THIRTY YEARS OF RESEARCH: INTERPRETATIONS OF THE EQUAL SIGN IN CHINA AND THE USA [1]

ROBERT M. CAPRARO AND MARY MARGARET CAPRARO

Texas A&M University

MEIXIA DING   XIAOBAO LI

University of Nebraska–Lincoln   *Western Carolina University*

Summary.—This study examined students' conceptions about the equal sign in light of historical findings with an international comparison group. Textbooks for preparation of students as mathematics teachers were examined. Participants were sixth-grade students from the USA ($n = 105$) and China ($n = 145$). About 98% of the Chinese children correctly answered all items by providing conceptually accurate explanations, but only 28% of the U.S. sample did. Textbooks for education majors who would teach in the USA rarely discussed the equal sign as equivalence while the Chinese texts introduced the "equal sign" in a context of relationships and discussed it as "balance," "sameness," or "equivalence."

Teachers and researchers have long acknowledged that students are apt to misunderstand the equal sign as an operator sign, that is, a signal for "doing something" rather than a relational symbol of equivalence (Thompson & Babcock, 1978; Behr, Erlwanger, & Nichols, 1980; Sáenz-Ludlow & Walgamuth, 1998; National Council of Teachers of Mathematics, 2000). Falkner, Levi, and Carpenter (1999) reported that all sixth graders in their study solved "$8 + 4 = \square + 5$" with 12 or 17. This finding was consistent with that of Kieran (1981), who pointed out students' beliefs that the equal sign was a "signal to do something." Popular use of calculators may reinforce this misconception. Moreover, children may be predisposed to think of equality as calculating answers and this misconception could persist until they receive direct instruction (Capraro, Ding, Matteson, Li, & Capraro, 2007). Similar results were found by Knuth, Stephens, McNeil, and Alibali (2006) and McNeil, Grandau, Knuth, Alibali, Stephens, Hattikudur, and Krill (2006).

Seo and Ginsburg (2003) observed students' understandings of the equal sign were context-related. And, McNeil, *et al.* (2006) examined a textbook series for middle school and suggested that students' interpretations of the equal sign were likely to be shaped by context. Therefore, developing an appropriate conception of the equal sign has been related to cognitive development, teachers' instruction, and classroom materials.

After more than three decades of applying the suggestions provided by research, do students' still have a misconception regarding the equal sign? Accordingly, the purpose of this study was to examine the relation between explanations in textbooks for preparing undergraduates to teach mathematics and students' understanding of the equal sign in the USA and China.

METHOD

Participants were U.S. students in Grade 6 ($n = 105$) who represented various ethnic minorities and sixth graders ($n = 145$) in schools located in widely differing areas of China: rural, urban, and suburban. Four items were administered to these students ($N = 250$) to assess their understanding of the equal sign. Textbooks for undergraduates aspiring to teach mathematics were examined from both the USA and China.

RESULTS AND DISCUSSION

The comparison of U.S. and Chinese students' number of correct responses to the four items are provided in Table 1. Inspection indicated that the Chinese sample gave a significantly higher percentage of correct responses than the U.S. sample across items. Comparing results of previous researchers, the present U.S. sample indicated an improved understanding of the equal sign. However, misconception of the equal sign was still present

TABLE 1
PERCENTAGE OF CORRECT ANSWERS BY TWO GROUPS IN GRADE 6

Group	Question 1 $6+9 = \square +4$	Question 2 $\square +8 = 12+5$	Question 3 $\square +3 = 5+7 = \square$		Question 4 $6+8 = 3+11$	n
			1st Box	2nd Box	True or False	
USA	28.6	28.6	23.8	86.7	47.6	105
Chinese	98.6	96.6	98.6	97.9	91.7	145

and has untold implications for students' successes and failures in algebra. An examination of textbooks for preparation of students to teach mathematics showed the equal sign was presented in Chinese books as a relational symbol of equivalence which contrasted with the presentation in U.S. books. Hence the comparison perhaps reflected pedagogical issues as a possible basis for the disparate results. It is suggested that curriculum editors, educators, and teachers provide appropriate contexts that emphasize "equivalence" for students to learn the equal sign.

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