Uncertainty premia for small and large risks

Pavel Savor*  Mungo Wilson†  Martin Puhl‡

This version: March 2015

Abstract

We identify an ambiguity premium for US stocks from increases in the option-implied concavity of preferences immediately before scheduled macroeconomic announcements. Our methodology relies on Skiadas’ (2013) critique of smooth ambiguity aversion models, which shows that ambiguity aversion has a negligible effect on small risks, defined as risks that are proportional to the holding period. We show that the same critique implies that the effect of smooth ambiguity aversion on large risks, such as macroeconomic announcements, should be of first-order importance. We test for the difference in the effect of ambiguity aversion on the two types of risk by studying the implied concavity of preferences for a representative agent, and confirm that such concavity indeed increases significantly ahead of announcements. Except for smooth ambiguity aversion, no other representative agent model predicts such an increase.

JEL Classification: G12
Keywords: Ambiguity aversion, risk aversion, options

* pavel.savor@temple.edu, (215) 204-6117. Fox School of Business, Temple University.
† Mungo.Wilson@sbs.ox.ac.uk. Said Business School, Oxford University, and Oxford-Man Institute.
‡ martin.puhl@aon.at. Oesterreichische Nationalbank and Vienna University of Technology.
I. Introduction

Robustness to uncertainty (Knight 1921), as opposed to risk, has been a central theme of much research in economics and other fields. Ambiguity aversion can rationalize experimental evidence that is not consistent with expected utility theory. In experiments, agents reveal a strong preference for lotteries with a known distribution of payoffs over those with the same mean payoff but an unknown distribution (Ellsberg 1961). Moreover, ambiguity aversion has the potential to explain a number of asset-pricing puzzles.\footnote{For example, Collard, Sheppard, Mukerji, and Tallon (2011) argue that smooth ambiguity aversion, which we describe below, can rationalize the otherwise puzzlingly high risk aversion required to explain the equity premium puzzle. Mankiw, in his discussion of Parker (2001) implicitly argues that some form of ambiguity aversion can rationalize the puzzlingly low rates of equity participation in the USA (Parker 2001, p. 335). It is likely that other puzzles, such as the home bias puzzle of French and Poterba (1991) can also be attributed, at least to a certain extent, to aversion to uncertainty in the presence of learning.} One criticism of this research is that when risks are small, in the sense that they can be substantially reduced by updating and acting more frequently, ambiguity aversion may not be important (Skiadas 2013). A second criticism is that direct evidence showing that uncertainty matters to agents in the field, for example by affecting securities prices, is not as strong as the evidence from experiments. In this paper we argue that the first criticism offers an opportunity to address the second by focusing on differences in economic variables between small and large risks.

The intuition behind our argument is straightforward. Consider the following generic problem in economics: an agent tries to choose an action $\alpha$ to maximize the expected value of an objective function $f(\bar{Y}; \alpha, \bar{\theta})$ over some uncertain outcome $\bar{Y}$ dependent on $\alpha$ and with a probability distribution that in turn depends upon an uncertain parameter $\bar{\theta}$ with expectation $\bar{\theta}$. Assuming the problem can be well approximated by a second order Taylor expansion, the agent’s value function is then

$$V = \max_{\alpha} E \left[ f(\bar{Y}; \alpha, \bar{\theta}) \right] \approx V(\alpha^*, \bar{\theta}) + \frac{1}{2} V''(\alpha^*) \text{Var}[\bar{\theta}].$$

(1)

In a dynamic setting, an agent can sometimes adjust the action $\alpha$ periodically so that the risk borne in the time interval between each adjustment is proportional to the length of the time interval. In such cases, we show in the next section that the agent’s uncertainty...
\( \text{Var}[\theta] \) is proportional to the square of the time interval, and therefore, for sufficiently small time intervals, irrelevant to the agent’s welfare (unless \( V''_\theta \) is infinite, in which case the agent is infinitely averse to uncertainty). Our argument here is essentially the same as the one in Skiadas (2013), who first pointed this out.

However, in other cases, adjusting the action, no matter how frequently, will not materially reduce the risk borne in a given time interval by the agent, and then \( \text{Var}[\theta] \) is also proportional to the length of the time interval, and the effect of uncertainty on welfare will be first-order even for finite values of \( V''_\theta \). These cases involve events that the agent knows will occur at particular times (pre-scheduled events). Such events are different from standard Poisson jumps or diffusions (or their discrete-time analogs), and are unique to the social sciences, as, in order to matter, they require forward-looking agents.

An implication of this argument is that an impending pre-scheduled event will increase the concavity of an agent’s preferences across possible outcomes if and only if their aversion to uncertainty, which is proportional to the negative of \( V''_\theta \), is positive. Provided we can estimate the degree of concavity of an agent’s preferences over payoffs in different states, this insight provides us with a test for the existence of uncertainty or ambiguity aversion.

Savor and Wilson (2013, 2014) show that equity and bond risk premia around important scheduled macroeconomic announcements are very high compared to other ‘normal’ times. Sharpe ratios are also much higher, often ten to twenty times greater than in normal periods. This finding on its own, however, is not sufficient to show the existence of ambiguity aversion. For that, we further need to show that the concavity of preferences of a representative investor increases ahead of scheduled announcements. This approach represents a particularly sharp test of smooth ambiguity aversion because no other representative agent model predicts such an increase in concavity.

We use S&P 500 index option prices from 1996 to 2013 to infer the concavity of a representative agent’s preferences.\(^2\) We plot the resulting implied concavities in Figure 1, and do so separately for days immediately before scheduled macroeconomic announcements (blue line) and for ‘regular’ non-announcement days (black line). Each dot in the figure is a local

\(^2\)See Ait-Sahalia and Lo (2000), who show that the implied concavity of preferences for stock market investors can be derived from option prices.
estimate of concavity, with 95% confidence intervals shown as dotted lines of the same color. The main result is that there is a clear increase in implied concavity ahead of announcements over nearly the entire range of states, consistent with ambiguity aversion and inconsistent with ambiguity neutrality. The difference represents a lower bound on ambiguity aversion, since if non-announcement day risks are not all small (perhaps because investors cannot trade as rapidly as they would like), ambiguity aversion will then matter on non-announcement days as well.

[FIGURE 1 ABOUT HERE.]

Estimating one global measure of concavity, rather than many local ones, requires making further assumptions about the form of expected utility under the null of zero ambiguity aversion. Assuming for example CRRA utility, these option prices imply an increase in relative risk aversion from 8.4 to between 10.2 and 11.6 (depending on the type of announcement) ahead of scheduled announcements. A t-test for the difference in these two estimates has a p-value of at most 0.3%. In sum, these results strongly reject the null of expected utility maximization over a known distribution of stock market returns, suggesting ambiguity aversion plays an important role in determining risk premia.

A number of studies attempt to identify ambiguity aversion at work in the field of asset pricing. These broadly fall into two categories. First, some studies identify the dispersion in a set of beliefs (most frequently professional analysts’ forecasts) as a proxy for uncertainty or ambiguity about the true distribution of returns (see, e.g., Anderson, Ghysels, and Juergens 2009; Drechsler 2013; Shi 2013; Ulrich 2013; and Antoniou, Harris, and Zhang 2014). We refer to these studies as the dispersed predictions set of studies. Second, other studies estimate a structural model of ambiguity aversion on a set of asset pricing moments (see, e.g., Ju and Miao 2012; Jeong, Kim, and Park 2014; Thimme and Völkert 2014; and Gallant, Jahan-Parva, and Liu 2014). We refer to these studies as structural models studies.3

Dispersed predictions are neither a necessary nor a sufficient indicator of aggregate uncertainty about the true distribution of asset returns. Ambiguity-averse agents with the same preferences and prior beliefs who receive the same information should reach the same condi-

3Brenner and Izhakian (2011, 2012) and Izhakian and Yermack (2014) use intraday stock returns to measure the degree of ambiguity as the variance of the probability of gain or loss.
tional beliefs about the world, even though they are ambiguity averse and there exists genuine uncertainty about the true distribution. Therefore, dispersion in beliefs is not necessary for ambiguity aversion. Non-ambiguity-averse agents who receive different information can reach different beliefs, and their predictions and actions need not fully reveal these beliefs, as shown in classic studies of noisy rational expectations models such as Grossman and Stiglitz (1980) or Diamond and Verrecchia (1981). If two expected-utility-maximizing agents receive different private signals, then they cannot necessarily infer the other’s private signal from his action, and so dispersion in beliefs can persist even though no one is ambiguity-averse. Thus, heterogenous forecasts or actions cannot readily proxy for uncertainty, and are not sufficient for ambiguity aversion.

Structural models proceed by fitting an assumed model of decision-making to the data. In the case of ambiguity aversion, they fall into two groups: models which assume Gilboa and Schmeidler’s (1989) ‘maxmin’ preferences, and those which assume smooth ambiguity aversion (of which Gilboa-Schmeidler preferences are an extreme special case). The results of models which assume Gilboa-Schmeidler preferences are clearly sensitive to the minimum (worst-case) distribution allowed in the support of prior distributions. Indeed, smooth ambiguity aversion models were developed to address the obvious weaknesses in the maxmin model, which is unable to separate beliefs (priors over distributions) from preferences (aversion to ambiguity). (See Klibanoff, Marinacci, and Mukerji 2005). Models which allow smooth ambiguity aversion are more robust, but also are vulnerable to the critique in Skiadas (2013), who shows that, given high-enough frequency trading, ambiguity aversion cannot affect risk premia for small risks, except in the extreme special case of maxmin preferences.

In the rest of the paper, we present our arguments and findings in detail. Section II outlines the formal arguments; Section III explains our empirical methodology; Section IV gives our results; and Section V concludes.

II. Smooth ambiguity aversion and asset prices
Until recently, the only formal model of ambiguity aversion in decision theory was that of Gilboa and Schmeidler’s (1989) ‘maxmin’ preferences, in which agents maximize expected utility over
the most pessimistic possible prior distribution of returns.\textsuperscript{4} Gilboa-Schmeidler preferences allow agents to assume the worst, which in principle can be very bad indeed, making participation, as opposed to non-participation, in risky assets such as equities hard to explain. Furthermore, such preferences are technically difficult to work with, as well as being extremely, and probably excessively, sensitive to beliefs about unlikely states of the world. Finally, such preferences do not allow the separation of the degree of ambiguity (uncertainty) from the degree of aversion to ambiguity, and so cannot address comparative statics issues such as: how will decisions change as agents’ aversion to ambiguity changes, holding the degree of uncertainty fixed?

Klibanoff, Marinacci, and Mukerji (2005) (KMM) propose a model of preferences satisfying ‘smooth’ ambiguity aversion. Given a prior distribution over future states of the world \(s\), \(\{p_{s|\theta}\}_{s=1}^{S}\) (or a continuous-state analog \(f_{\theta}(s)\)) and given prior probabilities \(\{q_{\theta}\}_{\theta=1}^{n}\) (or a continuous analog) that each of distributions \(p_{s|\theta}\) is the true distribution, agents try to maximize expected utility \(E_{\theta}[u]\) while accounting for uncertainty about the true distribution (Knight 1921). If the agent is ambiguity averse, he puts a lower weight on distributions that imply higher expected utility. Given a positive, increasing, concave, and differentiable function \(\phi(x)\), the agent tries to maximize

\[
\sum_{\theta=1}^{n} q_{\theta} \phi(\sum_{s=1}^{S} p_{s|\theta} u(x_{s})) = E[\phi(E_{\theta}[u(W)])].
\]

subject to the usual constraints. A linear \(\phi(.)\) implies that agents are standard expected utility maximizers, using the compound distribution \(\sum_{\theta=1}^{n} q_{\theta} p_{s|\theta} = E[p_{s|\theta}] = p_{s}\). Broadly speaking, as the relative curvature of \(\phi(.)\) increases, the preferences approach maxmin.\textsuperscript{5}

In a recent critique of smooth ambiguity aversion preferences, Skiadas (2013) points out that for ‘small risks’, i.e. risks from holding securities that are proportional to the holding period, the effect of smooth ambiguity aversion on risk premia is negligible. Essentially, his argument is that while risks (as opposed to uncertainty) associated with holding securities are first-order (that is, proportional to the time interval or holding period), uncertainty about

\textsuperscript{4}For an application see Maenhout (2004).

\textsuperscript{5}Gollier (2011) provides a very helpful discussion of such preferences, and the additional assumptions required under which smooth ambiguity aversion implies that an increase in ambiguity, or ambiguity aversion, increases risk premia.
the expected returns and the magnitudes of these risks are second-order (proportional to the square of the time interval), and therefore have negligible impact on investors who can rebalance sufficiently frequently, except in the extreme case of infinite ambiguity aversion (maxmin or Gilboa-Schmeidler ambiguity aversion).

Skiadas’ critique, however, seems to offer a test for the existence of smooth ambiguity premia in stock market risk premia. Some risks, such as those posed by scheduled announcements (market-relevant news announcements with known release dates), are not small in the sense defined by Skiadas. In the case of these risks, ambiguity aversion imposes first-order differences in risk premia relative to expected utility. In consequence, the concavity of the preferences of a representative agent should appear to increase ahead of scheduled announcements that impact risk premia and to decrease afterwards.

We next present our argument formally. First, we show how the concavity of preferences can increase ahead of pre-scheduled announcements if and only if a representative investor is ambiguity averse and requires a premium for bearing uncertainty. Then we explain why it is important to distinguish between unexpected information flows (small risks, in Skiadas’ sense) and scheduled announcements, which are not small risks.

II.A. Smooth ambiguity aversion and the option-implied concavity of the utility function

Consider an agent with KMM preferences allocating wealth across future states \( s = 1 \ldots S \). The agent allocates current wealth across a complete portfolio of Arrow-Debreu securities with prices \( \pi_s \) (state prices) and demands \( x_s \). The agent’s problem is given by

\[
\max_{x_1, \ldots, x_S} \sum_{\theta=1}^{n} \theta \phi(\sum_{s=1}^{S} \pi_s(x_s))
\]

subject to

\[
s.t. \sum_{s=1}^{S} \pi_s(x_s - W_s) = 0.
\]
The first order condition for all states $s$ is then

$$u'(x_s)\sum_{\theta=1}^{n} q_{\theta} p_{s|\theta} \phi'(\sum_{s=1}^{S} p_{s|\theta} u(x_s)) = \kappa \pi_s,$$

(4)

where $\kappa$ is the Lagrange multiplier associated with the budget constraint. In equilibrium, markets clear and $x_s = W_s$.

Gollier (2011) gives the following expression for state prices $\pi_s$:

$$\pi_s = \hat{p}_s u'(W_s),$$

(5)

where

$$\hat{p}_s = \sum_{\theta=1}^{n} w_\theta q_{\theta} p_{s|\theta}$$

and

$$w_\theta = \frac{\phi'(\sum_{s=1}^{S} p_{s|\theta} u(W_s))}{\sum_{\theta=1}^{n} q_{\theta} \phi'(\sum_{s=1}^{S} p_{s|\theta} u(W_s))}.$$

Because $\phi(.)$ is concave, distributions that imply higher expected utility (broadly speaking a lower $p_{s|\theta}$ for bad states, higher for good states) receive a lower $\phi'(.)$ than more pessimistic distributions, and so receive a lower weight $w_\theta$.

Note that $\sum_{s=1}^{S} \hat{p}_s = 1$. Under the null hypothesis of ambiguity neutrality, $\phi'$ equals a constant, and then

$$\hat{p}_s = \sum_{\theta=1}^{n} q_{\theta} p_{s|\theta} = p_s,$$

(6)

where $p_s$ is just the unconditional expectation over all priors of the state probability. In our empirical work, we equate this to the physical probability (or density in a continuous-state model). This is permissible under the null, where the agent acts as if he knows the true probability distribution of payoffs. When $\phi(.)$ is strictly concave, distributions that imply higher expected utility than some ‘middle’ distribution (not defined strictly) will receive $\hat{p}_s < p_s$ while those distributions which imply lower expected utility receive $\hat{p}_s > p_s$. 
The ratio of risk-neutral and physical state probabilities is then
\[
\frac{\pi_s}{p_s} = \lambda(s) = \frac{\hat{p}_s}{p_s} u'(W_s).
\] (7)

If the states are ordered from low to high \(W_s\), we can write the derivative of state prices with respect to wealth (this is just a change of index from \(s\) to \(W_s\) with \(p_s = p(W_s)\) etc.) as
\[
\frac{d}{dW} \lambda(W_s) = u''(W_s) \frac{\hat{p}_s}{p_s} + u'(W_s) \frac{d}{dW} \left( \frac{\hat{p}_s}{p_s} \right).
\] (8)

and then a concavity index \(CI(s)\) can be computed as follows:
\[
CI(s) = -\frac{W_s u''(W_s)}{\lambda(W_s)} + W_s u'(W_s) \frac{d}{dW} \left( \frac{\hat{p}_s}{p_s} \right) - W_s \frac{d}{dW} \ln \left( \frac{\hat{p}_s}{p_s} \right).
\] (9)

The first term is just relative risk aversion, while the second term depends on the distortion induced to state probabilities by ambiguity aversion \(\phi(.)\). Skiadas’ point implies that for small risks (risks that in the limit of continuous time are either Brownian motions or Poisson jumps) this term will be negligible. However, the same argument implies that for non-small risks (e.g., scheduled announcements of news that will affect stock market wealth), the second term will be positive if and only if \(\phi(.)\) is concave. Therefore, an increase in our concavity index ahead of an important scheduled announcement (and decrease thereafter) represents evidence for an ambiguity premium.

For discrete-state probabilities the derivative with respect to state is not well-defined. The solution is either to go to continuous-state probabilities or else to consider something which in the limit converges to this. Thus, we can think of this derivative as
\[
p'_{s|\theta} = \lim_{\Delta s \to 0} \frac{p_{s+\Delta s} - p_s}{\Delta s}.
\] (10)

or the equivalent left-hand side (LHS) derivative, and which is approximated by the ratio on the right-hand side (RHS) for small \(\Delta s\).
The concavity index is then

\[
CI(s) = \gamma(s) + W_s \left( \frac{1}{p_s} p'_s - \frac{1}{\bar{p}_s} \bar{p}'_s \right)
\]

\[
= \gamma(s) + W_s \left( \frac{1}{p_s} \sum_{\theta=1}^{n} q_{\theta} p'_{s|\theta} - \frac{1}{p_s} \sum_{\theta=1}^{n} w_{\theta} q_{\theta} p'_{s|\theta} + w'_{\theta} q_{\theta} p_{s|\theta} \right) \tag{11}
\]

But the weights \( w_\theta \) are independent of \( s \) so

\[
CI(s) = \gamma(s) + W_s \left( \frac{1}{p_s} \sum_{\theta=1}^{n} q_{\theta} p'_{s|\theta} - \frac{1}{p_s} \sum_{\theta=1}^{n} w_{\theta} q_{\theta} p'_{s|\theta} \right)
\]

\[
= \gamma(s) + W_s \left( \frac{1}{p_s} \sum_{\theta=1}^{n} q_{\theta} p'_{s|\theta} - \frac{1}{p_s} \sum_{\theta=1}^{n} p_s w_{\theta} q_{\theta} p'_{s|\theta} \right)
\]

\[
= \gamma(s) + \left( \frac{W_s}{p_s} \sum_{\theta=1}^{n} q_{\theta} \left( 1 - \frac{p_s}{p_s} w_{\theta} \right) p'_{s|\theta} \right) \tag{12}
\]

When \( \phi(.) \) is strictly concave, distributions that imply low expected utility have a high weight \( w_\theta \) and \( p_s > 1 \), while those that imply high expected utility have a low weight and \( p_s < 1 \), in consequence the summation term will be positive, and our concavity indicator will be higher than \( \gamma(s) \) because of an ambiguity premium.

Gollier (2011) points out that the prior distributions can be such as to have more ambiguity aversion behave in a perverse manner if the distributions are completely general. He provides fairly weak sufficient conditions under which this will not occur, which in our case are really conditions on the derivatives \( p'_{s|\theta} \). Essentially, a sufficient condition under which greater ambiguity or ambiguity aversion will increase concavity are that ‘worse’ distributions, which imply lower expected utility, are first- or second-order stochastically dominated by ‘better’ distributions. In practical applications these conditions will almost always hold. For example, if we assume agents are uncertain about the mean return, and have a support of prior distributions of increasing mean, higher-mean distributions dominate lower-mean distributions in the required way.\(^6\)

\(^6\)See Gollier for details.
II.B. Large versus small risks

The whole argument thus far depends on the weighting function

\[ w_\theta = \frac{\phi'(\sum_{s=1}^{S} p_{s|\theta} u(W_s))}{\sum_{\theta=1}^{n} q_{\theta} \phi'(\sum_{s=1}^{S} p_{s|\theta} u(W_s))} = \frac{\phi'(E_\theta[u(W_\theta)])}{E[\phi'(E_\theta[u(W_\theta)])]}, \tag{13} \]

where \( E'[\cdot] \) denotes the expectation of utility under the assumption that payoffs are distributed according to the probabilities \( p_{s|\theta} \) and \( E[\cdot] \) denotes the expectation over all distributions weighting each one by its prior probability \( q_\theta \). Over sufficiently small time intervals \( \Delta t \), absent a pre-scheduled announcement, the expected utility \( E_\theta[u(W_\theta)] \), as shown by Skiadas, is proportional to \( \Delta t \) and therefore we can write

\[ E_\theta[u(W_\theta)] = b_\theta \Delta t + c_\theta A_{t\in\Delta t} + o(\Delta t), \tag{14} \]

where \( b_\theta \), and \( c_\theta \) are constants and \( A_{t\in\Delta t} \) is an indicator variable that equals one if there is an important scheduled announcement at a date \( t \) in the time interval \( \Delta t \) and zero otherwise. For simplicity in notation, let \( b = E[b_\theta], c = E[c_\theta] \).

When there is no pre-scheduled announcement, we have

\[ \phi'(E_\theta[u(W_\theta)]) \approx \phi'(b\Delta t) + 0.5\phi''(b\Delta t)(b_\theta - b)\Delta t + o(\Delta t) \tag{15} \]

Therefore

\[ E[\phi'(E_\theta[u(W_\theta)])] = E[\phi'(b\Delta t) + 0.5\phi''(b\Delta t)(b_\theta - b)\Delta t + o(\Delta t)] \tag{16} \]

\[ = E[\phi'(b\Delta t)] \]
\[ \hat{p}_s = \sum_{\theta=1}^n w_\theta q_\theta p_{s|\theta} \]
\[ = \frac{1}{E[\phi'(E_\theta[u(W_\theta)])]} E[\phi'(E_\theta[u(W_\theta)])p_{s|\theta}] \]
\[ = \frac{1}{E[\phi'(b\Delta t)]} E[(\phi'(b\Delta t) + 0.5\phi''(b\Delta t)(b_\theta - b)\Delta t + o(\Delta t))p_{s|\theta}] \]
\[ = \frac{1}{E[\phi'(b\Delta t)]} [E[p_{s|\theta}(\phi'(b\Delta t))] + 0.5\phi''(b\Delta t)E[(b_\theta - b)\Delta tp_{s|\theta}] + E[o(\Delta t))p_{s|\theta}]] \]
\[ = p_s + 0.5 \frac{\phi''(b\Delta t)}{E[\phi'(b\Delta t)]} E[(b_\theta - b)p_{s|\theta}]\Delta t + o(\Delta t) \]

The trick here is to notice that the state probabilities \( p_{s|\theta} \) are themselves of the order \( \Delta t \).

Given a state \( s \), (with, say, a payoff of 1) halving the time interval roughly halves the probability of that state occurring in the smaller time interval for a small risk. (This argument can be made precise, and is essentially the argument used to show how a binomial distribution converges to a normal distribution when you allow the time interval to go from 1 to infinitely small and the number of states to increase. See Skiadas (2013) for details.) Thus, the term \( E[(b_\theta - b)p_{s|\theta}]\Delta t \) is also of order \( \Delta t^2 \) and is negligible provided agents can rebalance their portfolios sufficiently frequently. But in that case

\[ \hat{p}_s = p_s \]

in which case

\[ CI(s) = \gamma(s). \]

For scheduled announcements, there is a ‘minimum’ time interval in which \( p_{s|\theta} \) is not of the same order as the time interval, and trading more frequently will not reduce the risk by any material amount (shown below). In that case, the term \( E[(b_\theta - E[b_\theta])p_{s|\theta}]\Delta t \) is of order \( \Delta t \), and then the second term in the concavity index will be positive provided agents actually are
ambiguity averse.

\[
CT^A - CT^N = \left( -\frac{W_s \frac{d}{dx} \lambda(W_s)}{\lambda(W_s)} \right)^A - \left( -\frac{W_s \frac{d}{dx} \lambda(W_s)}{\lambda(W_s)} \right)^N
\]

\[
= \frac{W_s}{p_s} E \left[ \left( 1 - \frac{p_s}{p_s} w_\theta \right) p'_s|\theta| A_t \in \Delta t = 1 \right]
\]

\[
= \frac{W_s}{p_s} E \left[ \left( 1 - \frac{p_s + 0.5 \phi''(b\delta t)}{p_s \phi'(b\delta t)} E[(b_\theta - b)p_s|\theta] \delta\Delta t \right) \times \left( 1 + 0.5 \phi''(b\delta t)(b_\theta - b) \delta\Delta t \right) \right] E[p_s|\theta] A_t \in \Delta t = 1
\]

Since we observe the LHS, \( W_s \) and \( p_s \), we can, for a specified set of prior distributions \( \{p_s|\theta\}_{s=1}^S \) and a pre-specified functional form for \( \phi(\cdot) \), determine how large the ambiguity premium \( \frac{\phi''(b\delta t)}{E[\phi'(b\delta t)]} E[(b_\theta - b)p_s|\theta] \) has to be to explain our results.

II.B.1. Announcement versus non-announcement periods

We next justify the claim that important scheduled announcements do not constitute a small risk. We give a simple example, which amply illustrates the general principle that announcements represent non-small risks.

**Non-announcement periods**

Assume that for non-announcement periods there is a single risky asset with initial continuous payoff \( X_t \) whose future payoff is given by

\[
X_{t+\Delta t} = X_t \exp \left\{ (\tilde{\theta} \Delta t + \tilde{\varepsilon} \sigma_{\tilde{\varepsilon}} \sqrt{\Delta t}) \right\},
\]

where

\[
\tilde{\varepsilon} \sim N(0, 1).
\]

The variance \( \sigma_{\tilde{\varepsilon}}^2 \) is assumed to be common knowledge, while prior beliefs about mean log growth in the payoff are uncertain, with mean \( \mu \Delta t \) and normally distributed around this mean.

Given a prior belief \( \tilde{\theta} \),

\[
E[\Delta \ln P_{t+\Delta t}] = \tilde{\theta} \Delta t
\]
and

$$E_\theta[\Delta \ln X_{t+\Delta t} - \tilde{\theta} \Delta t]^2 = \sigma^2_e \Delta t. \quad (21)$$

Thus, for a given prior, risk (i.e., the variance of return) is first order, as assumed. But,

$$E[\Delta \ln X_{t+\Delta t}] = \mu \Delta t, \quad (22)$$

while accounting for the uncertainty about the mean gives the overall mean square error:

$$E[(\Delta \ln X_{t+\Delta t} - \mu \Delta t)^2] = E[(\tilde{\theta} \Delta t + \varepsilon \sigma \sqrt{\Delta t} - \mu \Delta t)^2] = \sigma^2_e \Delta t + E[(\tilde{\theta} - \mu)^2] \Delta t^2. \quad (23)$$

Therefore, the uncertainty about the mean adds a term to the mean square error of order \( \Delta t^2 \) as claimed. We can then write:

$$\tilde{\theta} \Delta t \sim N(\mu \Delta t, \sigma^2_e \Delta t^2). \quad (24)$$

We can use an exactly analogous argument to show that any additional uncertainty about the variance \( \sigma^2_e \) is also o(\( \Delta t \)).

The distribution assumed for payoffs arises naturally from time-aggregation of a geometric Brownian motion in continuous time. However, adding random (Poisson) jumps to the underlying continuous time model in fact does nothing to alter the relative unimportance of uncertainty about the distribution of returns. This is one of the central points of Skiadas (2013), so we do not pursue it here. If we think of our example as the discrete-time aggregation of a continuous-time Brownian motion as the underlying continuous-time model, all of our central results are robust to inclusion of a Poisson jump in that underlying continuous-time model. Therefore, for simplicity of exposition, we ignore Poisson jumps from now on. Skiadas (2013) provides all the additional proofs required.

**Announcement periods**

Around announcements, define an interval of a fixed length \([t_1, t_2]\) which contains an announcement ‘date’ (an exact time) \(t_A\). The date \(t_A\) is assumed to be common knowledge to all relevant participants. Suppose announcement news about log growth in a random payoff is drawn from
the same distribution (i.e. there is a jump with a deterministic arrival time which is drawn from the uncertain distribution)

\[
\Delta \ln X_A \sim N(k\tilde{\theta}, k\sigma^2_{X,A}), \tag{25}
\]

where \(k\) is an arbitrary constant used to scale the expected magnitude of the jump relative to the interval \(\Delta t = t_2 - t_1 = 1\). That is, given volatility of \(\sigma_\varepsilon\) of say daily payoff growth in a non-announcement period, the jump moves the payoff by an average of \(k\tilde{\theta}\) with a variance of \(k\sigma^2_{X,A}\). This allows the Sharpe ratio of the jump to vary relative to the Sharpe ratio of non-announcement periods. Now further suppose the interval \(t_2 - t_1\) is such that relative to this interval, \(k = 1\), without loss of generality.

Consider a trader who divides the interval into three equal sized intervals, so that \(\Delta t = (t_2 - t_1)/3 = 1/3\) and rebalances his portfolio three times, instead of just once, before \(t_2\). Again, without loss of generality let \(t_A\) be in the middle of these three periods, \(t_A \in [t_1 + (1/3)(t_2 - t_1), t_1 + (2/3)(t_2 - t_1)]\). There is then no announcement risk in the first and third periods, so variance of payoff growth is \(\sigma^2_\varepsilon/3\) and expected payoff growth is \(\tilde{\theta}/3\). In the middle period, however, the variance of payoff growth is affected by the presence of the announcement, and so is equal to \((\sigma^2_\varepsilon/3) + \sigma^2_{X,A}\), while expected growth is \((4/3)\tilde{\theta}\).

If he instead divides the interval into \(m\) subintervals, each subinterval that does not contain the announcement has the distribution of log change in payoff \(N(\tilde{\theta}/m, \sigma^2_\varepsilon/m)\). The subinterval containing the announcement, however, is drawn from the distribution \(N(((m+1)/m)\tilde{\theta}, (\sigma^2_\varepsilon/m) + \sigma^2_{X,A})\).

In one of these subintervals not containing the announcement, the expected squared error of the unconditional forecast of payoff growth (which accounts for uncertainty about the mean) is then

\[
E[(\tilde{\theta}/m + \tilde{\varepsilon}_{t_1+m} - \mu/m)^2] = (\sigma^2_\varepsilon/m) + (\sigma^2_{\tilde{\theta}}/m^2), \tag{26}
\]

while in the interval containing the announcement, the mean squared error is

\[
E[((m+1)/m)\tilde{\theta} + (\tilde{P}_{At_A} - \tilde{\theta}) + \tilde{\varepsilon}_{c,t_A} - ((m+1)/m)\mu)^2] = (\sigma^2_\varepsilon/m) + \sigma^2_{X,A} + ((m+1)/m)^2\sigma^2_{\tilde{\theta}}. \tag{27}
\]
(Here the variable $\tilde{A}_{t,A}$ is the pure effect of the announcement on the payoff.)

Thus, for small enough subintervals around the announcement, the uncertainty about the mean $\tilde{\theta}$ will be the same order as the variance of log price growth. For investors who can rebalance their portfolios sufficiently often, non-announcement uncertainty is second order relative to risk, whereas uncertainty about the outcome of a scheduled announcement is first-order.

It is not necessary to observe variables over these very small subintervals, however. Given a division of $[t_1, t_2]$ into $m$ subintervals, consider a ‘ball’ of $a$ subintervals including the subinterval containing the announcement. Then compared to the previous ball or the subsequent ball of $a$ subintervals, the announcement ball has mean squared error

$$\frac{a}{m} \sigma^2_{\tilde{\varepsilon}} + \sigma^2_{X,A} + \left(1 + \frac{a + 2m}{m^2}\right) \sigma^2_{\tilde{\theta}}$$

versus

$$\frac{a}{m} \sigma^2_{\tilde{\varepsilon}} + \frac{a}{m^2} \sigma^2_{\tilde{\theta}}$$

for non-announcement subintervals, with the difference

$$\sigma^2_{\varepsilon,A} + \left(1 + \frac{2}{m}\right) \sigma^2_{\tilde{\theta}}.$$

This difference is independent of $a$, the size of the ball relative to the size of the subinterval containing the announcement. Therefore, even if the jump is fully contained by, for example, a 1-minute interval, the difference in the importance of uncertainty for daily payoff growth will be first order for days containing announcements versus other days.

Letting $m$ become large, the term $\sigma^2_{\tilde{\theta}}$, which measures uncertainty about the mean expected payoff growth, only appears in the expression for concavity for pre-announcement periods, and is $o(\Delta t)$ for post- or non-announcement periods. Thus, concavity should be higher ahead of important scheduled announcements than immediately after or than in normal (i.e. non-announcement) periods.
III. Empirical procedure and results

III.A. Data

Our study uses S&P 500 Equity Index options traded on the Chicago Board Options Exchange (CBOE). The S&P 500 is one of the most actively-traded indexes in the world and represents a good proxy for the aggregate US stock market. S&P 500 Index options are European, cash-settled options and are available for a wide array of expiration dates and a dense grid of strike prices.

S&P 500 Index options have three maturity types. The first and predominant type has monthly maturity intervals. The expiration date is always set on the Saturday after the third Friday in the respective month. The second type also offers monthly maturities, but uses the last trading day of the month as the expiration date. The third type offers weekly maturities with expiration dates occurring every Friday. The two latter types were introduced in 2007 and have lower trading volumes. Our study focuses on the first type, since those options have the most trading activity and exhibit most liquidity. Maturities up to 12 months are available on a monthly basis. Longer maturities are offered at lower frequencies going up to five years.

Our sample covers the period starting in January 1996 and ending in August 2013, over which we collect 4,077,580 bid and ask prices. The average daily trading volume during this time period was about 300,000 contracts, and the most active contracts are typically those with shortest maturities. We obtain option prices, required data on the underlying, dividend yields, and risk-free rates for different maturities from Optionmetrics.

We exclude from our analysis options that have a bid price below $0.5, as these usually have prohibitively high spreads, low volume, and low informational value. We also exclude sets for a given day that contain fewer than eight available strike values for puts and calls, as well as deep-in-the-money options, since their trading volumes are very low. We replace the removed data points for deep in-the-money put options with the corresponding values for out-of-the-money calls using put-call parity, and vice versa for deep in-the-money calls. We further exclude options where implied volatility cannot be calculated, which typically happens because of violations of no-arbitrage conditions. Option contracts with maturities of fewer than eight days often show erratic behavior and consequently we also omit these contracts. Finally,
due to data availability reasons, we remove options with maturities above three years. Since
option trades do not occur continuously for all maturities and strikes and to ensure that prices
have information content, on a given day we study only options that have a positive trading
volume.

**[FIGURE 2 ABOUT HERE.]**

Figure 2 plots the average number of contracts we include in our sample for a given maturity.
Clearly the majority of available data are concentrated in maturities of up to three months.
Therefore, we focus our analysis on that maturity interval.

### III.B. Methodology

A set of options with a common expiration date represents a snapshot of aggregated market
beliefs, which allows us to extract the risk neutral density (RND) for that maturity. We can
then apply the idea in Ait-Sahalia and Lo (2000) to infer the concavity of preferences (relative
risk aversion, under the null of zero ambiguity aversion) from RNDs (together with the physical
probability density). The approach we adopt follows the methodology in Figlewski (2008) and
also incorporates ideas by Breeden and Litzenberger (1978), Banz and Miller (1978), Shimko
(1993), and Bliss and Panigirtzoglou (2004).

#### III.B.1. Interpolation and smoothing

Before we can extract an RND that is suitable for further analysis, we clean the data to remove
possible inconsistencies and eliminate extreme outliers with very low volume and high spreads,
so as to prevent our estimated RNDs from displaying theoretically impossible features such as
negative density values. Many direct smoothing approaches lead to erratic fluctuations near the
at-the-money strikes. To address this issue, Shimko (1993) proposes an algorithm that converts
option prices into implied volatilities based on the Black-Scholes model. The implied volatilities
are then smoothed and interpolated, before prices for a dense set of equally-spaced strike prices
are re-converted back to prices as a basis for the extraction of the RNDs. Shimko (1993)
stresses that the transformation does not assume the correctness of the Black-Scholes model,
but merely uses the Black-Scholes formula as an algorithm that allows researchers to smooth
and interpolate the given option set in a space that, though being theoretically inconsistent, preserves the volatility structure and information content comparably well. In this study, we try to use as little smoothing and adjustment of the data as possible. However, in order to aggregate the data by degrees of moneyness, we need comparable data points that do not necessarily correspond to available strike levels and thus require the use of interpolation.

We evaluated several smoothing methods. Higher order polynomials already provide a high degree of flexibility and are able to fit a large part of the volatility surface. However splines, as for example those used by Figlewski (2008), allow an even more accurate replication of the features of the curves. Cubic and fourth order splines run mandatory through all \( k \) data points and fit the resulting curve by \( k - 1 \) aggregated polynomials of third (or fourth) order. Apart from the function value itself, all derivatives up to the second (third) derivative are aligned at the transition points between the polynomials. The \( 6^{th} \) order polynomial imposes its curvature on the data points and leads partly to comparably large deviations. The cubic spline on the other hand has to run through all data points and also incorporates outliers that should be removed in the fit. In order to address these weaknesses, we also test smoothing splines, which allow the curve to not exactly go through all data points. The amount of acceptable deviation is defined by a smoothness parameter as proposed by Bliss (2002). Nevertheless, outliers in one area can still negatively affect the whole fitted curve.

In this paper, we thus employ B-splines to reduce drastic outliers. B-splines are a generalization of Bezier curves and are closely related to smoothing splines. They also do not require that the curve runs through all data points. A set of knot points is chosen and these act as transition points between areas in which the curve has different characteristics. In between those knot points, a polynomial function is fitted with a least square algorithm. To minimize the impact on the information content, we calibrate the process to employ a dense set of knots and tolerate only small deviations from the actual data points. We employ this smoothing technique at the level of the extraction of the risk neutral distribution and smooth the curve before it is transformed into the risk-neutral probability density (RND). As this method already enables us to extract suitable RND estimates, we refrain from using smoothing in volatility space in our results. We only use the method suggested by Shimko as a comparison method. Figure 3
provides an illustration of the impact of the smoothing method we employ.

[FIGURE 3 ABOUT HERE.]

III.B.2. Risk-neutral probability density function extraction

Figlewski (2008), following Breeden and Litzenberger (1978), exploits the fact that the RND function \( \pi(x) \) allows us, assuming risk neutrality, to express the value of a security as the value of its expected payoff at maturity \( T \). For a call option, this is the probability-weighted difference between the stock price \( S_T \) and the strike \( K \) at maturity \( T \):

\[
C = \int_{K}^{\infty} e^{-rT} (S_T - K) \pi(S_T) dS_T. \quad (31)
\]

\( C \) represents the call price, \( P \) the put price, \( S_t \) the price of the underlying asset at time \( t \), \( K \) the strike price, \( r \) the risk-free rate, and \( T \) the maturity. Correspondingly, we can write the price of a put option as:

\[
P = \int_{0}^{K} e^{-rT} (K - S_T) \pi(S_T) dS_T. \quad (32)
\]

As shown in Figlewski (2008), it is possible to extract \( \pi(x) \) and \( \Pi(x) \) (the risk neutral distribution function \( \pi(x) = \int_{-\infty}^{x} \pi(z)dz \)) from the partial derivatives of option prices with respect to \( K \).

\[
\frac{\partial C}{\partial K} = -e^{-rT} \int_{K}^{\infty} \pi(S_T) dS_T = -e^{-rT} [1 - \Pi(K)] \quad (33)
\]

Solving for \( \Pi(K) \) and taking the second derivative yields:

\[
\Pi(K) = e^{rT} \frac{\partial C}{\partial K} + 1 \quad \text{and} \quad \pi(K) = e^{rT} \frac{\partial^2 C}{\partial K^2} \quad (34a,b)
\]

For put options, the analogous expressions are given by:

\[
\Pi(K) = e^{rT} \frac{\partial P}{\partial K} \quad \text{and} \quad \pi(K) = e^{rT} \frac{\partial^2 P}{\partial K^2} \quad (35a,b)
\]

We show the typical shape of the risk neutral distribution function \( \Pi(x) \) and the RND function \( \pi(x) \) in Figures 4 and 5. The risk neutral distribution function runs S-shaped from 0
to 1. It resembles a distorted cumulative normal distribution function. The RND (see Figure 5) resembles a normal distribution. It is however typically leptokurtic and skewed.

[FIGURES 4 AND 5 ABOUT HERE.]

Given a set of options with a common maturity spanning all realistic strike prices $K$, it is possible to estimate the RND and risk-neutral distribution function using numerical derivatives (see Equations 33a and b).\textsuperscript{7} The denser the grid of strike prices that is provided by the option set and the smaller the bid/ask-spreads, the more accurate the extracted functions.

III.B.3. Option-implied concavity of preferences

Given our estimates of the RND and physical densities $p(S_T)$, and letting $\lambda(S_T) = \pi(S_T)/p(S_T)$, our concavity index can be estimated using equation (9) where

$$C.I.(S_T) = -\frac{S_T \frac{d}{d S_T} \tilde{\lambda}(S_T)|_{S_T}}{\tilde{\lambda}(S_T)}$$

(36)

IV. Results: Changes in implied concavity for large risks

Savor and Wilson (2013) show that asset prices behave differently on days of pre-scheduled macroeconomic announcements about inflation, unemployment, and interest rates. In this section, we estimate the effect of such announcements on the physical probability densities of equity indices and on the option-implied risk-neutral probability densities. We then show that option-implied concavity on days with macroeconomic announcements is higher than concavity for regular non-announcement (non-a) days.

As in Savor and Wilson (2013), we consider announcements on the Producers Price Index (PPI), unemployment (Emp), and Federal Open Markets Committee (FOMC) interest rate decisions. Our sample includes 761 announcements in the period starting in January 1996 and ending in August 2013. We use the S&P 500 equity index and its index options as a proxy for the US equity market.

\textsuperscript{7}The given equations assume the strike price grid to be equally spaced. They can, however, be easily adapted for varying strike distances.
IV.A. Physical probability density

Figure 6 plots the physical probability density of S&P 500 returns, aggregated over the sample period from 1996 to 2013. We use daily returns and compare announcement days to ‘regular’ days without announcements. Daily returns on non-announcement days are clearly concentrated in a narrow area above at-the-money. Returns strongly deviating from this area are rare and the observed physical probability density falls away very quickly for rising or falling moneyness.

[FIGURE 6 ABOUT HERE.]

The behavior of the physical density is different on days with announcements, with lower clustering around the center of the distribution and much higher mass on higher-return outcomes, as well as somewhat higher mass on lower-return outcomes. (In fact, although not shown here, the distribution of FOMC-day market returns first-order stochastically dominates that of non-announcement day returns). Results are similar for all a-days together.

IV.B. Implied risk-neutral probability density

Option markets offer insights into the aggregated beliefs of market participants. In particular, risk-neutral probability densities (RND) extracted from liquid option prices offer a sensitive instrument to visualize market expectations. Figure 7 shows the reaction of the market to an especially dramatic announcement. On September 29, 2008, the U.S. House of Representatives voted against the Emergency Economic Stabilization Act of 2008, commonly referred to as a bailout of the financial system, putting the planned liquidity infusion in jeopardy, with a predictably negative effect on the markets. On the next day, the Senate approved the bailout as an amendment to an already existing piece of legislation and, as a result, managed to calm the markets. Figure 7 presents the implied RND on the Friday preceding the House vote, on the day of the vote, and the day thereafter. On the day of the vote the RND drastically widened, illustrating the increased uncertainty. On the next day, after the bailout passed the Senate, the RND went back to its prior shape. Only a small residue of increased uncertainty is visible in the form of a decreased maximum and a wider distribution relative to September 29.

[FIGURE 7 ABOUT HERE.]
Physical and implied probability distributions can differ considerably. Figure 8 compares the physical probability distribution to the average implied RND. The implied curves, aggregated from 1996 to 2013, are distinctly wider and show that the market incorporates a higher degree of uncertainty in its expectation than historically observed, which is the well-known variance risk premium.

[FIGURE 8 ABOUT HERE.]

We now examine the effect of announcement days on the implied RNDs. In Figure 9, we show the average RND on non-announcement days, implied by options with maturities up to three months and an average maturity of about 44 days. We compare this RND to announcement-day RNDs, separately for PPI, Employment, and FOMC, as well as the aggregate of all three. Once more, we document an increase in the width of the curve and more uncertainty reflected in the option market.

[FIGURE 9 ABOUT HERE.]

Table 1 provides statistical analysis of the effect of announcement days. We perform a simple t-test, a Kolmogorov-Smirnov test, and a Mann-Whitney test. The t-test examines whether the mean of the non-announcement distribution is different than the mean on announcement days. The Kolmogorov-Smirnov test compares two RNDs and analyses whether the one on non-announcement days is stochastically larger than the one on announcement days. The Mann-Whitney test determines whether the two distributions differ by a positive location shift.

[TABLE 1 ABOUT HERE.]

We conclude that there is significant impact of announcements on the market. p-values for all three tests are quite low, and indicate that the RND on announcement days is distinctly different than on regular non-announcement days. In particular, announcements lead to a measurable reduction of the RND’s maximum and to its widening.

IV.C. Implied concavity

In this section we investigate the effect of announcement days on the implied concavity of preferences of a representative investor in the market. As described in Subsection II.A, we derive the concavity of preferences from the pricing kernel $\lambda(W_s) = \pi_s/p_s$ where $\pi_s$ is the
implied RND and $p_s$ is the physical probability density. We can then estimate the implied concavity as follows:

$$CI(s) = -\frac{W_s \frac{d}{dt} \lambda(W_s)}{\lambda(W_s)} \quad (37)$$

Figure 10 shows the average pricing kernel based on the extracted RND and the observed physical density used in this study. It is U-shaped around a minimum around the at-the-money level. As discussed above, we transform the pricing kernel to obtain an estimate of the implied concavity. Note that the time horizon used for the estimation is based on options with a mean maturity of 44 days or about 6 weeks. Consequently a 1% variation in moneyness corresponds to a significantly larger annualized price fluctuation.

[FIGURE 10 ABOUT HERE.]

In order to show that market implied concavity differs significantly for days with and without announcements, we investigate each announcement type separately and compare the respective implied concavity to non-announcement days. We furthermore conduct t-tests, Kolmogorov-Smirnov tests, and Mann-Whitney tests to show that differences are statistically significant. Figures 11 through 13 show the concavity index values for FOMC, PPI, and Employment announcement days, respectively. (Figure 1, which we discussed above, plots the concavity index values for all three announcement types together.) We also always present the concavity estimate for non-announcement days (i.e., days other than PPI, Employment, or FOMC days).

[FIGURES 11-13 ABOUT HERE.]

As expected, we find a clearly visible increase in our concavity index ahead of scheduled announcements. On days of FOMC or PPI announcements, the concavity index (CI) increases on average by 17%. The increase is 4-7% lower for Employment announcements. These increases are distinctly visible only on announcement days. On post-announcement days, the implied concavity reverts back to its normal levels.

The graphs depict average concavity values and are based on an average maturity of 44 days. We provide statistical analysis in Table 2, where we aggregate concavity estimates over a predefined moneyness area, spanning the range from 0.98 to 1.02, to form one concavity value per day and maturity. We then compare the distributions of those aggregated concavity values
on announcement days to the respective values of the distributions on non-announcement days. We especially focus on the area around at-the-money, as it provides highest data quality and the largest amount of available data.

All three announcement types separately, as well as all announcements together, result in an increase in implied concavity. For FOMC announcements, PPI announcements, and all announcements, p-values for the t-test are below 5%, and for the Mann-Whitney test between 5 and 6%. FOMC announcements also reach p-values of 5% for the very demanding Kolmogorov-Smirnov test and generally show the highest level of statistical significance. For PPI and all announcements, we observe KS p-values of 11.5 % and 19.6 %, respectively. For employment announcements, our results are not statistically significant (and the in-the-money options typically show a decrease rather than an increase in concavity).

In summary, with the exception of employment announcements, which we include for completeness, these results provide strong evidence for an increase in concavity around announcement days and therefore for the existence of a significant ambiguity premium.

[TABLE 2 ABOUT HERE.]

V. Conclusion

In this paper we argue that a distinction between large risks (risks that cannot be materially reduced by trading more frequently) and small risks (risks that can be so reduced) allows us to identify evidence for smooth ambiguity aversion in stock market data. Our argument proceeds by taking Skiadas’ (2013) critique of smooth ambiguity aversion seriously, and showing when this critique does not apply. In particular, we show that only in the presence of ambiguity aversion should agents’ preferences become more concave ahead of pre-scheduled macroeconomic announcements.

We then demonstrate how to estimate this concavity for a representative agent by combining the idea in Ait-Sahalia and Lo (2000) with more recently developed techniques for extracting RNDs. Our resulting empirical findings provide strong evidence for an increase in estimated concavity ahead of macroeconomic announcements and thus for the existence of an ambiguity risk premium.
Of course, there are alternative hypotheses that can explain our results. Perhaps the most obvious one is that a representative agent simply becomes more risk-averse ahead of announcements. On its face, this idea seems very ad hoc, but if heterogenous agents reallocate risks ahead of announcements, then it is conceivable that the risk aversion of the marginal holder of the market might be higher. Such a model would be much more complicated than our simple, tractable model of a representative agent with smooth ambiguity aversion, and to our knowledge no such model has ever been developed.

A second possibility is that our estimates of physical probability are subject to small sample bias. However, this is more likely to be a problem in the tails of the distribution, as opposed to the center, and our results actually are the strongest for the center of the distribution.
References


Figure 1: Implied concavity on non-announcement vs. all announcement days, 1996-2013.
Figure 2: Data availability over option maturity.
Figure 3: Raw vs. smoothed distribution function.
Figure 5: Risk-neutral probability density function (RND). Example taken from the S&P 500 on September 29, 2006. S&P 500 level equals 1335.85.
Figure 6: Physical probability density of S&P 500 returns over a 1-day horizon, 1996-2013.
Figure 7: Risk-neutral probability density (RND) before, on, and after September 29, 2008.
Figure 8: Physical probability density vs. implied RND, 1996-2013.
Figure 9: Implied risk-neutral probability densities (RND) for different types of days, 1996-2013.
Figure 10: Average pricing kernel over moneyness, 1996-2013.
Figure 11: Implied concavity on non-announcement vs. FOMC days, 1996-2013.
Figure 12: Implied concavity on non-announcement vs. PPI days, 1996-2013.
Figure 13: Implied concavity on non-announcement vs. employment days, 1996-2013.
Table 1
Risk-Neutral Density

This table presents the mean and standard deviation of the implied risk-neutral density at its maximum on announcement and non-announcement days. The announcement days include days when the FOMC is releasing its decision (FOMC), when PPI numbers are released (PPI), and when employment numbers are released (Emp). The last three columns show p-values of a t-test, Kolmogorov-Smirnov (KS) test, and Mann-Whitney (MW) test for the difference between announcement and non-announcement days.

<table>
<thead>
<tr>
<th>Type</th>
<th>Mean</th>
<th>Std. Dev</th>
<th>t-test</th>
<th>KS</th>
<th>MW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Ann.</td>
<td>8.2</td>
<td>2.3</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>FOMC</td>
<td>7.7</td>
<td>1.8</td>
<td>2.3%</td>
<td>3.3%</td>
<td>4.8%</td>
</tr>
<tr>
<td>PPI</td>
<td>7.6</td>
<td>2.5</td>
<td>0.2%</td>
<td>0.1%</td>
<td>0.1%</td>
</tr>
<tr>
<td>Emp</td>
<td>7.7</td>
<td>2.4</td>
<td>1.2%</td>
<td>0.6%</td>
<td>0.6%</td>
</tr>
<tr>
<td>All Ann.</td>
<td>7.8</td>
<td>2.3</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.1%</td>
</tr>
</tbody>
</table>
Table 2
Concavity Index
This table presents the mean and standard deviation of the concavity index on announcement and non-announcement days. The announcement days include days when the FOMC is releasing its decision (FOMC), when PPI numbers are released (PPI), and when employment numbers are released (Emp). The last three columns show p-values of a t-test, Kolmogorov-Smirnov (KS) test, and Mann-Whitney (MW) test for the difference between announcement and non-announcement days.

<table>
<thead>
<tr>
<th>Type</th>
<th>Mean</th>
<th>Std. Dev</th>
<th>t-test</th>
<th>KS</th>
<th>MW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Ann.</td>
<td>8.3</td>
<td>7.5</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>FOMC</td>
<td>9.7</td>
<td>6.7</td>
<td>4.1%</td>
<td>5.0%</td>
<td>5.7%</td>
</tr>
<tr>
<td>PPI</td>
<td>9.7</td>
<td>7.7</td>
<td>2.5%</td>
<td>11.5%</td>
<td>5.4%</td>
</tr>
<tr>
<td>Emp</td>
<td>8.6</td>
<td>7.5</td>
<td>31.9%</td>
<td>39.6%</td>
<td>35.6%</td>
</tr>
<tr>
<td>All Ann.</td>
<td>8.9</td>
<td>7.1</td>
<td>4.1%</td>
<td>19.6%</td>
<td>6.3%</td>
</tr>
</tbody>
</table>