This supports the assumption of neglecting recharge in the present problem (Fig. A2).

## References

- Abbaoui, K., Cherruault, Y., 1994. Convergence of Adomians method applied to differential equations. *Comput. Math. Appl.* 28 (5), 103–109.
- Adomian, G., 1991. A review of the decomposition method and some recent results for nonlinear equations. *Comput. Math. Appl.* 21 (5), 101–127.
- Adomian, G., 1994. *Solving Frontier Problems of Physics – The Decomposition Method*. Kluwer Academic Publishers, Boston.
- Adomian, G., Serrano, S.E., 1998. Stochastic contaminant transport equation in Porous media. *Appl. Math. Lett.* 1191, 53–55.
- Bear, J., 1979. *Hydraulics of Groundwater*. McGraw-Hill, New York, NY.
- Biazar, J., Babolian, E., Kember, G., Nouri, A., Islam, R., 2003. An alternate algorithm for computing Adomian polynomials in spectral cases. *Appl. Math. Comput.* 138, 523–529.
- Gabet, L., 1993. The decomposition method and linear partial differential equations. *Math. Comput. Model.* 21 (6), 11–22.
- Gabet, L., 1994. The decomposition method and distributions. *Comput. Math. Appl.* 27 (3), 41–49.
- Hantush, M.M., MariZo, M.A., 1997. Stochastic solution to inverse problem I groundwater. *J. Hydrol. Eng. ASCE* 123 (12), 1139–1146.
- Hanush, M.M., MariZo, M.A., 2002. Hydraulics of stream flow with bank storage. *J. Hydrol. Eng. ASCE* 7 (1), 76–89.
- Hussein, M., Schwartz, F.W., 2003. Modeling of flow and contaminant transport in coupled stream aquifer systems. *Water Resour. Res.* 65 (1–2), 41–64.
- Jagucki, M.L., Finton, C.D., Springer, A.E., Bair, E.S., 1995. Hydrogeology and water quality at the management System evaluation Area near Piketon, Ohio. US Geological survey Water Resources Investigation Report, 95-4139, 117 p.
- Kirkham, D., 1966. Steady state theories for drainage. *J. Irrigat. Drain. Div., ASCE* 92 (1), 19–39.
- Lal, A.M.W., 2000. Numerical errors in groundwater and overland flow models. *Water Resour. Res.* 36 (5), 1237–1247.
- Moench, A.F., Barlow, P.M., 2000. Aquifer response to stream stage and recharge variations. I Analytical step response functions. *J. Hydrol.* 230, 192–210.
- Norris, S.E., 1983a. Aquifer tests and well field performance, Scioto river valley near Piketon, Ohio, Part I. *Ground Water* 21 (3), 287–292.
- Norris, S.E., 1983b. Aquifer tests and well field performance, Scioto river valley near Piketon, Ohio, Part II. *Ground Water* 21 (4), 438–444.
- Norris, S.E., Fidler, R.E., 1969. Hydrogeology of the Scioto river valley near Piketon, South central Ohio. US Geological Water Supply Paper 1872, 70 pp.
- Nortz, E., Bair, E.S., Ward, A., White, D., 1994. Interaction between an alluvial aquifer well field and the Scioto River, Ohio, USA. *Appl. Hydrogeol.* 4, 23–34.
- Polubarinova-Kochina, P.V., 1962. *Theory of Groundwater Movement*. Princeton University Press, Princeton, NJ.
- Serrano, S.E., 1992. Semi analytical methods in stochastic groundwater transport. *Appl. Math. Model.* 16, 181–191.

- Serrano, S.E., 1995a. Analytical solutions to nonlinear groundwater flow equation in unconfined aquifers and effect of heterogeneity. *Water Resour. Res.* 31 (11), 2733–2742.
- Serrano, S.E., 1995b. Forecasting scale dependent dispersion from spills in heterogeneous aquifers. *J. Hydrol.* 169, 151–169.
- Serrano, S.E., 1997. *Hydrology for Engineers, Geologists and Environmental Professionals: An Integrated Treatment of Surface Subsurface, and Contaminant Hydrology*. HydroScience Inc., Lexington, KY.
- Serrano, S.E., 1998. Analytical decomposition of the non-linear infiltration equation. *Water Resour. Res.*, American Geophysical Union 34 (3), 397–407.
- Serrano, S.E., 2001. *Engineering Uncertainty and Risk Analysis: A Balanced Approach to Probability, Statistics, Stochastic Modeling, and Stochastic Differential Equations*. HydroScience Inc., Lexington, KY.
- Serrano, S.E., 2003. Propagation of nonlinear reactive contaminants in porous media. *Water Resour. Res.* 39 (8), 1228, 13-1-14.
- Serrano, S.E., Adomian, G., 1996. New contributions to the solution of transport equations in porous media. *Math. Comput. Model.* 24 (4), 15–25.
- Serrano, S.E., Unny, T.E., 1987. Semigroup solutions of the unsteady groundwater flow equation with stochastic parameters. *Stoch. Hydrol. Hydraul.* 1 (4), 281–296.
- Serrano, S.E., Workman, S.R., 1998. Modeling transient stream–aquifer interaction with the nonlinear Boussinesq equation and its analytical solution. *J. Hydrol.* 206, 245–255.
- Srivastava, K., 2005. Modeling the variability of heat flow due to random thermal conductivity of the crust. *Geophy. J. Int.* 160 (2), 776–782.
- Srivastava, K., Singh, R.N., 1999. A stochastic model to quantify the steady state crustal geotherms subject to uncertainty in thermal conductivity. *Geophy. J. Int.* 138, 895–899.
- Strack, O.D.L., 1989. *Ground Water Mechanics*. Prentice-Hall, Englewood Cliffs, NJ.
- Tabidian, M.A., Pederson, D., Tabidian, P.A., 1992. A paleo valley aquifer system and its interaction with the big Blue river of Nebraska during a major flood. In: Borden, R.C., Lyke, W.L. (Eds.), *The Future Availability of Groundwater Resources*. American Water Resource Association, pp. 165–172.
- Van De Giesen, N.C., Parlange, N.Y., Steenhius, T.S., 1994. Transient flow to open drains: comparison to linearized solution with and without Dupuit assumptions. *Water Resour. Res.* 30 (11), 3033–3039.
- Wazwaz, A.M., 2000. A new algorithm for calculating Adomian polynomials for nonlinear operators. *Appl. Math. Comput.* 111, 53–69.
- Wazwaz, A.M., Gorguis, A., 2004. Exact solutions for heat like and wave like equations with variable coefficients. *Appl. Math. Comput.* 149, 15–29.
- Weissmann, G.S., Fogg, G.E., 1999. Multi scale alluvial fan heterogeneity modeled with transition probability geostatistics in a sequence stratigraphic framework. *J. Hydrol.* 226, 48–65.
- Weissmann, G.S., Carle, S.F., Fogg, G.E., 1999. Three dimensional hydrofacies modeling based on soil surveys and transitional probability geostatistics. *Water Resour. Res.* 35, 1761–1770.
- Workman, S.R., Serrano, S.E., 1999. Recharge to Alluvial Valley aquifers from overbank flow and excess infiltration. *J. Amer. Water Resour. Associat.* 35 (2).
- Workman, S.R., Serrano, S.E., Liberty, K., 1997. Development and application of an analytical model of stream/aquifer interaction. *J. Hydrol.* 200, 149–163.