Reflections on Undergraduate Research in Mathematics

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Eric Rawdon changed my life. Sitting in his Honors Calculus 1 as a first semester freshman, I came to understand that mathematics was more than calculation. The truths written into our texts are the fruits of inspired thought and careful logic. I suspect that many of his students have had similar realizations. But I was lucky beyond this.

Dr. Rawdon needed students to run simulations related to his knot theory research. Code had been written to use polygons to simulate physical knots. By then (2004), computers were powerful enough to manipulate somewhat big polygonal knots with many vertices. Assertions about tight or “ideal” configurations could be informed with these experiments. Biologists and Physicists were finding this information useful as physical knots could be found throughout nature.

Duquesne University had a student lab with about 30 computers in the Math and CS Department. When these computers were not in use by students, they could manipulate digital knots. It was a thrill to sit there minding the experiments: watching in person or remotely for unused computers, starting runs, hurriedly pausing them if a computer went into use, gathering data from the run, completing it on a different machine if needed, and then collecting the final results back to a home directory. It was like harmless piracy in the name of science.

Beyond the fun work and the much appreciated student paycheck, I got a privileged glimpse at the culture of research in mathematical sciences. As a second semester Freshman, I attended an AMS special session that Eric co-organized in Pittsburgh. I was not far enough along at the time to justify travelling to an out of town conference. But this one was two miles from Duquesne’s dorms. I was a fly on the wall (okay, the 4th row) as impressive researchers gave 30 minute updates on their work and occasionally faced real challenges from their peers.

I was hooked. Eric had 3-5 undergrad researchers working for him at any given time. And these bright people became surrogate brothers and sisters. Many have gone on to academic careers. All, I think, drew significant inspiration from this work.

Throughout graduate school, I imagined leading such an effort some day. I studied broadly, having seen how many subjects came together in the knot theory group. I learned how to solve deep mathematical problems from Chris Lennard, who also allowed some latitude for my first efforts advising research. I had two great students: Andre Schrock and Matt Stoffregen who worked hard and presented talks at an MAA meeting in Morgantown, WV. I learned so much about the adviser’s role while doing this.

Humans are inherently curious. We want to know about the world around us. We observe that which we cannot describe and then collaborate to describe and understand those observations. We know that even in the mundane world of physical material, there are deeper truths than the one that informed our most recent calculation. Students are inspired to become involved with the effort of deducing, modelling, and explaining those truths.

And of course theoretical mathematics inspires questions in exactly the same way. Most college students are familiar with the “unreasonable effectiveness of mathematics” at modeling the physical world from their physics, chemistry, and other classes. But we frequently save the astonishing ways that abstraction can reveal new truth and allow for new application until the

Bachelor-level courses in algebra, analysis, number theory, and topology. Guided research projects are a great way for undergraduates to get their hands on that clay.

Three weeks after defending my PhD thesis, I started as a full time teaching professor at Temple University. With a full teaching load and one young child at home followed by another, my research program lost all momentum. I did attempt to advise student projects a little bit during my first couple years at Temple. All such efforts are worthwhile as they provide learning experiences for everybody. Beyond these lessons, the only thing I had to show for the first few years of such efforts was a much appreciated friend and racquetball partner named Adam.

Lacking the ability to craft an accessible inroad to funded and active research, I needed a collaborator on campus. I was lucky that Prof Charles Osborne was willing to sit through a collection of half baked pitches of research projects and select one that could bear fruit. Charles also has the intellectual discipline to stay on course and consolidate those insights that come along with time. We found that focusing on an intractable but accessible problem - the Collatz Conjecture in our case - allows the time and opportunity for even otherwise busy people to develop and explain such insights.

Charles and I advised some very talented students - Nick Sisko, Garrett Bowser, Sarah Hafer, and Joe Franks - who wrote and ran code and wrote down all sorts of interesting and rigorously justified observations about phenomena related to that conjecture. Their presentations and posters have been among the greatest moments of my career.

I also had the great fortune of advising some one-off projects with other students - Kat Osadchuk, Gillian McGuire, and Julianna Sims. These projects - on Fixed Point Theory, Voting Theory, and Benford’s Law respectively - each evolved on their own terms. In each case the student took ownership of the material and discovered their own angles of inquiry. As I reflect on enjoying these projects culminating with excellent presentations at conferences and symposia, I see the joy of a coach after a win rather than that of a player.

With all of these experiences, I have started to notice the advising practices that allow these projects to flourish.

Undergraduates tend to have a lot on their plates. American universities generally encourage breadth in students’ coursework and in campus life. We cannot expect our research students to have the dedication that we summoned to complete our graduate theses. We will sometimes have to re-cover ground just as we do in teaching our undergraduate classes. One helpful mantra comes from my colleague Boris Datskovsky: “Repetition is the mother of learning”. Particularly when we are working with students and uncover some insight that we are too excited about to imagine forgetting, we may need to summon some patience to reiterate that exciting truth with our students a week or ten weeks later.

We need to be willing to invest time and energy into projects that will not come to fruition. Students who show a spark of interest may find deeper interests elsewhere. Accepting this will help smooth out projects that are eventually finished and those that are not. About a third of the students I was convinced would complete projects lost interest before doing so.

But persistence, patience, and effort pay off. We can model the best of our fields and of scholarship generally. We can be punctual, prepared, and share what we’ve learned. We can show how fun the subject can be and how to work past frustration and mistakes. We can help our students develop as thinkers, writers, speakers, researchers, problem solvers, and collaborators. And we can learn a lot from these experiences ourselves.