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US and Chinese elementary teachers' noticing of cross-cultural mathematics videos

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Abstract
This study aims to explore teacher noticing differences of the sampled US and Chinese elementary teachers from cross-cultural mathematics videos. A total of 34 expert teachers commented on 25 video clips online. We coded what and how teachers noticed from the videos both quantitatively and qualitatively. Findings reveal teachers' strong interests and profound reflections, especially in the teaching domain including representations, communication, and teacher questioning/guide. Cross-cultural differences in teacher noticing were identified, which were discussed based on possible cultural influences. Implications in research and practice, teacher support, and methodology are discussed.

Keywords Cross-cultural videos · Teacher noticing · Expert teachers · Elementary mathematics

Introduction

Mathematics instruction around the world is culturally different, which results in students' diverse learning experiences. These experiences can serve as powerful resources for the field to develop and enhance teachers' expertise, but only if the field understands what teachers notice and how they interpret it when facing culturally different classroom lessons. The current

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study aims to contribute to this endeavor by exploring differences between US and Chinese elementary expert teachers' noticing when they are exposed to a set of cross-cultural videos on early algebra lessons. In general, past studies have revealed that cross-cultural videos are a promising tool to support teacher learning (e.g., Christ et al., 2014). However, little is known about what US and Chinese elementary teachers may notice from cross-cultural mathematics videos under educational reforms and how they may reason about what they noticed. Findings from this study are expected to inform the field about teachers' cross-cultural noticing differences, which will inform subsequent teacher support in respective countries.

Review of literature

Teacher noticing as teaching expertise

Teacher noticing is an important indicator of teachers' expertise (Dreher et al., 2020). It mainly refers to teachers' ability to attend to and interpret worthwhile classroom events so as to make important instructional decisions (Jacobs et al., 2010; Sherin et al., 2011). Teacher noticing is also called teachers' professional vision (Seidel & Stürmer, 2014; Sturmer et al., 2013; Van Es & Sherin, 2002, 2008) or "situation awareness" (Miller, 2011), which are notions developed in other fields (Endsley, 1995, 2006; Goodwin, 1994). To study teacher noticing, prior studies provided various frameworks, though without consensus (see the review of Stahnke et al., 2016). Some focused on "what" teachers noticed (Star & Strickland, 2008), while others focused on "how" teachers noticed (van Es & Sherin, 2002, 2008) and still others focused on both dimensions (Seidel & Stürmer, 2014; Stürmer et al., 2013; van Es, 2011). For the current research, we take the two-dimension construct (what and how) to explore differences between US and Chinese teachers' noticing skills.

With regard to the "what" dimension, Star & Strickland (2008) viewed this as the foundation for exploring teachers' noticing. Prior studies suggested different aspects. For instance, Miller & Zhou (2007) developed a coding system that categorized teacher noticing into both "the teacher" (e.g., teacher personality, student participation, classroom environment) and "the lesson" (e.g., content, lesson tool, lesson structure, student understanding, teacher questions). van Es (2011) in her "what teachers notice" dimension suggested four levels of noticing. At the baseline level, teachers mainly attend to whole class behavior and classroom climate. When the level of noticing increases, teachers' attention shifts from general to specific pedagogy as well as to students' mathematical thinking, which eventually progresses toward the relationship between teaching and learning. Slightly different, Stürmer and colleagues (Seidel & Stürmer, 2014; Stürmer et al., 2013) considered three aspects in their model of teachers' professional vision—goal clarity and orientation, teacher support and guidance, and learning climate—as main components of teachers' noticing skills. To these researchers, all of these components, including the "learning climate," are important because they represent a balanced view of teaching and learning. As they stated, "The learning climate in a classroom is of particular relevance for student learning since it provides an important motivational and affective background in which learning takes place" (Seidel & Stürmer, 2014, p.743). In this sense, Stürmer and colleagues' stress on non-subject-specific aspects of the classroom learning climate did not articulate different levels of noticing, which is in contrast to van Es (2011). The above varied aspects in terms of "what" teachers may notice provides a reference for this study in analyzing teachers' noticing.

With regard to the "how" dimension, there were also varied levels proposed in the literature. In their *Learning to Notice Framework*, van Es and Sherin (2002, 2008) viewed teachers' noticing as (a) identifying important events in a teaching situation, (b) using knowledge to reason about that specific event or situation, and (c) linking the specific event and situation to broader learning and teaching principles. Aspect (a) describes what was noticed without interpretation, while aspects (b) and (c) indicate knowledge-based reasoning at two levels: specific and general. This noticing framework was further detailed into four levels of "how" teachers notice with a similar trend as the above (van Es, 2011). Similarly, Jacobs et al. (2010) studied teachers' noticing about children's mathematical thinking and defined three interrelated skills that included attending to children's strategies, interpreting their understanding, and then deciding how to respond. It seems that Jacobs et al. combined aspects (b) and (c) in van Es and Sherin's framework and added a new aspect of "deciding how to respond." Furthermore, Stürmer and colleagues (Seidel & Stürmer, 2014; Stürmer et al., 2013) classified teachers' reasoning into levels of description, explanation, and prediction, which is similar to the three levels of the "situation awareness" model (Endsley, 1995, 2006). Note that "prediction" refers to predicting the consequence of an event on student learning based on general principles. Despite the variation of "how" teachers notice in the literature, we see a clear differentiation between *describing* without any reasoning and knowledge-based *reasoning* involving either specific or general aspects, which will be taken into consideration in the current study.

Teacher noticing, cross-cultural videos, and cultural dependency

Teacher noticing has been mainly studied through classroom videos (Borko et al., 2008; Santagata, 2009; Zhang et al., 2011). Given that teaching is a cultural event, cross-cultural videos of classroom teaching are found to be particularly powerful in eliciting teachers' noticing skills and prompting teachers for deep thinking (e.g., Christ et al., 2014; Moran et al., 2015). Through viewing videos of international peers, teachers may face different pedagogical decisions, which may prompt them to compare the varied practices with their own. Such comparisons may lead to discussions regarding how culture plays a role in different pedagogical choices (Moran et al., 2015) and results in profound reflections on and questioning about one's pedagogy, beliefs, and engrained practices (Hollingsworth & Clarke, 2017; Kleinknecht & Scheider, 2012; Moran et al., 2015). Likely due to the power of cross-cultural videos in supporting learning, TIMSS released 53 cross-cultural videos to the public with the goal of enhancing mathematics teaching (e.g., Hiebert & Stigler, 2004; Roth & Givvin, 2008). However, these videos were all from eighth grade classrooms, which is not suitable for elementary teachers. In addition, these lessons were randomly selected with different topics, which may limit direct comparisons among the lessons.

When viewing the same set of cross-cultural videos, teachers from different countries likely have different responses. For instance, Jacobs & Morita (2002) used videos from both US and Japanese classrooms for teacher professional development and found that overall, US teachers formulated more positive comments on videos from both countries while Japanese teachers were much more critical. The authors concluded that Japanese teachers clearly had a cultural script that contained a set of ideas about the "correct" way to carry out a mathematics lesson, which did not seem to exist within US teachers. Similarly, Miller & Zhou (2007) briefly reported that when watching the same mathematics lesson episodes, US teachers tended to notice general pedagogical issues while Chinese teachers noticed more mathematics content.

The above findings indicate that teacher noticing has cultural dependency, meaning that what teachers notice is likely shaped by their knowledge, beliefs, and cultural experiences (Clarke & Hollingsworth, 2002; Louie, 2018; Stigler & Hiebert, 1999, 2009). Louie (2018) argued that teacher noticing is not just a cognitive orientation (e.g., attending, interpreting, and responding to student thinking), but is also affected by teachers' cultural views. Recent cross-cultural empirical studies have also shown that cultural norms on quality teaching have played an important role in shaping teachers' noticing (Dreher, et al., 2020; Yang et al., 2019, 2021). Dreher et al. (2020) further argued that cultural norms should be evident in expert teachers' views. To explore this assumption, these researchers analyzed German and Taiwan mathematics education researchers' responses to student thinking. For the same teaching vignette of secondary school mathematics, while German experts expressed concerns that students' existing thinking was not valued and encouraged enough by the teacher, Taiwan experts were concerned that students' inadequate thinking was not further prompted.

Cultural differences of teacher knowledge, beliefs, and experiences in the USA and China

Since teacher noticing has cultural dependency, a review of cultural difference of US and Chinese teachers' knowledge, beliefs, and pedagogical experiences helps situate the current study. It is widely agreed that Chinese education is mainly rooted in Confucius Heritage Culture (CHC), which is sometimes misperceived as passive learning due to primarily whole class instruction and the authority of teachers (Biggs, 1998; Tran, 2013). Nevertheless, researchers surprisingly found that mathematics teaching in CHC classrooms (e.g., in China, Japan, Hongkong, Singapore, Korea, Taiwan) was "constructive" in that teachers focused on posing thoughtful questions to facilitate students' thinking while students worked hard to overcome any learning difficulties and to understand the abstract concepts (Li, 2007; Steve & Stigler, 1992; Stigler & Hiebert, 1999; Tran, 2013). In fact, experienced Chinese teachers believe that the core of teaching is to understand mathematical concepts (Cai & Ding, 2017; Li & Huang, 2013); thus, concrete representations such as manipulatives and modeling should serve as tools to learn abstract concepts (Cai, 2005; Ma, 1999). These teaching styles may be traced back to Confucius himself who taught in an almost Socratic manner and valued hard thinking (Lee, 1996). In addition, expectations for teachers' content knowledge in CHC classrooms are higher than those in Western countries (Kim et al., 2011; Sun, 2011), which can be confirmed by prior findings about Chinese and US elementary teachers' knowledge differences (Ma, 1999). Chinese teachers' pursuit of knowledge improvement likely has its' cultural roots in Confucius who commented that "Among three, there will always be one who can serve as my teacher (三人行必有我师)." It is common for Chinese teachers to learn from their peers through Chinese lesson studies and public lessons (Li & Huang, 2013) and to keep improving their planning and teaching. This is different from their US counterparts who tend to feel more satisfied with their lessons even if they spend much less time on planning (Fernandez & Cannon, 2005).

Compared to China, the overall US mathematics teaching style was found to be procedure-driven (Stigler & Hieber, 1999, 2009) and focused on pleasurable learning (Leung, 2001). Culturally, the USA is a more individualistic society, which is illustrated by the fact that US teachers spend more time differentiating for diverse learners in the classroom. In a belief survey study, Correa et al. (2008) reported that US elementary teachers were mostly concerned about students' learning styles and favored using hands-on approaches to learn mathematics. On the one hand, this potentially points to a difference in the hierarchy of the roles of teacher and learner, a criticism often made to Chinese classrooms (e.g., Ni et al., 2014). On the other hand, it contradicts with Chinese teachers pushing for abstract mathematical thinking and using manipulatives only as tools to assist learning (Cai, 2005; Ding et al., 2019; Li, 2007; Stigler & Perry, 1999). In fact, there is substantial variation in teaching quality among US classrooms (Hiebert & Stigler, 2004) with two extremes of teaching styles: student-centered classrooms with minimal teacher guidance (Kirschner et al., 2006) and teacher-led classrooms with little student input (Stigler & Hiebert, 1999). Both styles may lead to little learning (Anderson et al., 2000; Kirschner et al., 2006). The above observations seem to contradict with the learning theories (e.g., constructivism) and the governing standards (National Council of Teachers of Mathematics [NCTM], 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers [NGAC & CCSSO], 2010) that have uniformly emphasized deep learning. One of the reasons may be due to the individualized culture in which theories and standards are interpreted and used differently by individuals (Leung, 2001).

Despite cultural influences, both US and Chinese mathematics education systems have been seeking educational reforms. In China, mathematics education reform took place in 2001 to address the possible shortcomings criticized in the literature (e.g., overemphasizing basic concepts and skills and lacking enough student autonomy) and to emphasize more mathematical processes and affective demeanor (Ministry of Education, 2001; Ni et al., 2011; Xu, 2017). Consequently, reformed Chinese textbooks were found to systematically situate worked examples in real-world contexts to support students' sense-making (Ding & Li, 2010, 2014), which was also evident in Chinese expert teachers' videotaped lessons (Ding et al., 2019, Ding, 2021). The reformed spirit was also reflected in teachers' beliefs in that Chinese teachers under reform were more concerned with making connections between mathematical content and real-life situations and developing students' interest in mathematics (Correa et al., 2008). In a similar vein, the US mathematics education system is cognizant of the teaching challenges and has been seeking reform for decades (NCTM, 2000; NGAC & CCSSO, 2010). Although there were reported successes, some researchers found that the core features of mathematics teaching in US classrooms have "remained the same for a century or more" (Hiebert & Morris, 2012, p. 96). The above cross-cultural differences in teachers' knowledge, beliefs, and experiences provide a context to understand US and Chinese teachers' noticing when watching the cross-cultural videos.

The current study

In this study, we explore differences of the sampled US and Chinese expert elementary teachers' video noticing based on cross-cultural mathematics lessons. In particular, we ask: *What do the sampled US and Chinese expert teachers notice from the cross-cultural videos and how do they reason about what they noticed*? By cross-cultural videos, we refer to a set of US and Chinese mathematics videos. Extended from prior studies (e.g., Dreher, et al., 2020; Miller & Zhou, 2007; Roth & Givvin, 2008; Yang, et al., 2019, 2021), our videos from both countries focused on the same elementary school mathematics topic, early algebra, with the selected video clips closely matching cross-culturally to enable direct comparisons. It is expected that these closely matched topics can help avoid construct-irrelevant variance due to different content (Dreher, et al., 2020). In addition, we are interested in exploring "expert" elementary teachers' noticing because experts' views likely resemble cultural norms of effective teaching (Dreher et al., 2020). It is expected that findings from this study will shed light on the cross-cultural differences of teacher noticing, further our understanding of cross-cultural teaching approaches, and thus have the potential to support teacher learning and improve mathematics teaching through international collaborations.

Methods

The participants and the project

This study is part of a large NSF-funded project on US and Chinese elementary expert teachers' mathematics teaching of early algebra. During the first two years, the project was devoted to videotaping teachers' lessons without researcher intervention. In particular, Year 1 focused on inverse relations (between addition and subtraction and between multiplication and division) and Year 2 focused on the basic properties of operations (commutative, associative, and distributive). Both are early algebra topics emphasized by the Common Core State Standards (NGAC & CCSSO, 2010). During Year 3 of the project, we analyzed all cross-cultural videos and selected 25 video clips, which were shared with teachers in both countries. The videos were uploaded to online platforms for one-month (called "video forum"), which led to the summer onsite intervention. After the intervention (Year 4), teachers in both countries re-taught their lessons. The current study explores teachers' noticing differences from the cross-cultural videos before the summer intervention.

For the large project, a total of 17 Chinese and 17 US expert teachers (grades 1–4) were selected to participate. These teachers had at least 10 years of teaching experience, and all had good teaching reputations. All Chinese teachers received national and/or province-level teaching awards, while the US teachers were recommended by the school district/principals and five of which were National Board-Certified Teachers. In each country, teachers came from a large urban city. Each of the 34 teachers taught four lessons on either inverse relations ($N=68$) or the basic properties of operations ($N=68$), resulting in a total of 136 videotaped lessons.

Due to unexpected changes, 15 out of the 17 original Chinese teachers and two project coordinators who were teacher experts participated in the video forum ($N_{\text{ChinaVideoForum}}=17$). Note that the two Chinese coordinators are not part of our research teams and thus have the same level of understanding of our research aims as the other Chinese teachers. In the USA, 13 out of the 17 original participants and four newly recruited teachers participated in the video forum ($N_{\text{USVideoForum}}=17$).

Data sources and procedures

Data sources mainly included teacher comments on 25 video clips (Number of videos: $N_{\text{China}}=12$, $N_{\text{US}}=13$; or $N_{\text{inverse}}=15$, $N_{\text{property}}=10$). Although each video clip individually was from either the USA or China, we consider these videos collectively as "cross-cultural" because they offered a cross-cultural experience to teachers. The selection of the 25 video clips resulted from rigorous evaluation based on teachers' use of worked examples, representations, and deep questions (Pashler et al., 2007). Aligning with these major aspects, our project developed a coding framework (Ding et al., 2021a), which helped identify quality video clips. As such, each selected video clip contained at least one aspect of merit and afforded discussion opportunities for better instruction. In addition, we ensured the inclusion of distinguished teaching strategies in the video clips. For instance, when selecting Chinese video clips, we purposefully included teachers' skillful discussions of "tape diagrams," which is a powerful representational tool widely used in East Asian classrooms (Ding et al., 2019; Cai & Moyer, 2008; Murata, 2008). Tape diagrams are also called bar models, featuring "appropriately sized rectangles" to illustrate mathematical quantities (Cai & Moyer, 2008, p. 284). To select the US video clips, we ensured the inclusion of "array" models, which occurred mainly in US but not Chinese lessons. Arrays are groups of objects displayed in rows and columns. When the squares of the array are pushed together with no gaps or overlaps, it becomes an area model (NGAC & CCSSO, 2010). Whenever possible, we also tried to include videos illustrating deep questions. On average, the length of the resulting US and Chinese video clips was about 10 and 13 min, respectively. Table 1 indicates all video clips. The selected video clips were then sent to the filmed teachers to grant permission to share with their peers through the video forum.

To overcome the language barrier, each of the selected videos was fully translated and annotated with subtitles using the peer language. The translation was first done by an undergraduate researcher, Wenda Luo, who was fluent in both English and Chinese and specialized in mathematics education. During this process, difficulties encountered were mainly due to the lack of corresponding mathematical terms in the other country. Prior studies also faced similar challenges. For instance, Mesiti, Clarke, and van Driel (2019) reported that within the country (e.g., Australia), there was basically common language to describe and prescribe classroom practices but not without discrepancy. When it comes to cross-cultural studies (e.g., German and Taiwan), challenges in finding a common language increases (Dreher et al., 2020). In our study, this difficulty was resolved by constant discussions between the translator and the project PI (first author of this paper), with ongoing consultation with teaching experts in the USA and China. For example, one of the US lessons is about "fact family," which refers to a group of related facts (e.g., 3 + 5 = 8, 5 + 3 = 8, 8–3 = 5, 8–5 = 3). Even though the Chinese textbook contains parallel lessons, it does not have a corresponding term and simply refers to this as a group of number sentences (算式组). If we used this Chinese term, the English word "family" would have been discarded. After discussion, we decided to use the phrase "算式家庭" (number sentence family), which was both understandable to Chinese teachers and kept the meaning of the English term. All translated scripts were double checked by the project PI and proofread by the native speakers in the research project. The finalized translation was then annotated as subtitles by the translator for all video clips.

Due to challenges in accessing one common online platform, we uploaded the video clips to YouTube for US teachers and to YouKu for Chinese teachers. The 25 video clips were uploaded in exactly the same sequence for each platform. Teachers in both countries received the same instructions to first watch the videos of interest and then to comment on each video observed based on the prompts: (1) What do you notice? What stood out to you? (2) What questions do you have in terms of this video or in general? And (3) Other comments? Teachers entered their responses on each video as one comment entry. Note that teachers could see their peers' responses when they entered their own comments. Although we assumed that expert teachers would express their own thinking, this could still be a limitation in terms of the "independency" of the collected data.

Data coding and analysis

A total of 233 comment entries (US: 120; China: 113) from the online video forum were compiled into a spreadsheet. One author transcribed all Chinese comments into English.

Table 1 Video clips presented to both US and Chinese teacher participants

ID	Name of video clips	Length	Topic
23	US_Grade 1_T2_Fact Family算式组NEW!!!	7′29″	Inverse Relation_Additive
1	US_Grade 2_T3_Fact Family算式组	9′15″	
2	US_Grade 2_T4_Fact Family算式组	13′42	
3	China_Grade 1_T1_Fact Family一图四式	14′11″	
4	China_Grade 1_T2_Fact Family一图四式	13′01″	
5	China_Grade 1_T2_Initial Unknown Problem求原来是多少	15′21″	
6	China_Grade 2_T4_Additive Comparison加减法比较题型	14′31″	
7	China_Grade 2_T4_Check for Subtraction检验减法	10′20″	
8	US_Grade 3_T5_Multiplicative Inverse (equal group) 乘除法的关系(均组)	9′21″	Inverse relation_Multiplicative
24	US_Grade 3_T6_Multiplicative Inverse (equal group) 乘除法的关系(均组) NEW!!!	8′48″	
9	China_Grade 2_T5_Multiplicative Inverse (equal group) 乘除法的关系(均组)	8′40″	
10	US_Grade 4_T8_Multiplicative inverse (array) 乘除法的关系 (方格)	14′32″	
11	US_Grade 4_T8_Multiplicative Comparison 乘除法比较题型	14′15″	
12	China_Grade 3_T7_Multiplicative Comparison乘除法比较题型	10′48″	
13	China_Grade 3_T8_Multiplicative Comparison乘除法比较题型	10′49″	
14	US_Grade 1_T9_ Commutativity of Addition (formal) 加法交换律(正式)	4′	Basic Properties
15	US_Grade 1_T10_ Commutativity of Addition (formal) 加法交换律(正式)	6′45″	
16	China_Grade 4_T16_ Commutativity of Addition (formal) 加法交换律(正式)	11′34″	
17	US_Grade 3_T14_Distributivity (informal) 分配律(渗透)	9′53″	
18	US_Grade 3_T14_Distributivity (informal) #2 分配律(渗透)	11′08″	
19	China_Grade 3_T13_Distributivity (informal) 分配律(渗透)	16′53″	
20	China_Grade 4_T16_Distributivity (formal) 分配律(正式)	15′51″	
21	US_Grade 4_T17_ Associativity/Distributivity (informal) 结合率/分配律(渗透)	11′08″	
25	US_Grade 3_T13 _ Associativity of Multiplication (formal) 乘法结合率(正式) NEW!!!	6′34″	
22	China_Grade 4_T15_Associativity of Multiplication (formal) 乘法结合率(正式)	11′27″	

Next, all authors read the teacher comments while memoing (Creswell & Poth, 2018). Memos are reflective notes that help develop codes and themes. Our collective backgrounds (from both US and China) helped ensure our memoing was culturally valid. We then shared our observations of the themes emerging from the teachers' online comments.

Using the teacher noticing framework in this study, we first coded "what" teachers noticed using a bottom-up approach. Our methodological choice is due to the wide variation in the "what" dimension in prior studies. Based on Chi (1997), we considered each independent and complete idea noticed by a teacher as a unit of analysis, which suggests typical codes. Consider, for example, the following comment:

> Great student talk, and I like the "thumbs up" for students who agree, so they can still feel as though they are getting credit for their thoughts. The cube is a great visual representation and the students were able to relate the visual to the math facts given. (US-T13 on video#1).

We coded the above comment into two noticing codes (a) I like the way children share information/format of talk, and (b) I like manipulatives (cube) as great visual/show concept information. Note that if a teacher repeats a complete idea twice in a video comment, we will consider both thoughts as one unit and assign it one noticing code. Of course, the same noticing code could be applied to different teachers' comments, resulting in different "units" for analysis.

Using what was described above, we identified a list of initial noticing codes based on US and Chinese teachers' comments. Two authors used the same coding list to independently code all teachers' comments on the same videos. The interrater reliability (Kappa = 0.724, $p < 0.001$) indicates substantial agreement between the raters. The discrepancies were resolved through discussion. For instance, we changed several codes with "like" to "like/notice" for broader capacity and went back to update our coding. Based on the shared understanding, one author went further to code all video comments using a constant comparative method (Glaser, 1965). That is, when there was a new "idea" (noticing unit) that could not be assigned to an existing code, we would generate a new code and add it to the list. As a result, we identified a list of 166 codes, which were compiled from the 634 video-noticing units ($N_{US} = 326$, $N_{China} = 308$) based on the US and Chinese video comments.

The 166 codes were then reduced into 41 subcategories. Since some of the subcategories closely resembled one another, we further collapsed subcategories into larger categories (e.g., we collapsed manipulatives, diagrams, tallies, number/shape, array, story situation, and so on into "representations"). The resulting 24 categories were then reduced into four large domains: mathematics, children, teaching and other. Figure 1 illustrates part of the excel sheet resulting from the four-round coding process. Given that it was not always straightforward to classify a comment into one domain, we developed operational definitions. For instance, we defined that the "children" domain should be driven by students while the "teaching" domain by pedagogy. The "children" domain also does not include comments on how a teacher supports students' learning, which belongs to "teaching." Table 2 illustrates the definitions along with example categories and noticing codes. Table 3 shows the distribution of the video-noticing units across different levels of categorization.

Next, we coded the "how" dimension of teacher noticing by considering each comment entry ($N = 233$) as a unit of analysis and coding it under "describing" and/or "reasoning." Note that "evaluation" without explanation was considered as "describing" only (van Es, 2011). For instance, "I like the way the children share information"

Round #4: Domain	Teaching	Teaching	Teaching	Teaching	Teaching	Teaching	Teaching	Teaching	Teaching
Round #3: Category	Representation	Representation	Representation	Representation	Representation	Representation	Representation	Representation	Representation
Round #2: Subcategory	Purpose	Diagram	Diagram	Diagram	Diagram	Tallies	Number/Shape	Number/Shape	Chart
Round #1: Code	Notice S drawing picture to find the answer	Notice/Like the use of diagrams (bar model, number line/counter)	Wonder if tape diagram similar to number line or bar model	Like discussions prior to students completing the bar model	Notice T had S to write up the equation under bar model	Notice drawing tallies	Like the combination of number and diagram (数形结合) / understand number sentence based on diagram	Wonder about the connection between 口诀 and 数形结合	Like/notice using the chart to analzye the story problem

Fig. 1 An illustration of the Excel sheet resulting from the four-round coding process

was coded as "describing," while "The use of the cubes is a great visual for children to understand the members of the number family do not change" was coded as both "describing" and "reasoning." During the coding process, brief memos were also written to note the typical aspects involved in teacher reasoning. To check interrater reliability, comments on the first 10 videos were double coded. Due to the straightforward nature in coding the "how" dimension, there was an almost perfect agreement between the two raters (Kappa $=0.959$, $p=0.000$).

To obtain an overall picture of teacher noticing, we first analyzed the distribution of teachers' comment entries ($N=233$) across cross-cultural videos, which provides an overview of teachers' video-noticing interests and preferences. Next, we analyzed the distribution of video-noticing units ($N=634$) across the four domains to understand "what" teachers noticed. Given that teachers in both countries commented on the teaching domain most frequently, we compared various categories under this domain. Furthermore, we analyzed "how" teachers noticed by computing the percentage of US and Chinese teachers' level of reasoning and identified patterns in their reasoning. The above overall findings (mainly quantitative) were checked back against the narrative comments for enriched understanding. For instance, focusing on each of the top three teaching categories: representations, communication style, and teacher question/guide, we identified cross-cultural features of teacher noticing based on the full set of videos and the same videos from one country.

Results

An overview of cross-cultural differences in teacher noticing

Interests and preference

Teachers in both countries demonstrated strong interests in their international counterparts' videos. Figure 2 indicates that there is a preponderance of teacher comments (US: 66.7%, China: 83.2%) left on videos of international rather than domestic peers.

Table 2 Operational definitions for four domains with example categories and codes

Domain	Operational definition	Example categories (subcategories)[1]	Example codes
Mathematics	•The primary content of the comment must be in regard to mathematical concepts •Mathematics content can include comments about the task, mathematics vocabulary as well as comments regarding mathematics curriculum/standards	○Depth ○Curriculum standards Relationships ○Connect connection ○Math task ○Vocabulary	– Like the depth of conversation – Notice the discussion of quantitative relationships – Noticed high-order challenging task – Like the term of "fact family"
Children	•The primary content of the comment must be about either the learning, actions, behaviors or verbalizations of students (this does NOT include comments regarding how the teacher introduces or supports students' verbalizations) •The students must be the driving force behind these observations	○Engagement ○Explanation ○Learning Process	– Like student engagement/participation/excited – Notice children's engagement and deep understanding of math – Wonder about in-unison answers – Notice students having a natural sense of the properties
Teaching	•The primary content of the comment must be regarding pedagogy •This includes how the teachers communicate with students, their use of representations as a tool for teaching, how teachers guide students in the classroom and the goal/focus of an individual teacher (NOT including big-picture curriculum)	○Communication ○Language ○Representation (manipulatives, purpose, diagram, tallies, number/shape, chart, story situation), ○Teacher questions/Guide (deep questions, comparison, tackle mistakes, use student thinking, support productive struggle) ○Teacher traits	– Like the teacher patiently listen to students – Notice T providing sufficient time for thinking – Like T's emphasis on the meaning of quantities, – Like teachers' suggestion to restate/revise – Notice teacher only accepting one solution – Like the way dealing with mistakes – Notice T is very knowledgeable
Other	•The primary content of the comment must not be in regard to mathematical concepts, student/child learning behavior or teaching pedagogy •These comments still contain thought/ideas relevant to the observation based on the videotaped lessons	○Environment (board writing, technology), ○Classroom management, ○Lesson format (time allot)	– Like teachers' chalkboard use – Notice T's skillful use of technology – Notice T gets students back on task quickly

[1]This does not represent a full list

Table 3 Distribution of teachers' noticing units under four domains at varied levels

	Mathematics	Children	Teaching	Other	
Domain	1	1	1	1	$N_{total} = 4$
Categories	7	4	10	3	$N_{total} = 24$
Subcategories	7	4	24	6	$N_{total} = 41$
Codes	30	13	105	18	$N_{total} = 166$
Units	92	83	396	63	$N_{total} = 634$

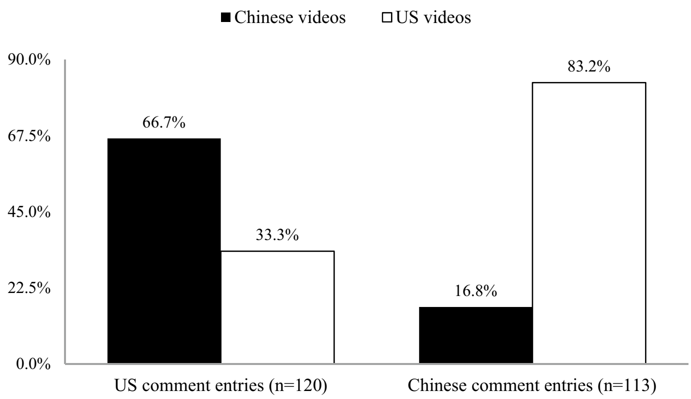


Fig. 2 Teachers' comment distribution and the sources of comments on the videos

What teachers noticed

Figure 3 indicates that across domains, both US and Chinese video comments mostly focused on teaching aspects.

Focusing on the video-noticing units under the teaching domain ($N_{US} = 326 \times 59.2\% = 193$; $N_{China} = 308 \times 65.9\% = 203$), teachers on both sides appeared to comment most frequently on, in order, representation, communication style, teacher question/guide, and goal/focus (see Table 4). Figure 4 further illustrates that teachers' comments in each of the top three categories were mainly devoted to their international peers' videos, which is consistent with the overall pattern.

How teachers noticed

Results indicate that most teachers on both sides first described what they noticed. Descriptions were slightly more frequent from US teachers (US/China: 96.7% vs. 92%, see "describing" in Table 5). For knowledge-based reasoning, there were many more Chinese comments that contained reasoning beyond descriptions (US/China: 51.7% vs. 85.8%, see "reasoning" in Table 5).

A closer analysis of how teachers reasoned about their noticing indicates cross-cultural differences. For US teachers, their reasoning mainly entailed: (a) posing clarifying

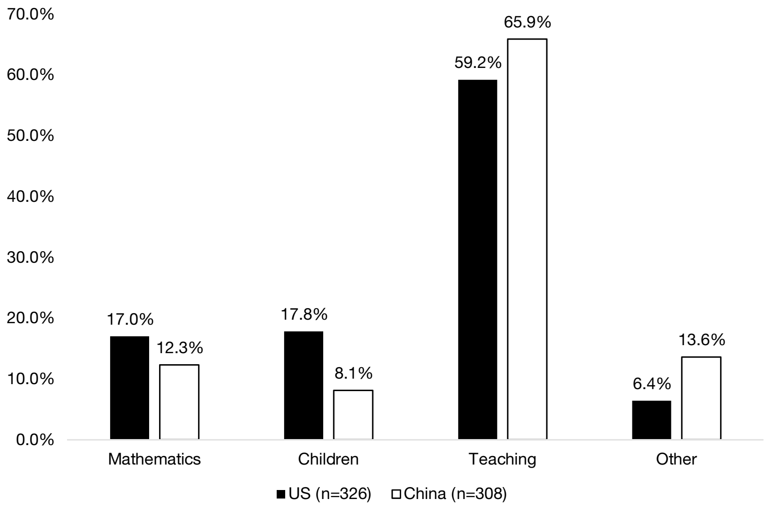


Fig. 3 Distribution of teachers' video-noticing units

Table 4 US and Chinese teachers' video-noticing units under the category of "teaching"

Teaching	US noticing units		China noticing units	
	Frequency	Percent (%)	Frequency	Percent (%)
Representation	66	34.2	107	52.7
Communication style	65	33.7	46	22.7
Teacher question/guide	48	24.9	20	9.9
Goal/focus	8	4.1	17	8.3
Other	6	3.1	13	6.4
Total	193	100	203	100

questions about the observed video, (b) providing an alternative teaching strategy based on factors of student involvement and diversity, (c) relating observations to their own teaching, and (d) expressing an interest in trying out certain strategies. For Chinese teachers, their reasoning mainly included (a) analyzing the observed teaching based on student learning or concept development, (b) comparing the observed lesson to a Chinese way of teaching, (c) providing alternative teaching strategies incorporating the Chinese approach, (d) articulating cross-cultural differences in teaching approaches and possible teaching beliefs, and (e) critically reflecting upon possible consequences of certain teaching approaches on student learning. An interesting note is that US teachers tended to compare observations with their personal teaching style, while Chinese teachers tended to refer to the common Chinese teaching approach that is familiar to them. We will further discuss this subtle difference in later sections.

Below, we will report teachers' noticing differences in the top three teaching categories (see Fig. 4a, b, and c). Note that the results were mainly drawn from teachers' comments on their international peers' videos; however, domestic noticing data will be reported to enrich cross-cultural comparisons.

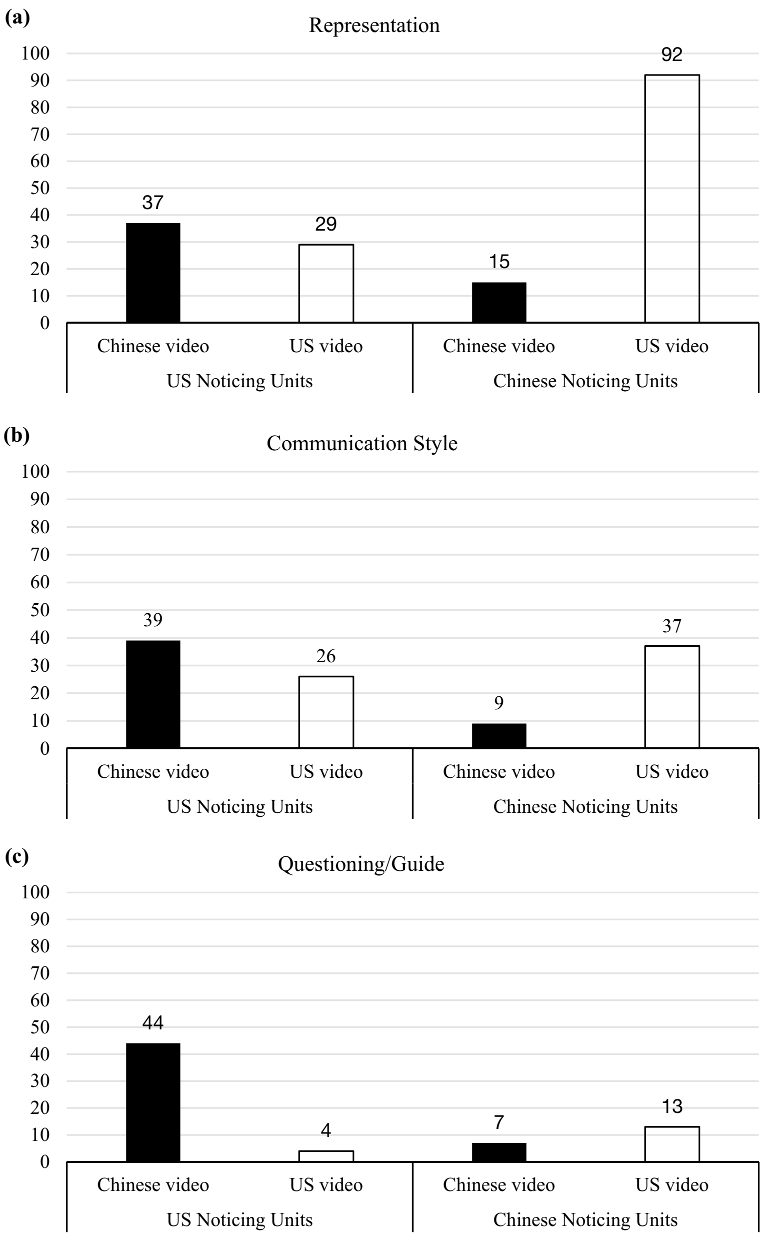


Fig. 4 Number of teachers' noticing units in the top three categories of teaching domains. Black and while bars represent the units devoted to Chinese and US videos, respectively. $N_{\text{US units}} = 179$ $(66 + 65 + 48)$; $N_{\text{Chinese units}} = 173$ $(107 + 46 + 20)$, also see Table 4

Table 5 US and Chinese teachers' level of noticing in online video comments

	Describing (%)	Reasoning (%)
US ($n = 120$)	96.7	51.7
China ($n = 113$)	92.0	85.8

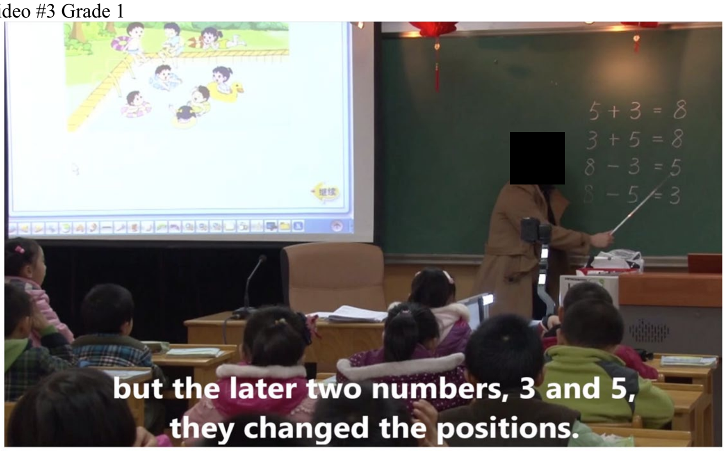
Cross-cultural differences of teacher noticing about representations

US teachers' noticing: real-world situations and linear models in Chinese videos

As indicated by Fig. 4a, US teachers paid greater attention to Chinese lessons' representation uses than that of their own peers [37/(37 + 29) = 56%]. In fact, the noticing units on Chinese lessons' representation uses mainly came from the US teachers [37/(37 + 15) = 71.2%]. Among the 37 US units, US teachers were mostly interested in Chinese lessons' use of real-world situations (19 units). Figure 5 (top) indicates a first-grade example (video #3). The teacher started the lesson with a swimming pool situation involving 3 children inside and 5 children outside of the pool. She then asked students to pose relevant story problems based on this real-world situation, which were solved as 5 + 3 = 8, 3 + 5 = 8, 8–3 = 5, and 8–5 = 3, resulting in an instance of inverse relations (also called "fact family").

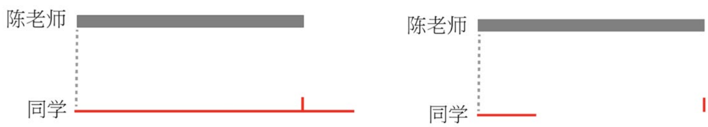
Typical US comments on video #3 included, "I noticed that the picture of the pool effectively elicited both addition and subtraction stories and equations from the children" (US-T17). While such comments were mainly descriptive, a few teachers provided reasoning. US-T13 stated, "Having students create word problems based on a picture shows higher level thinking skills." US-T2 linked her observations to regular US lessons and reflected on the role of a real-world situation in supporting students' meaningful learning:

Video #3 Grade 1



Video #6 Grade 2

The first problem stated, "Teacher Chen's favorite number is 45. This student's favorite number is 3 bigger than 45. What is this student's favorite number?" The second problem replaced "3 bigger" with "35 smaller."



(Translation: 陈老师-Teacher Chen; 同学-The student)

Fig. 5 Chinese teachers situating new learning in real-world situations (top) and using tape diagrams to facilitate concreteness fading (bottom). Image of Video#6 was redrawn by Mohen Li

> Wonderful how the concept began with an illustration of a real-world situation, (which) was verbalized as a story problem by a student, and only then was represented as an equation. Too often, math in the US stays very theoretical. It was clear from the lesson that the numbers were numbers OF something meaningful.

Interestingly, while the US comments focused on the role of real-world context itself, some Chinese teachers (e.g., Ch-T2, Ch-T9, Ch-T13) noticed differences in representational sequences in the parallel US and Chinese lessons. Below is Ch-T13's comments on video #3:

> US teacher shows the abstract "number triangle" first, and then picks student-favored images to create real-world scenario for each math fact; Chinese teacher would show real-world scenarios first, having the students talk about the meaning of the scenario and then trying to figure out the abstract math facts. …

The above comment on the "number triangle" is referring to a commonly used representation in US classrooms named "fact triangle." It is formed by a triangle shape with three numbers (e.g., 3, 5, 8) placed at each of the angles, which can be used to generate a fact family. Ch-T13 found that Chinese classroom representation uses were commonly different from the US lessons. Note that the sequence from real-world situation to numerical solutions is true for all Chinese worked examples, which is basically aligned with the "concreteness fading" approach recently recommended by the literature (Fyfe et al., 2015; McNeill & Fyfe, 2012). The representational sequence, however, did not draw the attention of US teachers, who mainly focused on the real-world situations itself.

US teachers also showed interest in the linear models (e.g., tape diagrams, number lines) in Chinese videos. Figure 5 (bottom) indicates a Chinese second-grade example (video #6) in which the teacher used a tape diagram to model a pair of comparison word problems. Among the 13 US codes about this model, teachers mainly acknowledged the tape diagram's ability to visualize the given quantities. US-T3 commented, "The teacher uses a bar graph to compare numbers that are bigger and smaller than her favorite number. … This is a great use of visualizing the value." Similarly, US-T10 stated, "Using the color bar to compare numbers was a good idea, it really helps the students visualize greater and less." For the same Chinese video, even though only two Chinese teachers commented on this video, their reasoning went beyond visualization of the individual quantities to emphasize the quantitative relationships and solution methods. Ch-T3 commented, "by using the tape diagram, she helped students visualize and understand the relationships among the numbers." Ch-T13's reasoning was similar: "… the use of visual representations decreased students' difficulties in understanding why different operations were used to solve the problems. This will enable students to summarize the relevant solution methods and comprehend the reasoning processes."

Chinese teachers' noticing: cubes, arrays, and tradeoffs in US videos

Chinese teachers' noticing about representations were mainly devoted to US lessons [92/(15 + 92) = 86%, see Fig. 4a]. In particular, they were most interested in the "cube" manipulatives (32 units) that were unique to US classrooms. Figure 6 (top) indicates two examples of cubes. In video #1, a second-grade teacher taught inverse relations. She first presented three number sentences of a fact family that involves 7, 1, and 8 and then asked students to figure out the last number sentence by using cubes. In video #14, a first-grade teacher formally taught the commutative property using the example

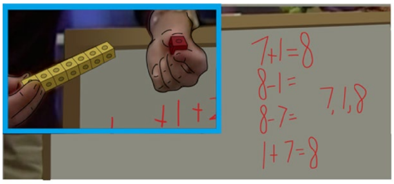
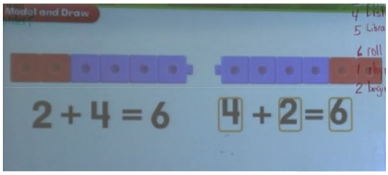
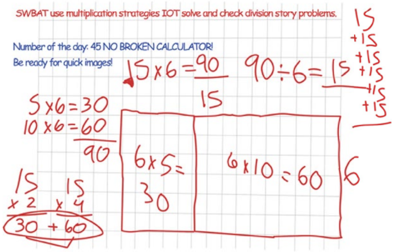
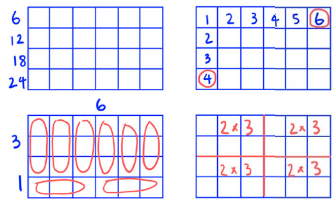
	Inverse Relations	Basic properties
Cubes	Video #1: Grade 2 	Video #14: Grade 1 
Arrays	Video #10: Grade 4 	Video # 17: Grade 3 

Fig. 6 Typical US classroom representations of interest to Chinese teachers. Images of Videos#1, #10, and #17 were redrawn by Mohen Li. In Video#10, 15×6 in the US means 15 groups of 6, which should be 6+6+6+…+6. However, the meaning of multiplication is often used without caution in the USA, as seen in this teacher's video. No Chinese and US teachers in this study questioned this aspect. In China, since the mathematics education reform in 2001, the Chinese curriculum has broadened the definition of multiplication; that is, 15×6 can be referred to both 15 groups of 6 and 6 groups of 15. This two-way definition has caused great debate, as it differs from the Chinese convention in which 15×6 refers to 6 groups of 15 or 15+15+15+15+15+15. For detailed information, please refer to Ding (2021a), Chapter 3

of 2+4=4+2, which was illustrated with cubes. Many Chinese teachers applauded this unique tool. For instance, Ch-T1 stated, "using the cubic trains can help students visualize the relationship between the four math sentences." Chinese teachers' positive descriptions share similarities with their US peers (10 units) who also thought the cube was a great visual representation.

However, while Chinese teachers acknowledged the cubes, their comments were relatively analytical and critical, considering the effect that the cubes may have on student learning. For instance, among 14 Chinese teachers who commented on the cubes in video #1, two suggested using a larger model: "However, in terms of the manipulatives, can it be bigger so the red and yellow parts could be clearly distinguished?" (Ch-T2). Five teachers suggested letting each student have their own manipulatives, "If a group of kids can do manipulatives together based on the teacher's directions, and then the teacher guides them to share their findings, I believe there will be more surprising results" (Ch-T16). The above critical but constructive comments only occurred with one US teacher (US-T1) who mainly concerned about student involvement rather than the learning effect: "Maybe the students next time could each have their own set of cubes so everyone feels involved." More insightfully, Ch-T12 questioned the purpose of using cubes in this lesson: "But what is the purpose of having students manipulate the cubes? Is it to help students have further understanding (of the inverse relations) or to double check (the computational answer)?" In fact, for video clip #14, Ch-T12 again raised a similar question: "Is the cube example used for students' discovery of the communitive property or is it used as an application of this property?".

Chinese teachers also demonstrated great interest (14 units) in the array/area model that was unique to US lessons. Figure 6 (bottom) illustrates two cases in which the teachers used grid paper to draw the arrays. If the grid was removed, the arrays would become area models. In video #10, a US fourth grade teacher used the array model to illustrate two related facts, $15 \times 6 = 90$ and $90 \div 6 = 15$. Based on this model students discussed various multiplication strategies such as $15 \times 6 = 5 \times 6 + 10 \times 6$; $15 \times 6 = 15 \times 2 + 15 \times 4$. In video clip #17, a US third grade teacher discussed various ways to break an array to compute 4×6 (e.g., breaking it into eight 1-by-3 arrays or four 2-by-3 arrays). In the online video comments, Chinese teachers not only uniformly acknowledged the array model but also linked what they noticed to a practice highly valued by Chinese mathematics education: Integrating numbers and diagrams (数形结合), which stresses connecting the concrete and abstract to support student learning. Typical comments on video #10 included, "The integration of multiplication and arrays not only infuses the distributive property but also lays a foundation for later learning of area" (CH-T1). Similar comments were made on video #17, "The use of arrays not only facilitates students' understanding of the distributive property but also develops their creative and divergent thinking" (CH-T15).

However, similar to the case of cubes, some Chinese teachers questioned the objectives of the lessons and the over-reliance on the arrays for computation. To Chinese teachers, the array model could serve as a great tool to illustrate the distributive property. However, both video clips focused on computation with little connection to this property. As such, Ch-T15 indicated that "I am not sure about the main objective of this lesson." Some teachers asked about the direction of the lesson, such as whether the big ideas were eventually revealed. Unfortunately, the selected video clips were representative of what the full lessons were targeting. Chinese teachers also discussed why the US classrooms depended on the array model to compute basic facts like $4 \times 6 = 24$. They question if it was because US classrooms lacked the multiplication Koujue, a powerful tool aiding instant recall of multiplication facts used in China from generation to generation (Zhang et al., 2019). For instance, Koujue "four six twenty-four" refers to $4 \times 6 = 24$ and $6 \times 4 = 24$. Multiplication Koujue is initially taught in second grade through a sense-making process and should eventually be mastered with automaticity. With the Koujue in mind, one Chinese teacher commented on the arrays in video #10 as the following:

> The integration of numbers and diagrams was used versatilely by this teacher. And, it seems that almost every American teacher is keen on this method. Is it because there is a lack of multiplication Koujue so the American teachers are "forced" to teach in this way? Or is it because they naturally favor this type of visual thinking? Chinese teachers won't use this approach because it is too time-consuming, or they may think it is unnecessary. As such, Chinese lessons appear to be less interesting or even boring. In this sense, multiplication "Koujue" is in fact a double-edged sword.

The above comment indicates Chinese teachers' knowledge-based reasoning through which they reflected upon the limitations and strengths of the arrays and multiplication Koujue, and the roles they play in supporting student learning. Chinese teachers' enthusiasm about the array models sharply contrasted their US peers, who made no comments on this model.

Cross-cultural differences of teacher noticing about teacher questions/guide

US teachers noticing: rigorous and deep questions/guide in Chinese videos

As indicated by Fig. 4c, US teachers' noticing on teacher questions/guide was mainly devoted to Chinese lessons [44/(44 + 4) = 91.7%] and the relevant noticing units on Chinese videos mainly came from US teachers [44 / (44 + 7) = 86.3%]. In fact, across the three categories of teaching—representation, questioning/guide, and communication style—US teachers demonstrated the highest interests in Chinese lessons' deep questioning/guide (see Fig. 4). US teachers noticed that Chinese teachers (a) asked purposeful questioning to elicit students' explanations of why (b) asked questions to facilitate meaningful classroom discourse, (c) asked comparison questions, and (d) used student generated work to further classroom discussions. Typical comments included the following: "There is true rigor in the line of questioning toward the end that requires the children to compare and contrast strategies that used addition and strategies that used subtraction to find an unknown" (US-T2 on video#5). "I noticed he had students explain and prove their thinking with counter examples and even challenged them to think of other representations besides numbers to fit this rule (commutative property)" (US-T5 on video #16). In the above comments, US teachers noticed that Chinese teachers requested comparisons, explanations, and even proof, which made their questions deep. US-T18 further commented on video#7 that Chinese teachers' questioning techniques enabled students to take ownership and responsibility of their learning:

> I noticed how the teacher employs the questioning technique to guide student learning. Even if a question answered is incorrect, she never just gives the students the answer. The teacher guides the students to figure out the correct answer and take ownership of their learning. She has set very high expectations of her students. She has created a safe environment conducive to accountability and responsible learning.

Although Chinese teachers commented less on their own videos (7 units from four Chinese teachers), they also pointed out the solid teacher guidance in their peers' lessons.

Chinese teachers' noticing: missing opportunities for deep questioning/guide in US videos

Teacher questions/guide in the US videos did not draw much attention (17 units). The main noticing units were from Chinese teachers [13/(13 + 4) = 76.5%, see Fig. 4c]. Overall, Chinese teachers' comments were critical in that they highlighted the missed opportunities for deep questioning and wondered about the underlying goals of the lessons. For instance, for video #1 (see Fig. 6), while the vivid metaphor of "fact family" was acknowledged by Chinese teachers, Ch-T4 commented that she would follow up with a deep question, "Why is it called fact family?" Other Chinese teachers commented on the missed opportunities for student thinking, "The students' responses were very commendable. They were thinking actively, resulting in a lot of methods. However, each time the teacher asked a question, there was not enough time for students to think it through" (Ch-T10 on video#21). In addition, some Chinese teachers compared the US and Chinese video clips and found that while Chinese teachers would explicitly point out the underlying concepts behind the worked examples, the US lessons often lacked such conclusive statements. For instance, when commenting on video #2, Ch-T15 stated, "US lessons have no conclusive statements and imitative practice, which is helpful for developing students' imaginative thinking. However, in comparison to Chinese lessons, such a teaching style is not beneficial for achieving the lesson objectives." By "no conclusive statements," Ch-T15 meant that there lacked a summary of the key concepts behind the worked example discussed, which differs from a common Chinese practice based on which Ch-T15 made this comparison. In fact, as shown by Table 4, Chinese teachers made 17 noticing units (8.3%) about lesson objectives, most of which wondered about the goal behind the worked examples taught in the US lessons.

In contrast to the above Chinese teacher comments, only 4 US units were categorized as teacher questions/guide, three of which indicated appreciation of their peers having students explain their thinking, which elicited different computational strategies.

Cross-cultural differences of teacher noticing about communication styles

In addition to noticing and reasoning about classroom representations and teacher questions/guide, both US and Chinese teachers expressed strong interests in the communication styles in US and Chinese videos.

US Teachers' noticing: structured and engaged communication style in Chinese videos

US teachers commented more about Chinese lessons' communication styles than their own peers' [39/(29 + 26) = 60%]. In fact, about 81.3% [39 / (39 + 9)] of the noticing units on Chinese lessons came from the US teachers (see Fig. 4b). In particular, US teachers noticed that Chinese lessons were very "structured," yet students were engaged in learning. Typical comments included, "The class is very structured; however, all students are on-task, engaged, and participate together at times" (US-T16 on video #3). "Very structured almost too structured lesson(s), but children do seem to be understanding the concept" (US-T7 on video #16). Some teachers (10 units) also noticed that Chinese teachers used various ways to engage students. US-T17 commented the following on video #3:

> What stands out for me is the various ways the teacher engages the students to think. I see that she directs them to make observations and connections independently, discuss ideas in a group of 4, and talk with "desk mates." I really like how she engages them in these various ways. The children are very involved and responsive.

Two US teachers used the term "turn and talk" to describe what they noticed, although this term was widely used to describe their own peers' videos (elaborated upon later). While the above comments mainly remained at a descriptive level, a few teachers highlighted patterns they noticed. For instance, US-T17 further commented on video #3, "I like how she empowers the students by calling the 'little teachers' to explain to the class and by saying 'this is your finding.'" This teacher made a similar comment on video #16 highlighting, "This is something I've noticed across the Chinese teachers' videos I've watched so far."

Chinese teachers' comments on the engagement of Chinese students' thinking in structured lessons, although less frequent, echoed the international patterns (9 units, 8.7%). However, a few Chinese teachers also elaborated on what was contained in a "structured" lesson that promoted student thinking. For instance, Ch-T13 commented on video #22, "The teacher takes full advantage of students' independent thinking, lets students discuss in groups of four, and share the ideas to class. Step by step, they explore the nature of the associative property, and then learn the differences and similarities between the commutative and associative properties by comparison. The logic is clear, and the students are very active." There were also a few domestic noticing units (n = 5) about Chinese teachers' logical thinking and language precision, which was not commented on by the US teachers.

Chinese teachers' noticing: relaxed and natural communication style in US videos

Chinese teachers demonstrated great interests in their international peers' communication styles [37 / (37 + 9) = 80.4%]. In particular, Chinese teachers frequently commented on the relaxed classroom climate, which was viewed as an indication of natural teacher-student relationships ($n = 11$ units). By "relaxed," Chinese teachers referred to US students sitting on the floor during learning. By "natural," Chinese teachers seemed to refer to the relatively casual teacher-student conversation style. Typical comments on video #1 included, "This is the first video that I watched, and I felt that the learning environment in US is totally different from the environment here, the kids are very relaxed. The teachers respect kids, and patiently remind the students who didn't understand her requirement" (Ch-T17). Ch-T11 articulated on this "natural" relationship when commenting on video #2, "It seems that in the USA, the relationships between teacher and students are very natural. That is, there aren't that many rules while teaching, it is mostly discussion-based. They can deal with any unexpected situation in a simple way." Other teachers were amazed by students' thinking in this relaxed climate, "The students sit on the ground relaxingly, but still highly focused on the teacher. They are willing to think and give their thoughts/responses to the teacher's questions" (Ch-T16). Noticing this was a quite different learning environment, Ch-T15 was curious about "how such little kids can focus their minds and thinking in this relaxed teaching style." In video #15, this teacher further wondered why such a natural classroom climate did not exist in the current Chinese classrooms and whether such an environment can be nurtured in China.

Chinese teachers' noticing of the relaxed and natural communication style was somewhat different from the domestic observations. Among the 26 domestic noticing units [26/ (26 + 37) = 41.3%], the US teachers most frequently (13 units) commented on the "turn and talk," in which students turn to each other to talk about something for a short time period. Typical comments included, "I liked the turn and talk" (US-T1) and "I notice the 'turn and talk' strategy invites all the children to share their thinking" (US-T17). There was only one noticing unit devoted to teachers' patience and positive reinforcement in classroom.

Discussion

In this study, we have invited US and Chinese expert elementary teachers to watch and comment on videotaped mathematics lessons. Consistent with prior research (Borko et al., 2008; Santagata, 2009; van Es & Sherin, 2008), we found cross-cultural videos facilitated teachers' deep reflections on their own practices (Hollingsworth & Clarke, 2017; Kleinknecht & Scheider, 2012; Moran et al., 2015). Differing from prior studies, our research involved mathematics lessons with matched concepts, which allowed teachers to make direct and focused comparisons. Below, we first discuss the cross-cultural teacher noticing based on the videos, followed by implications.

Similarities and differences of teacher noticing of cross-cultural videos

Using the two-dimension noticing framework ("what" and "how") established from prior research (e.g., Seidel & Stürmer, 2014; van Es, 2011), we found that what US and Chinese teachers' notice share similarities at a macro level. For instance, teachers in both countries pay more attention to the teaching domain, especially representation uses, deep questions, and classroom communications, which are the main components of teachers' balanced professional vision (Seidel & Stürmer, 2014; Stürmer et al., 2013). In particular, although teachers paid more attention to their international peers' lessons, teachers on both sides acknowledged the various visual supports for students' learning and valued deep questions that promoted students' thinking. In addition, teachers made positive comments on the different classroom climate or communication styles in peer countries, likely due to their common interests in students' engagement. In this sense, our findings are consistent with prior research that expert teachers tend to reason about the effect of teaching on student learning (Jacobs et al., 2010; van Es & Sherin, 2008).

Despite the similarities in teacher noticing at the macro level, there are cross-cultural differences at the micro-level. Besides the fact that US and Chinese teachers preferred to watch their international peers' videos that contained unique features, our comparison of what teachers noticed about the same videos indicated cross-cultural differences. As reported, while teachers in both countries commented on the "real-world situation" in Chinese classrooms, US teachers focused more on the concrete representations, while Chinese teachers attended to representational sequences (e.g., from concrete and abstract). When commenting on deep questions in the US lessons, US teachers found such questions effective in eliciting varied computational strategies while Chinese teachers noticed missing opportunities to elicit the underlying ideas. In addition to differences in "what" teachers noticed, teachers also differed in "how" they reasoned about their noticing. Overall, US teachers made fewer reasonings than their Chinese peers who attended more to the effect of teaching strategies on student learning. For instance, while both the Chinese and US teachers valued the cubes and linear models, US teachers focused on their ability to aid in visualization, either generally or of individual quantities; yet, Chinese teachers attended to how the models can help illustrate the embedded quantitative relationships and the underlying concepts. The above findings are consistent with prior research findings that Chinese teachers are more skillful in attending to mathematics content (Miller & Zhou, 2007) and the depth of students' mathematical thinking (Dreher, et al., 2020; Yang et al., 2019, 2021), which some may view as an indicator of teachers' deep noticing (e.g., Sherin, 2011; van Es. 2011).

The above cross-cultural differences in teacher noticing echo research assertions that teachers' noticing has cultural dependency (Dreher, et al., 2020; Yang et al., 2019, 2021), which lies in cultural differences of teachers' knowledge, beliefs, and experiences (Clarke & Hollingsworth, 2002; Louie, 2018). Prior studies indicate that Chinese elementary teachers generally possessed profound mathematical knowledge (Ma, 1999) and believed that concrete representations are tools for learning abstract concepts (Cai, 2005; Cai & Ding, 2017; Li, 2007). They also tended to ask deep questions in classrooms to facilitate abstract thinking (Ding et al., 2019; Ding, 2021), which are common features of CHC classrooms (Steve & Stigler, 1992; Stigler & Hiebert, 1999; Tran, 2013). Such knowledge, beliefs, and experiences may have triggered them to critically ask whether US teachers used representations to seek computational answers or to develop an understanding of big ideas. In contrast, US expert teachers in this study were more positive than their Chinese peers. As long as students were provided with visualization and engaged in mathematical learning (whether they were working on computation or learning about underlying ideas), teachers were generally satisfied. This could reflect US teachers' own knowledge limit or indicate a Western cultural norm of effective teaching that focuses on individual's learning needs (Correa et al., 2008; Dreher, et al., 2020; Jacobs & Morita, 2002; Louie, 2018; Yang et al., 2019). This is also consistent with prior findings that teachers from Western classrooms were more easily satisfied with lesson evaluation than their CHC peers (Fernandez & Cannon, 2005). Taken together, the above cultural difference suggests a balanced view between being satisfied with students' current acceptable level of learning and the necessity to push them towards deep thinking (Louie, 2018).

Cross-cultural differences in teacher noticing also seems to align with what has been valued in educational reform in each country. As reviewed, current Chinese mathematics education reform calls for attention to students' learning process and affection (Ministry of Education, 2001; Xu, 2017). Consequently, Chinese teachers in this study expressed strong interests in the cube and array models that aid the learning process and enable integrating numbers and diagrams (数形结合). They also attended to the relaxed US classroom climate and natural teacher-student relationships, which is consistent with current emphasis on students' affection in China. Similarly, US mathematics education has undergone reforms to change from procedural- to conceptual-based teaching (NCTM, 2000; NGAC & CCSSO, 2010). Even though it is argued that teaching style is robust to change (Ball, 2000; Stigler & Hiebert, 1999), teachers' strong interests in Chinese lessons' rich representation uses and deep questioning/teacher guide signify an initial and important attempt to make changes.

Learning from teachers' noticing: implications and contributions

Findings about teachers' cross-cultural video noticing have implications on classroom research and practice, teacher support, and methodological choices. First, teacher noticing appears to be an ideal research site that integrates theory and practice organically. In this study, US teachers were mostly interested in how Chinese lessons situate new learning in real-world situations (e.g., swimming pool), while Chinese teachers were interested in US lessons' use of manipulatives (e.g., cubes). An integration of these insights regarding representation uses—from story context to manipulatives to number sentences—contributed practice-based examples to the cognitive psychology literature on "concreteness fading" (Fyfe et al., 2015). This integrated insight may also be beneficial for teaching practice in both countries. For instance, if Chinese teachers add the middle step of manipulatives, below average learners may have a better chance of making connections between concrete and abstract. Similarly, if US teachers start with real-world situations before using cubes or other manipulatives, students may have more opportunities to experience modeling and problem solving. In addition, Chinese teachers' wonderings about the relationship between arrays, multiplication facts, and multiplication Koujue may lead to discussions of the various dichotomies, including between process (reasoning upon the array) and product (mastery of the basic facts), between concept (using arrays to learn the distributive property) and computation (using arrays to find the answer), and between concrete (array model) and abstract (going beyond the array). Leung's (2001) argued that such dichotomies are deeply rooted in different cultural values and paradigms between East Asian and Western education system. Nevertheless, we think there is a shared teaching and learning culture that permits learning from cross-cultural resources. For instance, US students may benefit from learning Chinese multiplication Koujue after their reasoning with array models. In fact, many US teachers train students to skip count (e.g., six, twelve, eighteen, Ding, 2021). Instead of counting the products only, students could count both factors and the associated product, which is indeed the same as multiplication Koujue (e.g., One six six, two six twelve, three six eighteen). On the other hand, Chinese curricula mainly uses the equal-groups meaning of multiplication. They may consider incorporating the array model with the multiplication Koujue, which can further enhance students' ability to integrate numbers and diagrams, a skill valued by current Chinese mathematics education standards (Ministry of Education, 2001).

Teachers' video noticing in this study also provides insights into needed teacher support. For instance, US teachers did not spontaneously attend to the representational sequence in Chinese videos (from concrete to abstract), even though this is an effective approach supported by recent research on "concreteness fading" (e.g., Fyfe et al., 2015). This indicates a need for intervention from researchers. In fact, during our subsequent summer workshop, we highlighted this Chinese lesson feature and introduced "concreteness fading" to US teachers who were thrilled and implemented this approach in their future teaching (Ding et al., 2021b). Additionally, US teachers in this study noticed that there seemed to be a common "Chinese teaching approach," as evidenced by Chinese teachers' tendency to compare between what they noticed from the US lessons and what might occur in Chinese classrooms. These findings are similar to Jacobs & Morita (2002), where Japanese teachers also held only one ideal cultural lesson script of effective mathematics teaching. Why do teachers in high-achieving countries like Japan and China hold a similar script within the culture and why is a common lesson image lacking in the USA? One of the interpretations is that Japan and China are affected by CHC, which encourages teachers to learning from others through various opportunities (Li & Huang, 2013). Can such an approach be applied to US culture to improve teaching quality at a large scale? As Hiebert & Stigler (2004) pointed out, instruction among US mathematics classrooms has largely but unnecessarily varied not because of consideration of diversity but due to the lack of common teaching goals, approaches in reaching the goals, and sharable teaching materials, which in turn results in ineffective teacher learning. In our study, while the US expert teachers realized the importance of visualization, deep questions, and student engagement, they did not seem to possess a specific cultural script to evaluate classroom teaching. As such, is it possible to establish high quality cultural scripts to help enhance teachers' expertise (e.g., noticing skills) in an individualized society like the USA? Clearly, cultural differences may be a factor that hinders learning from cross-cultural resources. However, Kim et al. (2011) called for attention to the different dimensions between "culturally contextualized" and "semantically decontextualized." Based on these notions, we think some of the aspects attended to by Chinese teachers (e.g., deep questioning and representation uses that focus on quantitative relationships and mathematical concepts) are semantically decontextualized, which may inspire the field to develop a cultural script of effective teaching to support US teachers. In fact, given that our cross-cultural video clips are empowering for teachers in both countries, we think that annotated cross-cultural videos like ours may also serve as sharable materials to support teachers' classroom practice.

Lastly, findings in this study contribute methodological insights to cross-cultural video studies and teacher noticing research. In the literature, teachers' noticing about classroom climate was often reported as superficial (Borko et al., 2008; Christ et al., 2014; Jacobs & Morita, 2002; Santagata, 2009; van Es & Sherin, 2008). In our study, we found that Chinese teachers showed overwhelming interests in the relaxed US classroom climate and natural teacher-student relationship. According to much of the literature, this may be classified as a superficial level of noticing (e.g., van Es & Sherin, 2008), a finding that is counterintuitive to prior studies' indication that Chinese teachers had profound mathematical understanding (Ma, 1999) and tended to focus on mathematical concepts (Cai & Ding, 2017; Miller & Zhou, 2007). However, if we consider not only the "what" but also the "how" dimensions of teacher noticing (Seidel & Stürmer, 2014; van Es., 2011), we would find that Chinese teachers made deep reflections and reasoning on what they noticed, for instance, by asking why such a natural teacher–student relationship is missing from Chinese classrooms. In fact, in our later in-person Chinese summer workshop, a few teacher participants further asked why Chinese teachers need to look like an authority figure who controls the pace of learning? Is it because they have to complete the structured lesson to ensure instructional depth? Is it possible depth has been over-pursued at the cost of students' full development? Moreover, some Chinese teachers recognized the tension between what they liked and the reality of a large class size and rigorous lesson structure, which calls for policy makers and mathematics educators in China to explore learning environments that are beneficial for all students, both academically and affectively. Our findings about Chinese teachers' noticing of US classroom climate provides a convincing case for the importance of considering both the "what" and "how" dimensions when analyzing teachers' video-based noticing (van Es, 2011), especially in cross-cultural videos where teachers' cultural backgrounds shape their noticing (Dreher, et al., 2020; Louie, 2018).

Limitations and future directions

This study has some limitations in the sample choice, video source, and data collection and analysis. First, our sample only involved 17 expert teachers in each country. The small sample size and the specific teacher expertise limit the generalizability of this study. For instance, the relaxed US classrooms with focused students may be an indicator of expert teachers, which may not be true for all US classrooms, where some students exhibit frequent misbehaviors (Ding et al., 2008). Future studies may invite typical or novice teachers to observe typical cross-cultural videos, which may provide alternative findings of teachers' noticing differences. The second limitation is related to video sources. Teachers in this study were exposed to 25 US and Chinese video clips. Perhaps due to the amount of work and time commitment, teachers, especially Chinese teachers, mainly commented on their international peers' videos, resulting in limited domestic noticing data for analysis. Moreover, our annotated videos clips in this study centered on the teaching of worked examples, which average 10–13 min long. Teachers in both countries expressed a great desire to watch the whole lesson as they were curious about the follow-up practice activities. Future studies may present fewer videos (could be full lessons) but require teachers to watch them all, which could provide more balanced data for deep explorations. In addition, our translation of the videos could have utilized a more rigorous method (e.g., using back-translation, double-translation), which could have provided the teachers with even more validated data. Third, our data collection and analysis also have limitations. As mentioned earlier, when teachers in each country entered their video comments, they could see their domestic peers' entries. In this sense, data collection in each country was not independent. Future studies could improve this part of design to ensure independent data collection (e.g., one cannot see others' comments before own data entries). In addition, during our coding of "what" teachers noticed, we used a bottom-up approach that considered a wide range of noticing codes generated by teachers. While taking all the teacher noticing units into consideration may be a strength, future studies may focus specifically on teachers' responses to students' thinking, which is a critical and interesting aspect that has been widely explored in prior research.

Conclusion

Despite the above limitations, this study contributes to the literature by demonstrating that cross-cultural videos, especially with matching topics, are an effective tool in exploring teacher noticing. Teachers' profound reflections on classroom practices and their demonstrated learning interests in their international counterparts are informative for the field of teaching and teacher education. We conclude that cross-cultural videos with matching topics are powerful and may be further used in other learning settings, such as teacher professional development and methods courses, to facilitate teacher noticing and learning.

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