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Representational Implications for Understanding Equivalence

Inferiors revolt in order that they may be *equal*; equals revolt in order that they may be superior (Aristotle, 322BC).

Teachers and researchers have long recognized that students tend to misunderstand the equal sign as an operator, that is, a signal for “doing something” rather than a relational symbol of equivalence or quantity sameness (Behr, Erlwanger, & Nichols 1980; National Council of Teachers of Mathematics [NCTM], 2000; Sáenz-Ludlow & Walgamuth, 1998; Thompson & Babcock, 1978).

Students’ equal sign misconception has been researched for more than thirty years (Weaver, 1971, 1973) with little refinement in the theory. It was popularly believed that younger students were not developmentally ready to work variations of open numbers sentences, such as missing addend problems (Thompson & Babcock, 1978). In fact, misconceptions about the equal sign were identified in kindergarten students even *before* formal instruction (Falkner, Levi, & Carpenter, 1999). However, it is clear that with specific instructional guidance, elementary students can understand the equal sign expresses a relation (Baroody & Ginsburg, 1983; Carpenter, Levi, & Farnsworth, 2000; Saenz-Ludlow & Walgamuth, 1998).

These previous studies did not employ random selection, examine the phenomenon internationally, or explore how materials used with elementary teachers prepare them to teach the equal sign. A major benefit of international comparisons is that cross-cultural comparisons lead to more explicit understanding of one’s own implicit theories about how children learn mathematics (Stigler & Perry, 1988).

We examined variables that could contribute to students’ equivalence misconception and whether the

equal sign misconception was still manifest in a U.S. sample and present in a Chinese sample. First, six U. S. methods books were chosen and examined to determine what strategies were being presented to prepare U.S. elementary preservice teachers (PTs) to teach equivalence and the equal sign to their future students. Strategies ranged from nothing at all (Smith, 2001), to a single paragraph (Cathcart, Pothier, Vance, & Bezuk, 2006; Reys, Liguist, Lamdbin, Smith, & Suyday, 2004; Van de Walle, 2004), to an activity (Tucker, Singleton, & Weaver, 2006). Seemingly, the authors of these textbooks expect that PTs understand the issues related to the equal sign and the implications for their students. Both Reys et al. (2004) and Van de Walle (2004) alert PTs to the common misconception that the equal sign means “the answer is next.” Both authors dutifully inform readers that using the calculator reinforces the equal sign misconception since the answer comes after the equal sign is pressed. To counteract this misconception, a balance scale can help students develop the correct conceptual understanding of equality and the equal sign (Reys et al., 2004). Van de Walle (2004) suggests that teachers should use the phrase “is the same as” (p. 139) instead of “equals” as students read number sentences.

In addition we also examined first through sixth grade U.S. student textbooks series between 1970 and 2000 to locate caveats provided to the teacher, suggestions for instruction, and the types of activities dealing with the equal sign and the idea of equivalence. The textbooks from earlier years provided almost no background information to teachers about the equal sign even though there was a mean of 8 pages of suggestions for teaching greater and less than. In some textbooks no definition was offered for equal beyond the “the same as” and in many no definition was offered at

all. However, a greater variety of problem types was apparent in earlier textbooks but totally disappeared in current textbooks.

Certain selected textbooks have a powerful influence in China in terms of market share, thus three sets of textbooks from first to sixth grade and the corresponding first grade guidebooks were selected for examination. The most popular text was published by the *People's Education Press* (PEP) (Lu & Yang, 2005) and controlled more than 70% of the market until 2002 (Li, 2004). The second text examined was published by *Jiang Su Education Press* (JSEP) (Su & Wang, 2005a). Thirdly we examined texts published by the *Beijing Normal University Press* (BNUP) (Research Group of National Mathematics Curriculum Standards for Compulsory Education, 2005).

Two consistencies were found in Chinese textbooks and guidebooks. First, there were no great differences among the three editions of Chinese textbooks because China has a centralized educational system that has adopted the same standards and textbooks for many years. All of the textbooks introduce the equal sign in conjunction with “>” and “<” before introducing the concepts of addition and subtraction. Correspondingly, teachers are encouraged to teach the equal sign within various comparison contexts. Thus, Chinese students encounter the concept of the equal sign as a relational symbol from the very beginning. In addition, Chinese textbooks provide many non-standard contexts (Capraro & Capraro, 2006; McNeil et al., 2006) to develop students' understandings of the equal sign throughout grades one to six. Some examples are: operations without equal signs; continuous operations where “arrows” are used in place of the equal sign; completing a table containing the terms dividend, divisor and quotient with one number missing from one of the terms; and filling in missing numbers $1 + \square = \square$.

U.S. sixth graders' misconception about the equal sign were not limited to the form $a + b = \square + c$.

Our study (Capraro, Matteson, Capraro, Ding, & Li, 2007; Matteson, Capraro, & Capraro, 2007) supported the same historic findings for the misconception. Our U.S. sixth grade sample and previous U.S. samples lag far behind Chinese sixth grade samples (Ding, Li, Capraro, & Capraro, 2007), which may be indicative of pedagogical issues and an answer for the disparate results. Findings indicate that misconceptions are still manifest in the U.S., and textbooks do little to mitigate the problem in the United States while in China students are able to interpret the equal sign as a relational

symbol of equivalence. We found that the inclusion of multiple representations for equivalence in textbooks and guidebooks in China make a difference in assisting students to correctly interpreting the equal sign.

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