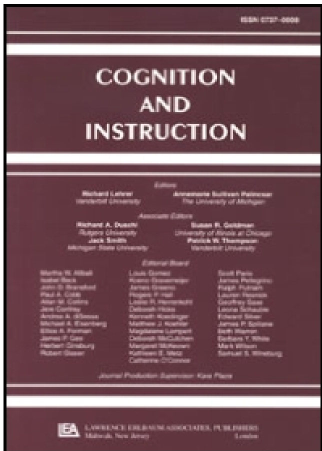

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Sources of Differences in Children's Understandings of Mathematical Equality: Comparative Analysis of Teacher Guides and Student Texts in China and the United States

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This study reports findings from comparative samples of sixth-grade Chinese and U.S. students' interpretations of the equal sign. Ninety-eight percent of the Chinese sample correctly answered 4 items indicating conceptions of equality and provided conceptually accurate explanations. In contrast, only 28% of the U.S. sample performed at this level. We examine how teacher preparation materials, students' textbooks and teachers' guidebooks treat equality in each country. U.S. teacher preparation textbooks rarely interpreted the equal sign as equivalence. On the contrary, Chinese textbooks typically introduced the equal sign in a context of relationships and interpreted the sign as "balance," "sameness," or "equivalence" and only then embedded the sign with operations on numbers.

Teachers and researchers have long recognized that students tend to misunderstand the equal sign as an operator, that is, a signal for "doing something" rather than a relational symbol of equivalence or quantity sameness (Behr, Erlwanger,

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& Nichols 1980; National Council of Teachers of Mathematics [NCTM], 2000; Sáenz-Ludlow & Walgamuth, 1998; Thompson & Babcock, 1978). Falkner, Levi, and Carpenter (1999) reported from their investigation in a single school of the problem "8 + 4 = a + 5", all 145 sixth-graders filled the box with 12 or 17. These types of errors were caused by students' misunderstanding the equal sign as a command to carry out calculations just like the "=" button on a calculator. According to Carpenter, Franke, and Levi (2003), students may have three different misconceptions of the equal sign: the equal sign may mean "the answer comes next" ignoring the rest of the problem (p. 10), that is, $8 + 4 = [12]$; students may "use all the numbers" (p. 11) such as $8 + 4 + 5 = 17$, arbitrarily restructuring the sentence; or they may put 12 in the box and "extend the problem" (p. 11) as $8 + 4 = [12] + 5 = 17$). These misconceptions about the equal sign were common from grade one through six. Less than 10% of the students at each grade level answered the above problem correctly (Carpenter et al., 2003). Further it was suggested the "do something signal" persisted from preschool through secondary and possibly even at the university level (Kieran, 1981). In order to achieve appropriate interpretations of the equal sign, it is necessary to understand why students have difficulty with interpreting the equal sign.

THE PROBLEM—THE THEORIES

Although many factors are related to children's mathematics achievement or understanding, there is little disagreement that curricula have a large impact on students' learning and teachers' teaching (Porter, 1989; Reys, Lindquist, Lambdin, Smith, & Suydam, 2003; Reys, Reys, & Chávez, 2004; Schmidt et al., 2001). In general, textbooks determine how and what teachers taught and students learned. One of the important ways textbooks affect students' construction of correct or incorrect understandings of the equal sign is in its presentation (McNeil et al., 2006; Seo & Ginsburg, 2003). Baroody and Ginsburg (1983) conducted a study of first, second, and third graders who used the *Wynroth* mathematics curriculum. In *Wynroth*, the "=" sign was defined as a literal translation "the *same* number as," which demonstrated relational understanding of the equal sign to avoid the phrase "the answer is," which conveys an operational connotation. Therefore, students were asked to say "3+5=" as "three plus five is the same number as?" The *Wynroth* curriculum introduced the equal sign situated within various forms (e.g., $3 + 1 = 2 + 2$; $3 + 1 \neq 4 + 2$; $3 + 1 < 4 + 3$) to emphasize a relational meaning rather than within the context of addition. All first graders in that study used the *Wynroth* curriculum for 7 months, as did the second and third graders, but the second and third graders used a traditional curriculum in earlier grades before the adoption of *Wynroth*. By testing the students' conceptions about equality, the researchers found a conceptual difference between first and second graders' responses. Thus, the researchers concluded first graders might have had an advantage over both second and third graders who were influenced by previous instruction in another curriculum. Therefore, some children may have insisted on maintaining an "action orientation" involving the use of their everyday addition strategies. Baroody and Ginsburg also suggested that teachers could cultivate students' relational views of the equal sign by emphasizing this view at the very beginning of formal mathematics instruction.

Seo and Ginsburg (2003) examined elementary textbooks and requisite workbooks for how the equal sign was presented. Their findings indicated the context in which the equal sign was introduced was limited to performing operations. Hence, the equal sign rarely appeared without plus or minus signs, and most number sentences were presented in canonical format; such as $a + b = c$ or $a - b = c$. Thus, Seo and Ginsburg concluded that most American textbooks did not support students' understanding of the equal sign with a relational meaning, but also reinforced students' misunderstanding of the equal sign as operation. McNeil et al. (2006) extended Seo and Ginsburg's (2003) study by examining middle school textbooks. They found equations with operations on both sides of the equal sign rarely appeared in any of four textbooks, which partially explained why middle school students still misinterpreted the equal sign.

The effect of instruction on understanding of "equal sign" was studied by Denmark, Barco, and Voran (1976) who designed a teaching experiment with the concept of equality as the focus. Their findings demonstrated that first graders were able to acquire the concept of the equal sign as equivalence in a form such as $6 = 4 + 2$ after two months of instruction. Thus, these researchers concluded that instruction was a contributing factor to understanding the equal sign as a relational symbol.

In 1991, Wolters reported the effect of structuralistic instruction on second-grade students' understanding of the equal sign. This instruction included three levels of activities: (1) concrete manipulative/verbal level—teachers took two objects such as a teaspoon and a knife and elicited various relational responses from students (e.g., the knife is bigger than the spoon); (2) perceptual representation level—teachers provided relational symbols ($=, \neq, <, >$) for students' perceptual comparison of objects; and (3) abstract symbolic level—teachers asked students to notate the relationship between two or three numbers (less than ten) using relational symbols. The results indicated that students who received structuralistic instruction demonstrated better understandings of the equal sign than their counterparts. Therefore, Wolters concluded that students' mastery of the concept of equivalence was significantly related to the type of instruction they received. Sáenz-Ludlow and Walgamuth (1998) identified this as a teacher factor. This teacher factor focused on at-risk third graders' understanding of the equal sign in a yearlong teaching experiment. Students initially interpreted the equal sign as an operator symbol but by the end of the school year were able to interpret it as "quantitative sameness." The researchers concluded that students' expansion of their conceptualization of the equal sign was due to their active role in classroom discussion, the properties of the mathematics task presented by the teacher, and the teacher's intellectual sensitivity to the balance between teaching and learning (Sáenz-Ludlow & Walgamuth, 1998).

RESEARCH NICHE

To further clarify the roles played by textbooks and teachers for students' understanding of equality, we compared the performance of Chinese and American students and closely examined how methods and student textbooks presented the equal sign. Our sample included students in China whose instruction and national curricula was similar to what Baroody and Ginsburg (1983), Wolters (1991), and Sáenz-Ludlow and Walgamuth (1998) attempted to attain. Thus, we asked the following questions: *Does the U.S. student sample misunderstand the equal sign as an operator? Is the equal sign misconception present in the Chinese student sample? How do teacher preparation/guidebooks in U.S. and China address the equal sign? How do Chinese student textbooks present the equal sign?*

METHOD

Participants

The terms U.S. and Chinese are used to distinguish between U.S. and Chinese samples and neither term is intended for broad generalizations to either nation. The U.S. sixth-grade sample was representative of various ethnic, SES , and urbanicity groups identified in 3 schools in one school district from 10 teachers' classes (Table 1). Students belonging to each group were selected (from 1,000 students) to attain the U.S. study participants ($n = 105$). Every sixth-grade student identification number was entered into SPSS and approximately 10% of the cases were selected using the select cases feature. Then consent was requested, and approximately five students were lost because parental consent was not returned. We chose Chinese sixth-grade students ($n = 145$) as a natural comparison group. The sample was obtained from three schools selected from widely differing areas, one rural, one urban, and one suburban of Jiang Su, a large province.

Instruments

Based on previous studies, we designed a 4-item instrument to diagnose students' conception of equality. The first item mirrored items used in previous studies,

TABLE 1
Demographics of the U.S. and Chinese Samples

	Ethnicity				*Gender*		*SES*			$N_{selected}$, ($N_{inDistrict}$) *Sample*
Group	*B*	*H*	*W*	*O*	*F*	*M*	*Low*	*Med*	*High*	*(Population*
U.S. Sample	44	34	12	15	59	46	31	11	63	105
Population	(1664)	(1539)	(541)	(415)	(2495)	(1664)	(1394)	(541)	(2224)	(4,159)
Chinese Sample	Homogeneous-Han race				74	71	28	77	40	145
Population					(561)	(539)	(209)	(583)	(308)	(1,100)

Note. For SES: low, medium, and high correspond to free, reduced, and self paid lunch for the U.S. sample and for the Chinese sample: low, medium, and high, correspond to perceptions based on social factors including city and parents' employment. U.S. District percentage comparisons are in parentheses and for the Chinese sample school percentages are in parentheses. B = Black, H = Hispanic, W = Caucasion—non-Hispanic, O = Includes multi-racial, Asian, American Indian, and Pacific Islander. Numbers in parentheses represent the population from which the sample was drawn.

where the missing addend followed the equal sign (e.g., $6 + 9 = a + 4$) (Carpenter et al., 2000; Falkner et al., 1999). A second item placed the missing addend at the beginning of the number sentence. The other two items included a true–false equivalency statement $6 + 8 = 3 + 11$ and a sentence with two equal signs and two missing numbers $a + 3 = 5 + 7 = a$. *Cronbach's alpha* was .89 for the full data set.

Textbook Analyses

We selected six U.S. preservice mathematics teacher preparation textbooks (i.e., Cathcart, Pothier, Vance, & Bezuk, 2006; Hatfield, Edwards, Bitter, & Morrow, 2005; Reys et al., 2003; Smith, 2001; Tucker, Singleton, & Weaver, 2006; Van de Walle, 2004) for review. The selected textbooks accounted for just over 78% of university-based elementary teacher preparation programs (data supplied by textbook publishers). Two Chinese mathematics methods textbooks (Shen & Liang, 1992; Ye & Zhao, 2000) and corresponding teacher guidebooks and student textbooks were examined. In addition, the *National Mathematics Curriculum Standards for Compulsory Education* (Ministry of Education, 2001) was also examined, because it has great impact on teaching in China (Ma, 1999). Every page of each book was examined for the introduction and use of the equal sign and teaching information provided to the teacher (e.g., teacher notes). Three Chinese textbook series, from grades 1 to 6, were also examined to find how the equal sign was presented in different contexts at different grade levels. Using McNeil et al. (2006) as a U.S. benchmark, we coded two categories of the equal sign for the Chinese textbooks: o*perations equal answer context* and *non-standard context*. The non-standard contexts were further divided into (1) equations with operations on both sides, (2) equations with operations on the right side of the equation, (3) equations without explicit operations on either side, (4) no equation (e.g., "Use <, =, or > to complete each statement), (5) computation without the equal sign (e.g., "3+4" where students are expected to include the equal sign as well as the answer), (6) name the parts of the equation, (7) use arrows or lines to connect operations and answers, and (8) fill in the two or more missing components (e.g.., $a+ 3 = a$ or $a - a = a$). To ensure dependability we used a process termed *reconciliation*. Reconciliation is where both coders come together to discuss their classifications and to achieve agreement about any discrepancies. This process was used to ensure that raters remained consistent to the framework and to themselves over time. During reconciliation, each categorization was justified and explained and discussion continued until agreement. For example, the raters initially disagreed on whether x+ ( ) *a* 50+( ) should be coded as "operation on both sides" or "use of relational symbols." Agreement was reached that it should be coded as "operation on both sides." After coding 145 pages jointly and developing clear and consistent understandings of each category, one of the two raters coded every page from the two series. The other rater randomly selected 10% of the pages (265 out of 2,645 pages) and recoded them. The dependability between the two raters for standard or nonstandard context was 95%. The dependability for without equal sign was 99%. Other consistencies for name part of the operations, using arrow to connect, fill in missing number, no explicit operations on either side, operations on the right sides only, operation on both sides, use relational symbols were 97%, 95%, 96%, 98%, 94%, 96%, and 92%, respectively.

RESULTS

Does the U.S. Sample Misunderstand the Equal Sign as an Operator?

Table 2 shows percentages correct by item by country. About 28% of the U.S. sixth-grade sample correctly solved the first and second items, which was similar to results indicated by other researchers in recent years. For example, Rittle-Johnson and Alibali (1999) found that 31% of fourth and fifth graders correctly solved problems such as $3 + 4 + 5 = 3 + \_\_\_$. In addition, Knuth, Stephens, McNeil, and Alibali (2006) found that 32% of sixth-graders offered a correct definition of the equal sign when asked. Although test items varied across these studies, there was no fundamental difference in the items used to assess students'

TABLE 2
Percentage of Correct Answers

Group	*Question 1* $6+9=\square+4$	*Question 2* $\square+8=12+5$	*Question 3* $\square+3=5+7=\square$ 1st Box	2nd Box	*Question 4* $6+8=3+11$ T or F	n
U.S. Grade 6	28.6	28.6	23.8	86.7	47.6	105
Chinese Grade 6	98.6	96.6	98.6	97.9	91.7	145

Note. In text and tables a "□" is to indicate a missing number for typesetting purposes when either an underline or nothing was in that space e.g., ___ $+ 3 = 5 + 7 =$.

understanding of the equal sign. Such consistent results among different studies that were completed independently by different researchers in different locations confirmed the dependability of results of this study.

In terms of theories of misconceptions, if students had misconceptions, the errors caused by misconceptions should consistently appear in different contexts (Anderson & Smith, 1987). Thus, in our study, students were examined, in addition to the problem type $8 + 4 = a + 5$, on three additional items. Are the students' errors caused by an equal sign misconception limited to the problem type like $8 + 4 = a + 5$? We initially checked correlations among the four items. Because students could correctly fill the second box in the problem $a + 3 = 5 + 7 = a$ even with a misconception (they simply calculated $5 + 7 = 12$), we did not consider student responses to the second box. In fact, sixth-grade students in the U.S. sample were able to correctly place a 12 in the second box but were unable to explain their answer. The same students' responses to the four problems were correlated ($p < .01$) ($R_{12} = 0.811, R_{13} = 0.688, R_{23} = 0.706, R_{14} = 0.523, R_{24} = 0.528$, $R_{34} = 0.485$). However, the correlations among the first three items were higher than among each of the first three and the fourth item. These results showed students' responses in the second and third items were similar to the first one, whereas the fourth item was somewhat different. This item was a true/false item, resulting in a 50% chance score. The U.S. sixth-grade sample's misconception about the equal sign was not limited to the form $8 + 4 = a + 5$ because the first three problems in our test reflected the same level of performance that has been interpreted as evidence of understanding the equal sign as an operator.

Is the Equal Sign Misconception Present in the Chinese Sample?

Almost all Chinese students were able to correctly answer all the problems. The question $6 + 9 = a + 4$ was correctly answered by 98.6% of these students. Table 2 shows a comparison of U.S. and Chinese students' percentage of correct responses to the four items. The table indicated the disparity in performance across items.

How is the Equal Sign Presented in Teacher Preparation/Guidebooks?

Methods' textbook analysis: United States

Six methods books were chosen and examined to determine what strategies were being used to prepare U.S. preservice elementary teachers to present the equal sign to their future students. Strategies ranged from nothing at all (Smith, 2001), to a single paragraph (Cathcart et al., 2006; Reys et al., 2003; Van de Walle, 2004), to one activity (Tucker et al., 2006). Seemingly, it may be assumed by the authors of these textbooks that preservice teachers understand the issues related to the equal sign and the implications for their students. This Equals That (Tucker et al., 2006) was presented as an activity to introduce the equal sign. Through the activity, the authors suggested saying to students that, "... we use the equal sign to tell how many blocks there are all together" (p. 98). Even though these authors presented an introduction to the equal sign, their explanation suggested that the answer follows the equal sign.

Cathcart et al. (2006) in one paragraph suggested using the equal sign interchangeably with words like "makes" (addition) and "leaves" (subtraction). This approach allowed students to obtain the correct answer to simple addition and subtraction problems but might lead to misconceptions for students when presented with problems such as $2 + 6 = \square + 5$. This verbiage also possibly leads students to misconceptions of the equal sign as an operator. In another paragraph in the same text, the authors suggested that the equal sign meant, "is the same number as" or "is another name for" (p. 145). Although this verbiage seemed to focus on equality as symbolizing a relation, it still does little to encourage students to balance both sides of the equal sign. Hatfield et al. (2005) developed the meaning of the four operation signs (add, subtract, multiply, and divide) but did not mention the equal sign. However, in the section on multiplication, these authors described the use of the equal sign in one sentence and then substituted "are" for the "=" sign in the sentence immediately following employing a literal translation of "are" for "=."

Both Reys et al. (2003) and Van de Walle (2004) alerted presevice teachers to the common misconception that the equal sign meant, "the answer is coming" (Reys et al., 2003, p. 345). Both of these textbooks' authors informed readers that using the calculator reinforced the equal sign misconception, because the answer came after the equal sign was pressed. To counteract this misconception, a balance scale can help students develop the correct conceptual understanding of equality and the equal sign (Reys et al. 2003). Van de Walle (2004) suggested that teachers should use the phrase "is the same as" (p. 139) instead of "equals" as students read number sentences. There were no alternate forms to the canonical use of equal sign presented in any of the methods textbooks examined.

Methods' (guidebook) analysis: China

Certain selected textbooks have a powerful influence in China in terms of market share. Thus, three textbooks and corresponding teacher's guidebooks were selected. Whereas in the U.S., the term *suggestion* is often dealt with cursorily, in China, teachers view this term as more of an imperative. Teachers view textbooks and guidebooks as authoritative, therefore, they feel they should diligently follow the suggestions to perfect their teaching and improve their content knowledge (Li, 2004). It is well known that Chinese teachers spend a great deal of time studying these materials intensively [*zuanyan jiaocai*], commonly referred to as lesson study (Ma, 1999). The teaching materials include student textbooks, teacher manuals (guidebooks), and a *teaching and learning framework* (renamed to National Mathematics Curriculum Standards for Compulsory Education after 2001) that is the Chinese equivalent to the NCTM standards but includes textbooks and teacher manuals (guidebooks). Thus, the aforementioned three types of books greatly influence how Chinese teachers teach certain concepts, increase their teaching knowledge, and expand their content knowledge. As a result, it was necessary to see how teacher guidebooks provided suggestions for Chinese teachers.

Chinese guidebooks usually include learning goals, explanations for certain units, and teaching suggestions. In the learning goal sections, a guidebook provides detailed goals that students should reach after studying a unit. In the explanation sections, the rationales behind the writing of a unit include: content sequence, use of examples and exercises, and design of context. This enables teachers to have deeper understandings of their textbooks. The pedagogical suggestions are also presented in a similar way. In the following sections, we provide information concerning how the three popular guidebooks offer suggestions for teaching the equal sign. Generally, instructions for the teacher indicated once students know the numbers and their corresponding value the equal sign is introduced using a comparison context.

The *People's Education Press* (PEP) guidebook (Curriculum Research Center and Elementary Mathematics Curriculum Research, 2005) highlights equality as a relation. It suggests teachers should elicit the concepts of "the same as," "greater than," and "less than" through a story of "small pigs helping rabbits to construct houses" where the animals are carrying logs. The comparison between logs and animals helps students understand the idea of "the same as." Teachers were also given suggestions to ask students questions such as "what other pairs can you compare?" According to the guidebook, it is important for teachers to use all possible concrete materials and life situations to help students make sense of "the same as," "greater than," and "less than." With regard to developing students' understandings of "the same as," "greater than," and "less than," the guidebook suggested that teachers should use "one to one" correspondence strategies to teach students.

Jiang Su Education Press (*JSEP)* guidebook (Su & Wang, 2005b) also employs similar suggestions. The guidebook clearly states the main goal of the unit is to learn and understand "=," ">," and "<." Students should know how to represent the relationship between two numbers. Like *PEP*, it suggests that teachers should use the "one to one" correspondence method to help students understand the concepts of "the same as," "greater than," and "less than."

Beijing Normal University Press (BNUP) guidebook (Research Group of National Mathematics Curriculum Standards for Compulsory Education, 2005b) also emphasizes learning the equal sign through the context of comparison. However, students are asked to extend the scope of comparison to "long and short," "high and low," and "light and heavy." The idea of "one to one" correspondence is emphasized in helping students' understanding of the equal, greater, and less than symbols.

How is the Equal Sign Presented in Chinese Student Textbooks?

Student textbook analysis: China

The most frequently adopted textbook series was published by the PEP (Lu & Yang, 2005) and controlled more than 70% of the market until 2002 (Li, 2004). The second textbook series and corresponding guidebooks were published by JSEP (Su & Wang, 2005a). Jiang Su is one of the largest and more developed provinces in China with a population of more than seventy million. The JSEP textbook series was also adopted by many provinces because of its high quality, when comparing to those of other provinces. The students in our investigation also used the JSEP textbook series and their teachers used the corresponding guidebooks. The third textbook series was published by Beijing Normal University Press (BNUP) (Research Group of National Mathematics Curriculum Standards for Compulsory Education, 2005a). The authors of National Mathematics Curriculum Standards for Compulsory Education (Ministry of Education, 2001) were the editors of the BNUP textbooks. As a result, BNUP textbooks have a strong potential influence on other textbooks. Therefore, these three textbooks exert a great deal of influence on Chinese education and are used by most teachers and students in China. Therefore, due to the overlap in content among the textbooks, the JSEP and PEP textbooks were used for the quantitative analysis and specific examples were cited from BNUP.

Introduction of the equal sign. The textbooks introduced the equal sign at grade 1. Before introducing the concept of the equal sign, the PEP textbook introduces the numbers less than 10. Multiple concrete contexts were presented to illustrate the concept "the same as" which was the verbiage they are taught to substitute for the "=" sign. For example, this textbook provides a context where different animals move different objects. Students are asked to understand the relationship between the objects and animals—in a context of one to one correspondence (see Appendix A, top two pictures; for color pictures of the pages see http://coe.tamu.edu/~rcapraro/C_I_Equality_Paper.htm). Additionally, students are asked to compare lengths and heights, making judgments about whether a comparison was fair under certain conditions, such as two students seem to be the same height but one of them is actually standing on his toes. Based on these structured educational experiences, the formal symbol of "=" was introduced in three ways simultaneously: concrete, symbolic, and verbal ("Deng yu hao" in Chinese is literally translated as "equal sign") together with two other relational signs *greater than* (>) and *less than* (<), such as, $3 = 3$, $3 > 2$, and $3 < 4$ (see Appendix A, bottom left picture). Again, concrete contexts like five rabbits and three carrots are presented where students were asked whether there were enough carrots for all the rabbits if each rabbit eats one carrot (see Appendix A, bottom right picture). After learning these comparisons, addition and subtraction was introduced. Problems such as $3 + 2 = \square$, $2 + 3 = \square$, $1 + \square = \square$, $4 - \square = \square$, $4 - \square = \square$ were presented within a context of familiar pictures.

The JSEP textbook introduced "=," ">," and "<" together with pictorial and written representations. Before introducing the formal mathematical symbol of "=," students were provided various comparison contexts such as comparing length and weight. In addition, a context called "animal sports conference in the forest" where students were asked to compare numbers of different animals to understand the concept of "the same as," using their informal understanding of equality (see Appendix B). The number of pages and types of exercises in this textbook series were similar to PEP textbooks. What follows is a typical exercise for students in this unit (Su &Wang, 2005a p. 19): "Fill the '$\bigcirc$' with >, <, or =; $1 \bigcirc 0$; $3 \bigcirc 3$; $2 \bigcirc 3$; $5 \bigcirc 4$". Additionally, students were asked to place the numbers in the boxes to make the sentence true (Su &Wang, 2005a, p. 20) "$4 > \square$, $2 < \square$, $\square + 5 = \square$, $\square + 1 < \square + \square$".

The BNUP textbook also employed the same sequence as the other two textbooks to introduce the equal sign in the context of comparison (see Appendix C) under the unit heading "comparison." This was the second unit in the textbook where these relational signs were introduced. The first page of that unit introduced "=." Similar to the other textbooks, this one used a context "Happy Zoo," where students compared various quantities of animals. The formal equal sign was initially presented in the equation "$4 = 4$" on that page. On the next page (see Appendix C), students were asked to fill in the boxes for following items:

$$7 \bigcirc 3,\ 4 \bigcirc 9,\ 3 \bigcirc 3,\ 5 \bigcirc 8,\ 10 \bigcirc 1,\ 2 > \square,\ 6 > \square,\ 6 = \square,\ \square = 8,\ \square < 9.$$

Thus, there were few differences between these popular Chinese textbooks when introducing the concept of the equal sign. That is, in these Chinese textbooks the formal symbol of "=" was introduced after the informal concept of "the same as." Then the "=" was introduced under the context of comparison, which emphasized the relationship of "equal to" together with "greater than" and "less than" within multiple representations: pictures, symbolic expressions, and words. After introducing the equal sign, addition and subtraction were introduced where students were provided with both standard and non-standard forms to understand both operations and the equal sign.

Developing understanding of the equal sign. From the page-by-page analysis of JSEP and PEP the predominant context for presenting the equal sign was *operation equals answer* (36.6%),numerous other contexts also appeared in the grades 1 through 6 textbooks (see Table 3). Nearly an equal percentage of items were presented in a non-standard format *without using the equal sign* (33.1%), that is, "compute $2 + 3$." This format may be considered closely aligned with the presupposition that the calculator may contribute to the misconception in the U.S. However, the "compute" problem type in China does not seem to give rise to the same equal sign misconception. The "compute" format indicated a problem type that was not necessarily linked with number operations. In numerous places, students performed operations without the equal sign $3\times2\ldots$ (see Appendix D). Another way of dealing with equality in this textbook was to *list the names of parts of the operation* (3.8%). For example, students were asked to complete a table containing the terms dividend, divisor, and quotient with one number missing from one of the terms, all without the use of an equal sign. In this tabular representation, students do not need to write "=" when they performed the computations. This format seemed to be an effective way to help students disconnect operations from the equal sign. Moreover, another frequently used format was continuous *operations where "arrows" instead of the "equal sign"* (4%) were used. For example, $12 \times 4 \rightarrow \square \div 3 \rightarrow \square \times 5 \rightarrow \square \div 8 \rightarrow \square \times 6 \rightarrow \square$ (see Appendix D). This type of exercise was likely to help students see the inappropriate use of the equal sign, resulting in a deeper understanding of the equal sign as a relational symbol.

Another readily encountered context was when students *fill-in missing numbers* (1.7%) such as: $1 + \square = \square$, $\square + \square = \square$, and $\square + 5 = \square$. These contexts have more than one possible solution, which may encourage more fluid and dynamic understandings of both the operators and equal sign. Although this more open format resembled the *operation equals answers* as defined by McNeil et al. (2006), students must demonstrate some relational thinking in order to obtain admissible solutions for this context. That is, if students only understand the equal sign as "to do something," they will be unable to solve this context because they cannot

TABLE 3
Percentage of Equal Sign by Context in Chinese Textbooks: Grades 1–6

		Grade											
		1		*2*		*3*		*4*		*5*		*6*	
Textbook		*JSEP*	*PEP*	*JSEP*	*PEP*	*JSEP*	*PEP*	*JSEP*	*PEP*	*JSEP*	*PEP*	*JSEP*	*PEP*
Equal Sign Contexts	Mean%												
Operations equal answer (Standard)	36.6	55.7	50.2	41.9	39.2	17.9	17.7	19.9	51.8	19.4	33.4	33.7	22.7
Without equal sign	33.1	14.7	14.3	35.1	28.6	64.9	49.0	54.5	28.1	40.6	17.2	38.5	32.8
Name part of the operations	3.8	2.7	10.6	4.5	5.9	1.5	3.6	3.9	1.6	0.5	3.1	0.0	0.0
Using arrow to connect	4.0	4.5	5.3	2.2	12.8	1.9	9.3	1.5	4.5	1.6	1.5	1.3	2.0
Fill in missing no.	1.7	13.8	8.0	5.9	5.9	1.7	5.0	2.4	3.8	1.2	1.9	1.9	2.2
No explicit operations on either side	6.6	0.4	1.2	0.8	0.6	5.6	6.3	5.0	5.8	22.5	20.8	11.4	13.5
Operation on the right sides only	5.0	0.3	0.2	0.0	0.3	0.6	1.3	1.3	0.6	4.2	8.1	2.5	5.3
Operations on both sides	4.2	2.6	0.9	0.8	2.8	0.6	1.3	10.8	0.6	3.6	7.9	9.2	16.3
Use relational symbols	6.1	5.2	9.3	3.1	3.9	5.4	6.6	0.4	3.2	6.3	5.8	1.3	5.0

Note. Due to rounding totals of contexts by grade by book do not equal 100%.

add "1" and "□" to get some other unspecified number. This context fosters *inductive bridging* that forms the basis for transitioning between the standard and non-standard contexts of the equal sign (cf., Capraro & Capraro, 2006; McNeil et al., 2006).

Another context, *equations without explicit operations on either side* (6.6%) dealt with measurement, conversion among different bases, fractions, and shapes. For example, the textbooks used equivalencies among the following: 1 kilogram = 1000 grams, 1 hour = 60 minutes, and 1 kilometer = 1000 meters. A related context was *equations with the operation on the right side only* (5%). The formula $v = a \times a \times a$(see Appendix D) was used to have students examine the relationships among height, length, and width of cubes. In the chapter on factoring, students were presented with numerous opportunities to practice problems like $12 = (\ ) \times () = (\ ) \times (\ ) = (\ ) \times (\ )$ (see Appendix D). This was another form of an open-ended problem requiring students to think about various equivalencies. Another open-ended context that received less page space relatively than previously mentioned contexts, but was represented in each textbook, was when students were required to make equivalent statements with *different operators on each side of the equal sign* (4.2%) or when students had to choose relational symbols ($<$, $>$, $=$) (6.1%) to make the statement true.

Contrasting Chinese to U.S. textbooks (e.g., McNeil et al., 2006), Chinese textbooks provide more diverse contexts for students to potentially develop more comprehensive understandings. Specific examples were provided from a single textbook, BNUP because of the similarity of presentation among the textbooks. The other two textbooks, JSEP and PEP, were used to provide the quantitative results (Table 4). The number of instances of equal sign occurrences decreases as the grade level increases. This is consistent across textbooks. The percentages were computed by dividing the number of instances any one equal sign context was used by the total number of instances for all contexts by grade level and series.

DISCUSSION

Although there are many possible explanations for the long-standing difficulties U.S. students experience with interpretations of equality as a relation, we believe one contributing factor for the disparity is textual presentations. Our U.S. sixth-grade sample and previous U.S. samples lag far behind the current Chinese sample, which clearly demonstrates that students' understanding of the equal sign in the U.S. can improve greatly.

Examination of Chinese teaching materials demonstrates that the lessons are related to understanding the equal sign among other relational symbols situated within a great diversity of problem contexts. Teachers are specifically encouraged to present a multitude of problems of various types and arrangements of missing

TABLE 4
Equal Sign Instances, by Grade Level and Textbook Series

Grade	*Textbook Series*	*Instances*	*Pages*	*Instances per Page*
1	JSEP	1481	198	7.48
	PEP	1297	217	5.98
2	JSEP	1247	226	5.52
	PEP	1001	249	4.02
3	JSEP	931	238	3.91
	PEP	637	264	2.41
4	JSEP	818	223	3.67
	PEP	313	255	1.23
5	JSEP	974	251	3.88
	PEP	726	267	2.72
6	JSEP	465	232	2.00
	PEP	539	245	2.20

Note. The instances were defined as any representation relevant to equal sign (i.e., $3 + 5 =$___ is a typical instance), however, some instances may not contain equal sign (i.e., compute 3+5) but were still included.

numbers, operators, or both. Additionally, for Chinese teachers, "it is intolerable to have two different values on each side of an equal sign" (Ma, 1999, p. 111). Unfortunately, U.S. teachers are more likely to accept student work like "$3 + 3 \times 4 = 12 = 15$," because in the U.S. the order of operations is paramount and students are able to get correct answers (Ma, 1999). In fact, in a recent study of U.S. teachers, Ding and Li (2006), found teachers paid little attention to students' errors like $3/3$ x $2 = 6/8$ during instruction. Additionally, teachers themselves made similar errors of equivalence by writing $360 \div 4 = 90 \times 3 = 270$ on the blackboard or overhead.

In contrast to Chinese methods textbooks, only two U.S. mathematics methods textbooks directly address the equal sign, and none include lesson examples or activities to help understand how the equal sign should be taught. Teachers can only be expected to teach their students what they themselves experience and understand. If teachers' only experiences with the equal sign have been less than exemplary and without any counter instruction in their teacher education program or continuing education, we can expect the issues with student interpretation of equivalency to persist. Textbooks play an important role in teaching and learning because they determine, to a great degree, how teachers teach and students learn mathematics (Confrey & Stohl, 2004; Reys et al., 2004).

Our results are consistent with findings from McNeil et al. (2006). That is, the contexts presented by textbooks positively correlate with students developing an appropriate understanding of the equal sign. The predominant Chinese students' textbooks, guidebooks, and Mathematics Standards introduce and develop students' understanding of the equal sign under the context of "comparison" and before introducing number operations. The informational knowledge of "the same as" is introduced before the formal symbol "=." It is important to introduce a new concept by building on students' past experiences. Chinese textbooks provide various contexts for students to develop progressively their understanding of the equal sign. However, our investigation revealed that nearly equal percentages of contextualized and decontextualized items are incorporated in Chinese textbooks. Other prominent U.S. researchers have surmised that the use of a decontextualized equal sign contributes to the U.S. misconception. The argument generally is, when students use calculators they enter the problem as 3+5 and press the equal sign and the answer appears giving rise to the belief that the equal sign is an operator. Our evidence seems to indicate even though the compute problem type in China is similar to the calculator use in the U.S., the decontextualized use of the equal sign is not sufficient to precipitate the equal sign misconception in Chinese students. It is possible that the combination of the two forms, *operation equals answer* and open-ended contexts helps Chinese students to develop understandings of the equal sign differently from students in the U.S. Perhaps it is through these more open-ended problems where students learn to make sense of operators and relational symbols simultaneously that helps Chinese students to develop more profound mathematical understandings that account for performance differences as identified in this study.

Considering the importance of textbooks in shaping teachers' teaching and students' understanding, in addition to high fidelity of teachers' teaching with approaches suggested by textbooks and guidebooks in China, it is plausible to conclude that Chinese textbooks are instrumental in Chinese students developing appropriate understanding of the equal sign. From these findings it is important to note that teacher preparation materials in the United States, student texts, and professional development should be aligned to focus-on equality as a statement of relation.

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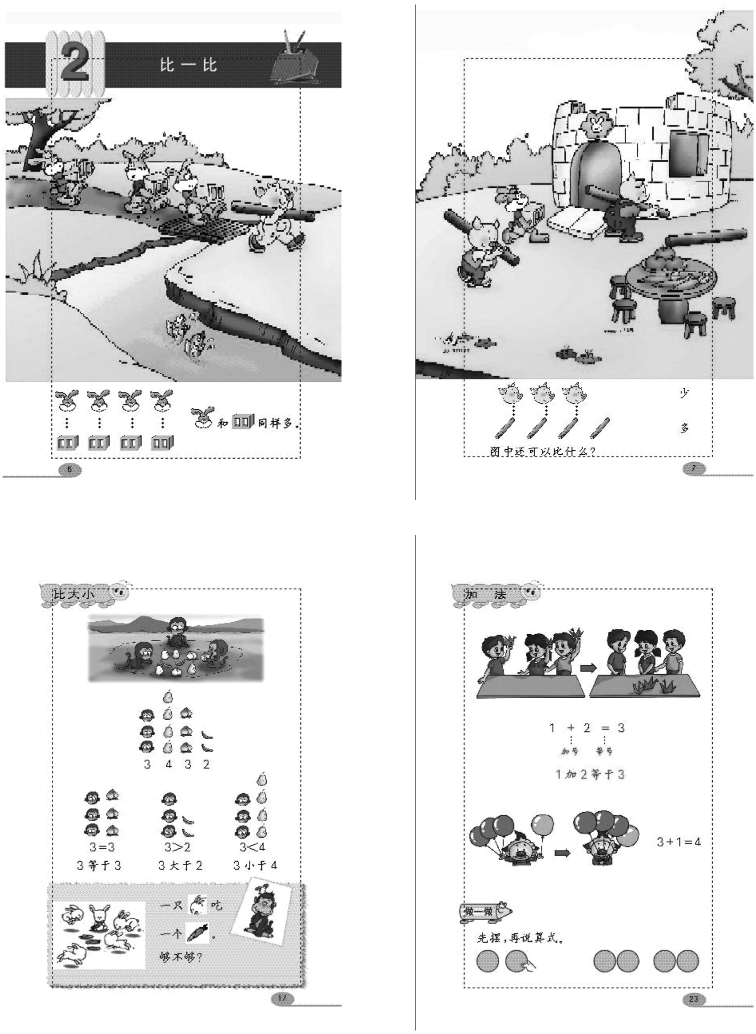
APPENDIX A

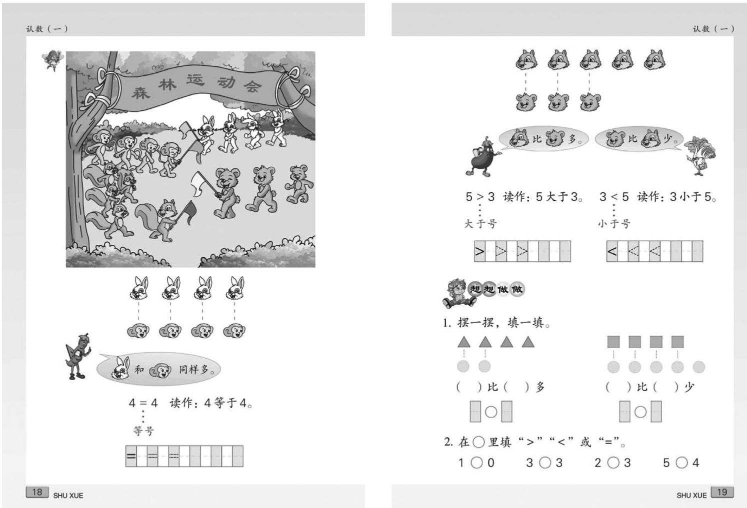
The context of introducing "equal sign" in Chinese PEP Grade 1 textbook (pp. 6–7, 17, 23).



Note. The top textbook pages (pp. 6–7) introduce the concept of "the same as," "greater than," and "less than"; the textbook page (p. 17, bottom left) formally introduces the "=," ">," and "<"; the textbook page (p. 23, bottom right) follows relational instruction with the equal sign to the context of addition.

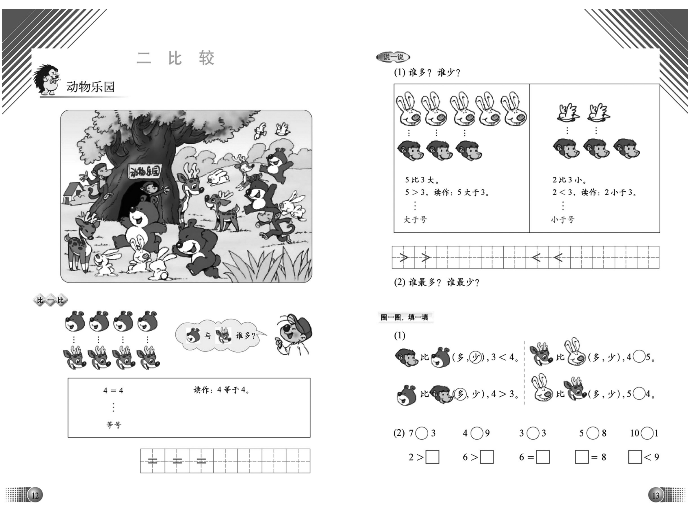
APPENDIX B

The lesson introducing “=” in Chinese JSEP Grade 1 textbook (pp. 18–19).



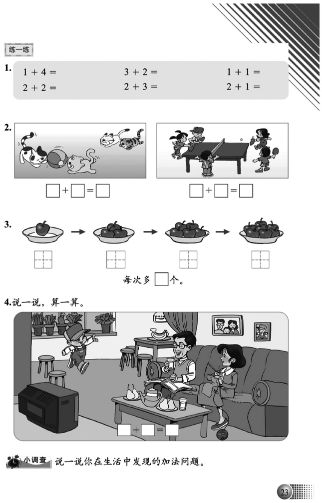
APPENDIX C

The lesson introducing “=” in Chinese BNUP Grade 1 textbook (pp. 12–13).



APPENDIX D

The lesson developing “=” in Chinese BNUP textbooks (pp. 12–13).



练一练

1. $1 + 4 =$ $3 + 2 =$ $1 + 1 =$

$2 + 2 =$ $2 + 3 =$ $2 + 1 =$

2. □+□=□ □+□=□

3. 每次多□个。

4.说一说，算一算。

□+□=□

小调查 说一说你在生活中发现的加法问题。

23

BNP Vol. 1, p. 23

一 乘除法

小树有多少棵

每捆 20 棵

□×□=□（棵） 答：________。

试一试

1. 4 捆一共有多少棵？ 5 捆呢？

2.

3×2	5×4	6×7
30×2	50×4	6×70
300×2	500×4	6×700

3.

30×4	50×8	9×600
40×5	60×7	800×4

BNP Vol. 3, p. 2

5. 在○里填上“>”“<”或“=”。

$48 \div 4$ ○ $84 \div 4$ $720 \div 8$ ○ $720 \div 9$

24×3 ○ 24×2 $26 \div 2$ ○ 26×2

6. 李老师给售货员 100 元，找回 4 元，每个足球多少钱？

7. 填表。

被除数	320		64		96		2800
除数	8	4	2	15	3	25	7
商		600		4		3	

8.

$84 \div 4 - 16$	$(54 + 36) \div 3$	$360 - 200 \div 4$
$36 \div 3 + 27$	$76 - 18 \times 4$	$720 \div 8 - 45$

9. 小丽借了一本故事书，共 150 页。照这样的速度，一个星期她能看完吗？如果不能按时看完，请你给她提个建议。

13

BNP Vol. 3, p. 13

三 千克、克、吨

有多重

称一称 掂一掂

1 千克 = 1000 克

千克可以记作“kg”，克可以记作“g”。

1 kg = 1000 g

18

BNP Vol. 3, p. 18

练一练

1. 在◯里填上“>”“<”或“=”。

96 ÷ 8 ◯ 96 ÷ 6　　120 ÷ 4 ◯ 240 ÷ 8

90 ÷ 5 ◯ 80 ÷ 5　　42 ÷ 3 ◯ 42 × 3

2. 先估计商是几位数，再计算。

6)846　　4)156　　7)364

3. 计算下面各题，并验算。

432 ÷ 6　　389 ÷ 9　　872 ÷ 4

902 ÷ 6　　618 ÷ 3　　824 ÷ 4

4. 至少要运多少次？

5. 同学们做了93朵花，每6朵扎成一束，最多可以扎多少束？

62

BNP Vol. 3, p. 62

3. 按要求连一连。

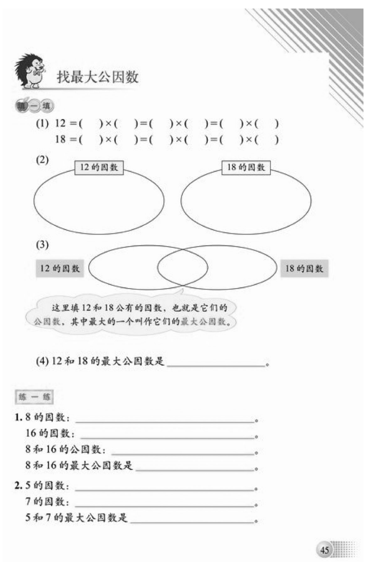
4. 小明想了一个数，这个数乘3后得数在380和400之间，这个数可能是多少？

5. 12 $\xrightarrow{\times 4}$ □ $\xrightarrow{\div 3}$ □ $\xrightarrow{\times 5}$ □ $\xrightarrow{\div 8}$ □ $\xrightarrow{\times 6}$ □

6. 某班要举行一次朗诵比赛，每位学生的朗读时间规定为3分。一位学生准备了一篇840个字的演讲稿，在赛前试读时，他用了5分，怎么办？

65

BNP Vol., p. 65



找最大公因数

填一填

(1) 12 = ( )×( ) = ( )×( ) = ( )×( )

18 = ( )×( ) = ( )×( ) = ( )×( )

(2)

(3)

(4) 12和18的最大公因数是______。

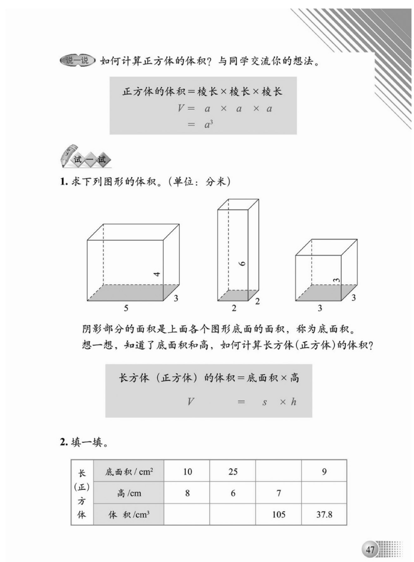
练一练

1. 8的因数：______。
16的因数：______。
8和16的公因数：______。
8和16的最大公因数是______。

2. 5的因数：______。
7的因数：______。
5和7的最大公因数是______。

45

BNP Vol. 5, p. 45



说一说 如何计算正方体的体积？与同学交流你的想法。

正方体的体积＝棱长×棱长×棱长

$V = a \times a \times a$

$= a^3$

试一试

1. 求下列图形的体积。（单位：分米）

阴影部分的面积是上面各个图形底面的面积，称为底面积。
想一想，知道了底面积和高，如何计算长方体（正方体）的体积？

长方体（正方体）的体积＝底面积×高

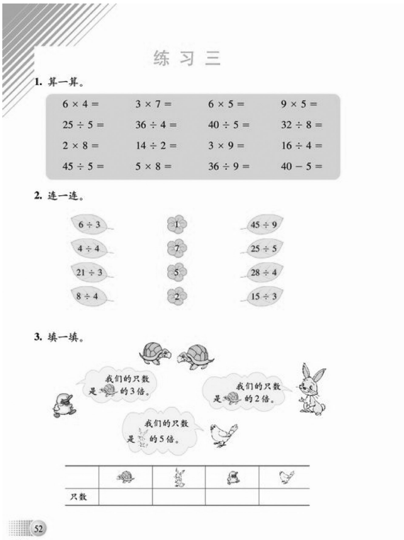
$V = s \times h$

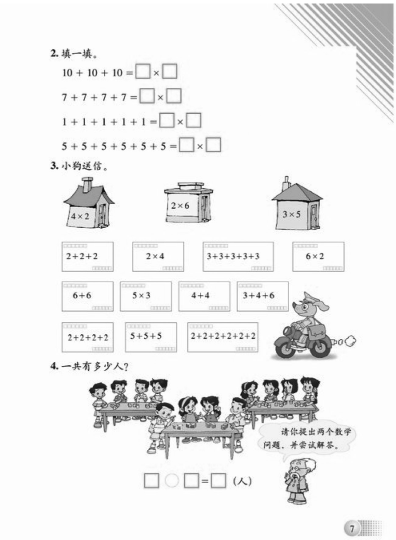
2. 填一填。

长（正）方体	底面积/cm²	10	25		9
	高/cm	8	6	7	
	体 积/cm³			105	37.8

47

BNP Vol. 5, p. 47





BNP Vol. 2, p. 52

BNP Vol. 2, p. 7