Analyzing a Circuit

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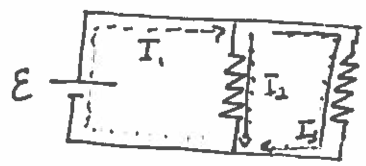
Temple University,
Fall 2017)

① Break circuit into V + I regions

• all touching wires must be at same V.

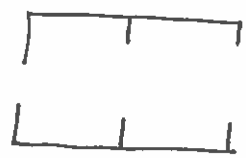
• all straight segments must have same current, that is, current may only change at junctions.

Ex



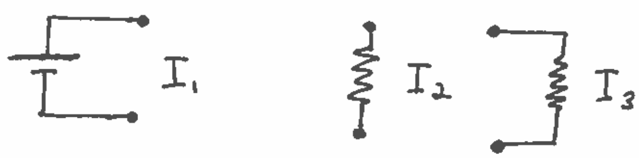
← $+\mathcal{E} = V$

← $0 = V$



$V = +\mathcal{E}$

$V = 0$



↑ 2 potentials

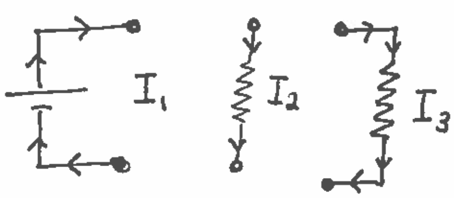
← 3 currents

② Pick positive direction for currents

- like picking directional axes; positive currents in same direction as our "axis", negative in opposite direction.
- Pick a direction and stay consistant!
- Any direction will be fine.

ball moving, can pick $+\hat{x}$ to be either direction

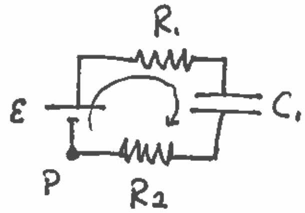
Ex



③ Kirchoff's Loop rule: $\Delta V = 0$ around a loop

- Each circuit element encounter has a ΔV_i associated with it. (Specifics later)

Ex



- Pick a starting point ("p")
- Pick a direction + path around loop
- "Walk around loop, writing down a term whenever a component is encountered.
- When you get back to start ("p"), write "= 0" at the end.

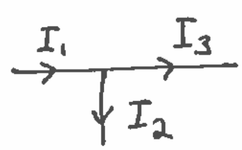
$$+\underset{\mathcal{E}}{\boxed{}} + \underset{R_1}{\boxed{}} + \underset{C_1}{\boxed{}} + \underset{R_2}{\boxed{}} = 0$$

(Boxes represent terms)

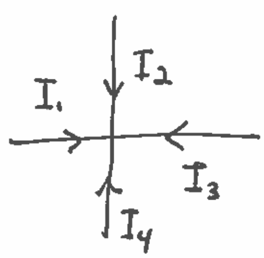
④ Kirchoff's junction rule: $I_{in} = I_{out}$ of a circuit junction

- Stick with the directions for current you picked earlier!

Ex



$$I_1 = I_2 + I_3$$



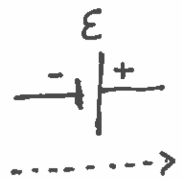
$$I_1 + I_2 + I_3 + I_4 = 0$$

We picked the directions to all be into this junction. This is ok, at least one will be negative (unless all are zero)

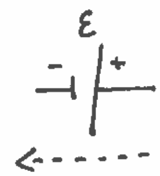
— What to do for each circuit element?

- Keep I directions consistant with earlier!
- Let "------>" indicate the direction we are "walking" along a loop.

- <u>Emf Source</u>:



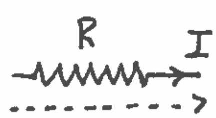
term $\boxed{+\mathcal{E}}$



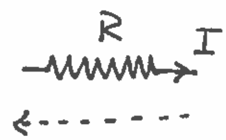
term $\boxed{-\mathcal{E}}$

  - We know the direction of (DC) emf, so the sign is whichever terminal we exit.
  - For AC emfs, pick "+" to be the direction we chose for the current through that emf.

- <u>Resistor</u>:



term $\boxed{-IR}$



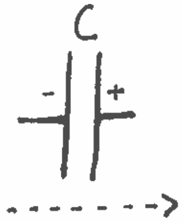
term $\boxed{+IR}$

  - Encountering a resistor in the direction of the current assumes we are going "downhill" $\Rightarrow \Delta V = -IR$
  - Encountering a resistor in the opposite direction of the current implies the opposite: "uphill" $\Delta V = -(-IR) = +IR$

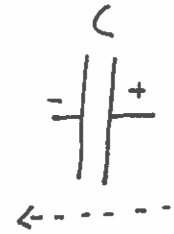
For any resistor, we can always write: $\underline{\underline{\Delta V = IR}}$

(For equivalent resistors, $R \rightarrow R_{eq} \Rightarrow \Delta V = IR_{eq}$)

• **Capacitor**



term $\boxed{+\frac{Q}{C}}$



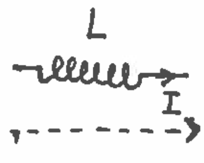
term $\boxed{-\frac{Q}{C}}$

- Capacitors are like emfs, need to know which side is "+"
- If we don't know, can write "Q/C"; $Q>0$ implies charge aligns with our loop; $Q<0$ implies charge opposes loop.
- Current direction doesn't matter! (But it does determine if the capacitor is charging or discharging)

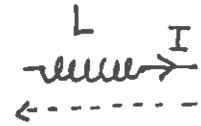
For any capacitor, we can always write: $\underline{\underline{Q = CV}}$

(For equivalent capacitors, $C \to C_{eq} \Rightarrow Q = C_{eq} V$)

• **Inductor**



term $\boxed{-L\frac{dI}{dt}}$

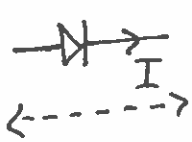


term $\boxed{+L\frac{dI}{dt}}$

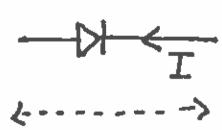
- Inductors oppose a changing current with a back emf.
- Sign is then relative to current direction.

For any inductor, can always write: $\underline{\underline{\mathcal{E}_L = -L\frac{dI}{dt}}}$

• **Diode**



$\boxed{\Delta V = 0}$

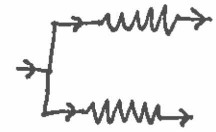


$\boxed{I = 0}$

- Diodes do not have a ΔV, but instead dictate the allowed current direction: one way only!

(5) Find equivalent components as needed.

- no choice for current => series
- equivalent components simplify sections of a circuit for easier analysis.



- choice for current => parallel

Note: May need to apply (3), (4), + (5) in different orders, depending on the circuit in question.

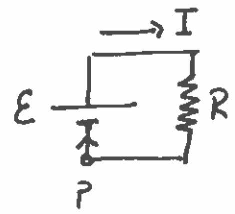
Note: Try to use loops + nodes that include the unknown(s) that you are solving for.

Note: You will need at least as many unique equations as unknowns.

Note: Knowns can often be simplified by replacing them with equivalent components.

Note: Playing around with a computerized circuit simulator will help you visualize how circuits work, without the need for an electronic store's supply of materials.

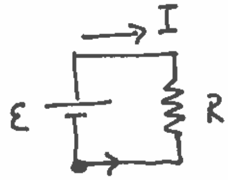
Example: Simple circuit



- I choose direction of I to be clockwise
- from p, loop: $+\mathcal{E} - IR = 0$ (clockwise loop)

$$\Rightarrow \quad \mathcal{E} = IR$$

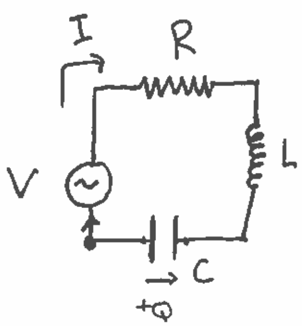
Ex 2



- I clockwise
- loop counter-clockwise

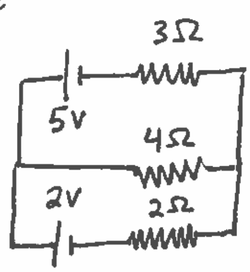
$$+IR - \mathcal{E} = 0 \quad \Rightarrow \quad \mathcal{E} = IR \quad \left(\text{same result!}\right)$$

Ex 3 RLC w/ AC emf

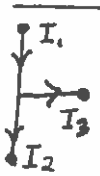


- I clockwise, loop will follow I
- loop: $$+V - IR - L\frac{dI}{dt} - \frac{Q}{C} = 0$$
- Note: I defined "+Q" to be in the opposite direction of my current, this implies that it is being charged by the emf.

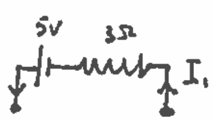
Ex 4

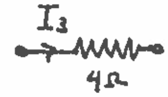


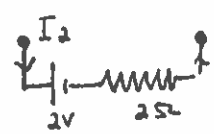
Find all three currents



- Current directions defined at left ↵ (cutaway of left side of circuit)

- Different currents: 





- Different potentials:

 V_1,  V_2, V_3,  V_4

(continued) 2→

2 → junction rule: $I_1 = I_2 + I_3$

top loop, counter-clockwise from 5V : $+5V - 4I_3 - 3I_1 = 0$

bottom loop, clockwise from 2V : $+2 - 4I_3 + 2I_2 = 0$

big loop, counter-clockwise from 5V: $+5 - 2 - 2I_2 - 3I_1 = 0$

☆

Rearranged: (i) $5 = 3I_1 + 4I_3$

(ii) $2 = -2I_2 + 4I_3$

(iii) $3 = 3I_1 + 2I_2$

note: with 3 unknowns we will only need 3 of these

junc. rule into (iii) → $3 = 3(I_2 + I_3) + 2I_2 = 5I_2 + 3I_3$

Solve (ii) for I_2 → $I_2 = 2I_3 - 1$

into iii again: → $3 = 5(2I_3 - 1) + 3I_3$

$$3 = 10I_3 - 5 + 3I_3$$

$$8 = 13I_3 \implies \underline{\underline{I_3 = \frac{8}{13} A}}$$

into (ii) → $2 = -2I_2 + 4\left(\frac{8}{13}\right)$

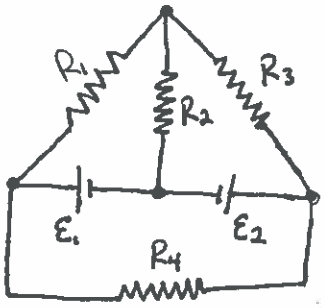
$$\implies \underline{\underline{I_2 = \frac{3}{13} A}}$$

Junc. rule: $I_1 = I_2 + I_3 \implies \underline{\underline{I_1 = \frac{11}{13} A}}$

Note that we (accidently) picked the correct directions for the currents, as they are all positive.

☆ Just algebra from this point

Ex 5



Find all currents

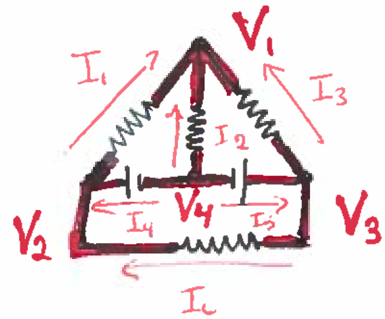
$\mathcal{E}_1 = \mathcal{E}_2 = 6V$

$R_1 = R_3 = 3\,\Omega$

$R_2 = 12\,\Omega$

$R_4 = 10\,\Omega$

Redrawing:



- 4 regions where potential can be different
- 6 currents

4 junctions:

$$I_1 + I_2 + I_3 = 0$$
$$I_4 + I_6 = I_1$$
$$0 = I_2 + I_4 + I_5$$
$$I_5 = I_6 + I_3$$

yikes! → 7 loops:

Start simple: bottom loop (▭), clockwise from $\mathcal{E}_2$:

$$+\mathcal{E}_2 - I_6 R_4 - \mathcal{E}_1 = 0 \implies (6V) - I_6(10\Omega) - (6V) = 0$$

$\implies \underline{\underline{I_6 = 0}}!$ (potential is same on both sides of R_4)

This means (from junc. rules): $I_1 = I_4$, $I_3 = I_5$, ↙ eliminates I_4 & I_5

Top-left loop (◿); clockwise from $\mathcal{E}_1$: $+\mathcal{E}_1 - I_1 R_1 + I_2 R_2 = 0$

Top-right loop (◺); counter-clockwise from $\mathcal{E}_2$: $+\mathcal{E}_2 - I_3 R_3 + I_2 R_2 = 0$

continued.

$2 \rightarrow$ Rewriting what we have:

$$I_1 + I_2 + I_3 = 0 \,, \qquad I_1 = I_4 \,, \quad I_3 = I_5 \,, \quad I_6 = 0\,A$$

$$\mathcal{E}_1 - I_1 R_1 + I_2 R_2 = 0$$

$$\mathcal{E}_2 - I_3 R_3 + I_2 R_2 = 0$$

Now: $\mathcal{E}_1 - I_1 R_1 + I_3 R_3 - \mathcal{E}_2 = 0$ (big triangle, $\triangle$)

$$\rightarrow (6V) - I_1(3\Omega) + I_3(3\Omega) - (6V) = 0$$

$$\rightarrow I_1(3\Omega) = I_3(3\Omega)$$

$$\Longrightarrow I_1 = I_3$$ ← another current down

junc. rule now gives: $I_1 + I_2 + I_1 = 0 \Rightarrow 2I_1 = -I_2$

Into top-left loop (Δ): $\mathcal{E}_1 - I_1 R_1 + (-2I_1)R_2 = 0$

$$\Rightarrow \mathcal{E}_1 - I_1(R_1 + 2R_2) = 0$$

$$\Rightarrow I_1 = \frac{\mathcal{E}_1}{R_1 + 2R_2} = \frac{6V}{(3\Omega) + 2(12\Omega)} = \frac{6V}{27\Omega} = \frac{2}{9}A$$

<u>So</u>:

$$I_1 = I_4 = \frac{2}{9}A$$

$$I_2 = -\frac{4}{9}A$$ ← picked opposite direction, here

$$I_3 = I_5 = \frac{2}{9}A$$

$$I_6 = 0\,A$$

Done! Not so bad?