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Situation Model Perspective on Mathematics Classroom Teaching:

A case study on multiplicative inverse relations

Ryan Hassler & Meixia Ding, Temple University

Introduction

Findings from recent international tests (TIMSS, 2013; PISA, 2013) reveal that during the Common Core State Standards era (Common Core State Standards Initiatives [CCSSI], 2010), U.S. students continue to exhibit a lack of mathematical understanding. Although the goal of mathematics instruction has arguably always been centered on learning for understanding (Hiebert & Carpenter, 1992; Hiebert et al., 1997; Stylianides & Stylianides, 2007; Silver, Mesa, Morris, Star & Benken, 2009), the field of mathematics education continues to place an ever increasing emphasis on the comprehension of fundamental mathematical ideas. According to the CCSSI, students must be able to form well-connected and conceptually grounded mathematical ideas in order to facilitate transfer of learning. This is supported by recent empirical evidence that indicates comprehension improves when conceptually relevant connections to prior knowledge are formed. (Sidney & Alibali, 2015). In most U.S. mathematics classrooms however, "instructional tasks tend to emphasize low-level rather than high-level cognitive processes" (Silver et al., 2009, p. 503) and curriculum materials generally lack connections within and across topics (Ding, in press; Schmidt, Wang, & McKnight, 2005). A U.S. instructional preference for procedural focused learning (Baroody, 1999; DeSmedt et al., 2010; Torbeyns et al., 2009) with few references to tasks that assess targeted concepts (Crooks & Alibali, 2014) may therefore be prohibiting connection-making opportunities during classroom instruction.

Although forming connections is a common theme across most current educational research on mathematical comprehension (Anthony & Walshaw, 2009; Barmby et. al, 2009; Blum, Galbraith, Henn & Niss, 2007; Businskas, 2008; Sidney & Alibali, 2015), few have comprehensively explored specific ways in which to facilitate connection-making. The purpose of this case study (Stake, 1995) is therefore to examine how two expert elementary teachers facilitate connection-making during classroom instruction. A situation model perspective is used to analyze classroom instruction on multiplicative inverses, a critical topic that lends itself to form numerous connections (Baroody, Torbeyns, & Verschaffel, 2009; Nunes, Bryant, & Watson, 2009). The findings from this study are expected to contribute to enhancing the classroom teaching of elementary inverse relations. In addition, since longitudinal empirical evidence (Baroody, 1987; Stern, 2005; Vergnaud, 1988) suggest that an elementary student's comprehension of inverse relations significantly predicts both algebraic and overall mathematical achievement in later years, this research hopes to contribute to the growing body of mathematical comprehension research. The coding framework developed for this study may be useful for future studies surrounding the comprehension of other fundamental mathematical concepts.

Literature Review

U.S. students exhibit a great weakness with cognitively high-demanding mathematical tasks (OECD, 2013), which may be largely attributed to the quality of their learning opportunities (Thompson, Kaur, Koyama, & Bleiler, 2013). An important task for mathematical comprehension is the ability to apply fundamental concepts that transcend across various contexts (Bruner, 1960; CCSSI, 2010). One such concept is inverse relations, as the ability to reason with inverses is critical across all levels of mathematics (Baroody, Torbeyns, & Verschaffel, 2009; Carpenter, Franke, & Levi, 2003; Nunes, Bryant, & Watson, 2009a).

The Case: Multiplicative Inverse Relations

In general, elementary school children lack a formal understanding of inversion (Baroody, Ginsburg & Waxman, 1983; De Smedt, Torbeyns, Stassens, Ghesquiere, &

Verschaffel, 2010; Resnick, 1983). Because the first formal teaching of inverse relationships occurs when forming connections between addition and subtraction, the majority of prior research on comprehension of inverse relations has mainly focused on additive inverses (Cowan & Renton, 1996; Squire, Davies & Bryant, 2004). However, because inverse operations have been identified as a critical piece of mathematical competency across all elementary grades levels (CCSSI, 2010), there also exists a need to examine the comprehension of multiplicative inverses. Although limited, research reveals that multiplicative inverses are a particular struggle for many elementary students (Robinson & Dubé, 2009b). For the purpose of this study multiplicative inverses refers to the complement principle, that if $M \times N = P$, then $P \div M = N$. With regards to this principle, both Grossi (1985) (as cited in Vergnaud, 1988) and Thompson (1994) found that elementary students were unable to recognize the appropriateness of using either equation when solving application problems. This is perhaps due to Ding and Carlson's (2013) claim that current instruction of inverse relations does not support conceptual connection-making or perhaps this indicates that elementary students have not yet developed a well-connected situation model for inverse relations.

Situation Model Perspective

Making connections in order to transfer knowledge into new situations represents conceptual understanding. In the case of inverse relations, this is supported by theoretical accounts which suggest that once a concept has been learned it represents general knowledge that can be applied more broadly (Baroody, 2003; Baroody & Lai, 2007; Lai, Baroody, & Johnson, 2008; Siegler & Araya, 2005). Too often, the instruction of elementary inverse relations tends to focus on procedural knowledge with few connections made to the underlying principles (De Smedt et al., 2010). This prohibits students' conceptual understanding (Torbeyns, De Smedt, Stassens, Ghesquiere, & Verschaffel, 2009), possibly because of an incomplete situation model. According to van Dijk and Kintsch (1983), a situation model is an internal network of connections that form a "cognitive representation of the events, actions, persons and in general the situation" (p. 11) which is to be learned.

Kintsch (1988) believes that the deepest level of comprehension occurs when students form situation models. This claim is based off of his Construction-Integration theory of reading comprehension which views reading as an inferential process of evaluating propositions in relationship to three types of mental representations that a learner forms while reading text. According to Kintsch (1986; 1988), the process of forming these mental representations begins with the reader creating an initial list of propositions based solely on the words that they are reading. This is known as the surface component (1), or a verbatim representation of the text in which words and phrases themselves are encoded into memory. The second component, a textbase (2), represents the semantic structure of the text in that it captures the linguistic relationships among propositions represented in the text. As the textbase is created, entire sentences are read and the reader begins to make meaning of the text. Because the first two components only involve direct translation of what is explicitly written, learners are not required to make inferences. Therefore, limited connections to prior knowledge are needed. If however, a reader draws on prior knowledge to create a more complete mental representation that can be used to make inferences between the situation the text represents and other contexts to which that text may be applied, then the final situation model (3) component has been created. A situation model is therefore deeply connected to prior knowledge in such a way that allows for a learner to use new content knowledge in "novel environments and for unanticipated problem solving tasks" (McNamara et al., 1996, p.4).

Kintch (1986) noted that in both a first grade and a college setting, once a situation model was formed for a mathematics based word problem, comprehension increased. This occurred because learners tended to make connections to prior knowledge and could reconstruct the problem using their situation model, as opposed to simply recalling the problem by use of the textbase component. Multiple other studies (e.g., Kintsch, 1994; Osterholm, 2006; Weaver, Bryant & Burns, 1995) have shown this important role that situation models have in altering the definition of learning from not what is simply to be remembered, but rather what conclusions can be drawn based on an inference making process. Because the mental representations on which recall is based differs from the representation on which inference is based, connection-making is especially important when learning a new mathematical concept (Sidney & Alibali, 2015). Several researchers agree that the most influential factor of comprehension is a coherent situation model (Graesser, Millis & Zwaan, 1997; Zwaan, Magliano & Graesser, 1995) and thus a framework that includes strengthening connections within learning opportunities is needed to promote transfer and enhance students' mathematical understanding

How to Facilitate a Situation Model

To create an effective situation model, a learner must implement a deep level of inference making that demands connecting implicit and explicit information to one's prior knowledge (Zwaan & Madden, 2004). Although the amount and the ability to activate conceptually relevant prior knowledge has been shown to be a significant and reliable predictor of comprehension (Langer, 1984; McNamara et al., 1996; Pearson, Hansen & Gordon, 1979), novice learners often struggle to make connections to relevant prior knowledge (Novick, 1988). Therefore, in order to best facilitate understanding for learners with little prior knowledge, curriculum and instruction should be as coherent and explicit as possible (Kintsch, 1994; Reed, Dempster & Ettinger, 1985).

In addition, analyzing experimental variables within learning opportunities that affect the ability for learners to make connections and draw inferences is essential in the pursuit of helping student enhance their ability to create situation models. According to *the Institute of Education Sciences (IES)*, these variables include the instructional tasks, types of representations, and the kinds of questions used during instruction (Pashler et al., 2007).

Instructional Tasks. A critical component in organizing instruction to improve student learning is to establish connections between instructional tasks and underlying principles (Pashler et al., 2007). Examples of instructional tasks include review tasks, instructional examples and practice problems. Just using a greater variability of instructional tasks however, does not guarantee transfer benefits (Paas & Van Merrienboer, 1994; Atkinson, Derry, Renkl & Wortham, 2000). Instead, according to a situation model perspective, instruction should be designed to form connections within and between instructional tasks in order to increase mathematical comprehension. Indeed, various instructional methods have been designed to develop these connections during mathematics instruction. They include interleaving instructional examples with practice problems (Pashler et al., 2007), using contrasting alternative solution methods (Rittle-Johnson & Star, 2007) and using both correct and incorrect examples during instruction (Booth et al., 2013). In addition, because the use of worked examples has been shown to increase initial comprehension within cognitively high demanding tasks (van Merriënboer, 1997; Renkl, 1997) they too have been extensively researched in mathematics education.

A worked example is "a step-by-step demonstration of how to perform a task or how to solve a problem" (Clark, Nguyen & Sweller, 2006, p. 190). The use of worked examples in mathematics instruction is supported by the belief that they serves as an expert mental representation and thus help increase comprehension (Chi & VanLehn, 2012; Sweller & Cooper, 1985). From the perspective of a situation model, the use of worked examples helps students develop a schema by facilitating connection-making between prior knowledge in order to increase the likelihood of transfer (Kirschner et al., 2006; Paas, Renkl & Sweller, 2003). Therefore, corresponding practice problems should have connections to the worked examples so as to practice the learned knowledge. Because worked examples and practice problems should be built on student's prior understanding, review tasks used during instruction should also provide opportunities to form connections to relevant prior knowledge. Unfortunately the amount of time allocated to review in U.S. mathematics classrooms is limited, maybe because an emphasis seems to be placed on allowing students enough time to work on practice problems (Jones, 2012; Stigler & Hiebert, 1999). In addition to lower amounts of instruction time devoted to worked examples, Ding and Carlson (2013) found that U.S. teacher lesson plans spend little time unpacking worked examples. It therefore seems as if there may be many opportunities to enhance connection-making within the instructional tasks used during mathematics instruction.

Representations. To allow students a hands-on exploration of mathematics, concrete manipulatives (e.g., blocks, rods, tiles) and concrete representations (e.g., story problems) are often used in elementary school classrooms (Clements, 1999). Martin and Schwartz (2005) believe that by interacting with concrete manipulatives, students form stronger connections to their mental representations which helps to increase mathematical comprehension. This has been empirically supported by Harrison & Harrison (1986), who provided descriptions of successful learning activities that utilized concrete objects such as rulers and place value cards. Although literature suggests that concrete representations are useful during initial learning (Resnick & Omanson, 1987), they also often contain irrelevant information that may prohibit students from making deep connections to the underlying principles (Kaminiski, Sloutsky, & Heckler, 2008). For instance, several studies (e.g., Gentner, Ratterman, & Forbus, 1993; Goldstone & Sakamoto, 2003; Son, Smith & Goldstone, 2011) have shown that using only concrete materials hinders transfer to unknown situations. It therefore is commonly believed that concrete representations alone do not guarantee comprehension (McNeil & Jarvin, 2007), and thus should not be the only representations used to facilitate situation models.

Problem solving by paper and pencil, without the use of manipulatives or drawings, is a common example of abstract representations in mathematics. Since abstract representations are purely symbolic, students who reason at the abstract level appear to do so as a result of interacting with a situation model. From the perspective of a situation model, abstract representations therefore need to be an integral part of instruction because they are essential in the inference making process of many advanced mathematical tasks (Fyfe, McNeil & Borjas, 2015). Novice learners however often struggle to attain mathematical comprehension when only abstract representations are used during instruction (McNeil & Alibali, 2000; Rittle-Johnson & Alibali, 1999). This was perhaps most famously noted when Carraher, Carraher and Schliemann (1985) found that the ability for Brazilian children street vendors to solve basic computational mathematics problems was dependent on the context and concrete representations of the problems. Therefore, there exists a need to facilitate connection-making between concrete and abstract representations.

Pashler et al. (2007) suggests that by integrating both concrete and abstract representations into instruction, students are better able to make connections to prior knowledge. In fact, instruction involving various representations has repetitively been shown to increase comprehension (Ainsworth, Bibby & Wood, 2002; Goldstone & Sakamoto 2003; Richland, Zur

& Holyoak, 2007). Specifically, using concrete representations for initial learning and over time replacing parts of these representations with abstract representations, has been suggested by both theorists (e.g., Bruner, 1966) and researchers (Fyfe, McNeil, Son & Goldstone, 2014; Gravemeijer, 2002; Lehrer & Schauble, 2002). Known as concreteness fading (Goldstone & Son, 2005), empirical evidence supports the notion that students' transfer ability increases when a combination of representations is used during instruction (McNeil & Fyfe, 2012). Since transfer has been linked to the coherence of a situation model, it is important to analyze both the type and the sequence of representations found in current learning opportunities.

Deep Questions. Classroom discourse, the use of language within social contexts (Gee, 2010), helps to facilitate the development of student conceptual understanding (Chin, 2007; Mortimer & Scott 2003; Franke et al., 2009). Costa (2001) and Swartz (2008) provide empirical evidence that students attain deeper comprehension when they are provided with opportunities to converse within instructional settings, which Greeno (1991) agrees may contribute positively to the development of mental representations. These opportunities include verbal interactions with teachers, which often involves the act of asking and answering questions**.** Questioning student understanding during classroom instruction is a critical learning opportunity that shapes student learning (van den Oord & Van Rossem, 2002) through eliciting students' explanations of underlying principles (Craig, Sullins, Witherspoon, & Gholson, 2006).

In order to help students build connections and improve learning, the *IES* recommends that teachers need to help students learn how to ask and answer deep question (Pashler et al., 2007). Defined as a question that elicits deep explanations, deep questions include questions that target "causal relationships" (p. 29) and that are structurally connected to underlying principles. These include questions such as "why, why-not, how, and what-if" (p. 29). The inferential nature of these questions force students to distance themselves from the present in order to think about past or future events (Sigel & Saunders, 1979) and thus have been shown to have a direct impact on the cognitive process (Chapin & Anderson, 2003; Chin, 2006; Morge 2005). From the perspective of a situation model, focused and deliberate deep questions (Rubin, 2009) help students to facilitate connection-making between and within mathematical principles. Unfortunately, few deep questions are being asked in today's classrooms (Khan & Inamullah, 2011; Wimer, Ridenour, Thomas & Place, 2001) which may partially be why U.S. students continue to exhibit a lack of mathematical understanding,

The reviewed literature clearly supports the notion that a critical component of comprehension is the inference process that occurs as a result of making connections to prior knowledge (Pearson et al., 1979). I argue that students are best supported in this process when they are presented with learning opportunities useful for connection-making. Those opportunities that appear to be the most contributing factor in the creation of a situation model include: (a) the presentation of instructional tasks (b) the types of representations and (c) the use of deep questions. In response to Linn's (2006) call for future empirical research to explicitly search for ways to facilitate children's connections to prior knowledge, the following research question has emerged: How does an expert elementary mathematics teacher facilitate connection-making during classroom instruction on multiplicative inverses?

Method

From the perspective of a situation model, this case study (Stake, 1995) investigates how two U.S. expert elementary teachers facilitate connection-making during mathematics instruction. The focused content is multiplicative inverses.

Participants

The two teachers (T1 & T2) in this study are participants in a five-year National Science Foundation (NSF) funded project on early algebra in elementary schools. Although they teach in different buildings, both are third grade teachers for the same large high-needs urban school district in Pennsylvania. The teachers were selected from grade 3 because according to the Common Core State Standards (CCSSI) this is where multiplicative inverses is first taught. Furthermore, both teachers are female and at the time of this study they had $n_{T1} = 27$ and $n_{T2} = 23$ students on their class rosters. Based on the criteria used to select participants for the above mentioned NSF project, both T1 and T2 are considered to be expert teachers. Specifically, they have both been teaching for more than 17 years and are both Nationally Board Certified Teachers (NBCT).

Data Sources

This study analyzes classroom instruction for the following two mathematical lessons involving multiplicative inverses: (L1) Multiplication and Division (L2) Solving Inverse Story Problems. The two teachers in this study agreed to be videotaped while instructing a lesson on each of these two topics (Table 1). The four total lessons were videotaped using two digital video cameras, one that followed the teacher throughout the lesson and one that was set up to capture student interactions. The teacher camera footage was used to code the connection-making opportunities that occurred during instruction. All lessons were enacted during the 2014-2015 academic school year and had an average length of $\bar{x}_{T1} = 54$ minutes and $\bar{x}_{T2} = 57$ minutes.

Data Analysis

With regards to a situation model perspective of comprehension, all four videotaped lessons were coded using a researcher developed framework (Table 2) This framework is based on the IES recommendations for establishing connections to underlying principles and was adapted from a teacher lesson planning rubric used by Ding and Carlson (2013). The current framework contains three main categories (instructional tasks, types of representations, deep questions) which have all been shown to affect a student's ability to form connections to relevant prior knowledge. Each main category consists of three subcategories (review, worked examples and practice problems; concrete, abstract and sequence of representations; prior, current and future knowledge questions). A scale of 0-2 was used to code the teacher's effectiveness for facilitating connection-making within each subcategory found in the framework.

To answer the research question – How does an expert elementary mathematics teacher facilitate connection-making during classroom instruction on multiplicative inverses? –each of the four enacted lessons were analyzed and individually scored based on the same connection-making framework. This included scoring each subcategory (review, worked examples and practice problems; concrete, abstract and sequence of representations; prior, current and future knowledge questions) based on a 0-2 scale for the level to which the teacher's instruction during that lesson had appeared to facilitate connection-making. All subcategory scores were summed, and a connection-making score per lesson was determined. Averaging the two lesson scores per teacher, yielded an overall teacher connection-making score. These scores, along with a qualitative analysis that includes typical ways in which each teacher facilitated connection-making, were used to determine the extent to which learning opportunities found within classroom instruction promote the situation model perspective. Reliability of the coding framework was checked by having a second researcher code one of the videotaped lessons (25% of the data). All 9 subcategory scores were found to be identical to scores given by the first researcher.

Results & Discussion

When applying the connections-making rubric to L1 (multiplicative inverses), T1 scored 17/18 points and T2 scored 12/18 points. In L2 (division story problem), T1 scored 18/18 and T2 scored 11/18. Individual subcategory scores for each lesson can be found in Table 4. The discussion below highlights the similarities and differences that were found with regards to how the two teachers used instructional tasks, representations and deep questions in order to facilitate connection-making.

Connection-Making with Instructional Tasks (IT).

During the review task in L1, T1 made explicit connections to the targeted content by using students' prior work to review multiplication as multiple sets of equal groups. In this instance, T1 used one students' problem involving placing shoes into shoe boxes, to review the concept of how many groups (boxes), how many were in each group (shoes) and how many there were altogether. Although not explicit (never used the words), these questions reviewed the previously learned abstract vocabulary (i.e., factor & product). When talking about solution strategies for this problem however, T1 made an explicit connection between addition and multiplication (skip-counting). Both This connection was later used to set-up the inverse relation of division, the targeted content. Even though T1 only discussed one students' work, it was clear that she had provoked personal connections as students recalled the specifics from their own problem. T2's connections to the targeted content during review tasks, were on the other hand implicit. During L1 for example, T2 helped students recall the explicit connection between addition and subtraction, but only used this connection for the purpose of having students recall that they had previously learned the concept of an inverse. Time was then spent reviewing addition and subtraction fact triangles, but again with no explicit connection to multiplicative inverses (the current to-be-learned content). Instead of implicitly reviewing vocabulary with a concrete example like T1 did, T2 had students "shed some light" by verbally defining relevant words such as multiplication, division and relationship. In general, T1 used concrete representations during review to make explicit connections to the targeted content (strategies for solving inverse relation problems) whereas the time spent on review by T2 seemed to only include non-contextual abstract ideas (i.e., vocabulary and fact triangles) that were only implicitly related to the targeted content. This difference is reflected in the subcategory scores found in table 4.

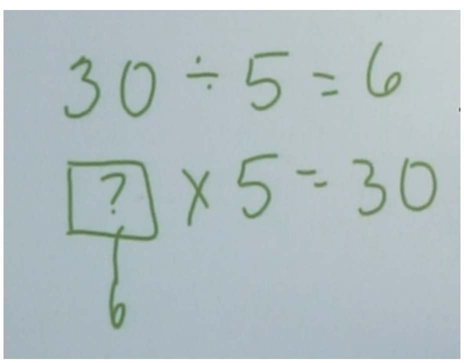
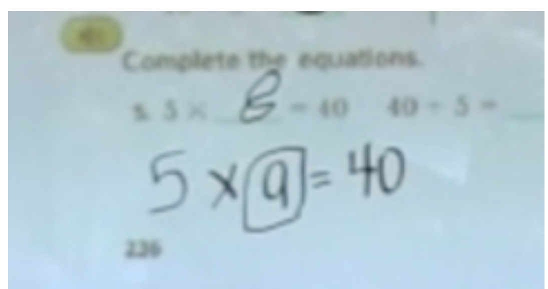
Across lessons, T1 continuously showed more explicit awareness to inverse relations during classroom instruction. For example, after working through the multiplication problem—"A robot has 4 hands. Each hand has 6 fingers. How many fingers does the robot have altogether?"—T1 facilitated a discussion about how this problem could be reversed. Using the ideas of "how many in each" and "how many in total," the class determined the division problem to be "There are 24 fingers from a robot. This robot hands. How many fingers in each hand?" Immediately following this worked-example, the students spent time writing their own inverse story problems and thus practice problems were explicitly connected to the worked example. This models T1's general trend of consistently alternating between worked examples and practice problems. Further, by using the same context for creating both a multiplication and a division problem, explicit connections between and within various IT occurred. In contrast, T2's worked examples were often not as connected and lacked depth. For instance, in T2's first lesson, students made a 3×4 array card and were asked to write a representative multiplication and division statement. Although most students were able to write both $3 \times 4 = 12$ and $12 \div 4 =$

3 , no explicit connection was made to how the array represented the two multiplicative inverse number sentences. Furthermore, when presenting students with the notion of reversing a story problem from multiplication to division, T2 showed an already reversed problem from the textbook which did not allow students the connection-making opportunities that T1's students had for this same task. T2 also demonstrated an inconsistent meaning of multiplication when in L1 she used 3×4 to represent a story problem involving 4 groups of 3, but later in a different worked example used 7×4 to represent seven groups of 4. In general, time was not spent unpacking T2's worked examples for the purpose of drawing deep connections to the targeted content. As a result, instruction at times seemed rushed, which most likely was due to the examples not containing much depth or variability. Although the practice problems were not interleaved into T2's instruction, they did seem to closely model the worked examples.

Connection-Making with Representations (REP).

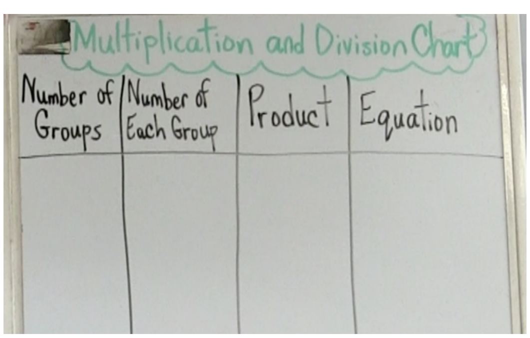
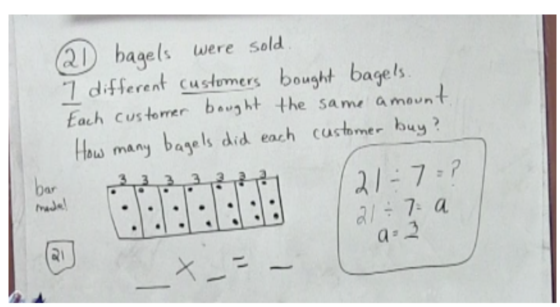
A common theme found between T1 and T2 was that they both promoted the use of multiple solution strategies (e.g., using pictures, skip-counting, repeated addition, equations) throughout each lesson. Multiple REP were therefore used during instruction in each of the four teacher lessons, however, the effectiveness of using these different REP for facilitating connections varied. With regards to concrete REP, T2 did not use them to effectively develop explicit connections involving multiplicative inverses. Specifically, when using an array model to represent 18 ÷ 3 during the first lesson, T2 used a count-up method to arrive at an answer of 6 for each row, but failed to make any reference to the inherent multiplicative inverse. Likewise, when using circles to represent 7×4 in the previously discussed worked example, T2 made no reference to the reason why 7 circles were drawn instead of 4. In sharp contrast, when using concrete representations to create groups of equal quantities, the students in T1's class were repetitively forced to think about the meaning of each number in the multiplicative inverse problem. Unlike T2, when repeated addition or skip-counting was used as a solution strategy, T1 often highlighted the connection to multiplicative inverses.

Both T1 and T2 effectively used abstract representations to establish connections to multiplicative inverses. This was evident by both teachers' use of side by side number sentences to explicitly connect multiplication and division. There was however a slight teacher difference in symbol use when writing these number sentences. While T1 used an empty box (or a box containing a question mark) as a place holder for a missing factor in a multiplication sentence, T2 actually made reference to Algebra and used the letter "a" to represent the unknown factor. Each teacher also showed students how to use the "house" notation for division, ultimately creating an implicit connection to the future concept of long division. The use of these alternative abstract REP may have been for the purpose of promoting abstract reasoning.

<table>
<tr><td>T1: Use of Abstract REP</td><td>T2: Use of Abstract REP</td></tr>
<tr><td>
</td><td>
</td></tr>
</table>

Both teachers mentioned the importance of using efficient strategies but neither explicitly connected efficiency to abstract reasoning. The desire for students to master multiplication facts in order to solve division problems was therefore non-existent in any of these lessons. In fact, one might even argue that T1 actually downplayed the need to know these facts because of how much she encouraged the use of multiple solution strategies. For instance, when students provided an abstract solution early on during the L1 instruction, T1 accepted the solution as valid however, she did not exploit it as the final preferred method. Instead, she solicited other strategies by asking questions such as –"What if you didn't known 4 times 4?"—or—"Can you use a different strategy? Repeated Addition?" On a surface level, this resulted in IT not always progressing from concrete to abstract REP, which is reflected in T1's L1 sequences of representations subcategory score.

At the beginning of L2, T1 encouraged the use of concrete REP with students who had already demonstrated a clear abstract understanding for multiplicative inverses when she said "even if you know your multiplication fact, what is another way to prove this?" During the discussion that followed, it became clear that T1 was attempting to draw explicit connections between various representations and solutions strategies. This desire to use increasingly more abstract REP became even more explicit during the middle of L2 when the T1 began using a multiplication and division chart for helping to turn a concrete story problem into a solvable equation. Filling this chart in from left to right indicates that student's encounter the concrete questions "how many groups" and "how many in each group" before having to reason with the abstract principles of product and equations.

<table>
<tr><td>T1: Sequence of REP

</td><td>T2: Sequence of REP

</td></tr>
</table>

Multiple solutions were also used throughout T2's instruction, but the sequence of REP used in these solutions seemed to always progress from concrete to abstract. As shown in the above worked example involving bagels, T2 solved the problem first by using a bar model, then by repeated addition and finally by abstract number sentences. The presentation of each IT by T2 used a similar approach which seemed to not be altered by student interactions. On the other hand, T1's IT and uses of various REP were dictated mainly by student reasoning that became apparent through classroom discourse.

Connection-making with Deep Questions (DQ).

Specific to facilitating connections to prior knowledge, both teachers used DQ when reviewing additive inverses and when discussing previously learned multiplication concepts. For the purpose of drawing connections to student's prior knowledge of additive inverses, T1 asked questions such as "What is different about this problem compared to problems that we had been working with before?"—and—"Do you think multiplication and division are related like addition and subtraction?" Illustrating the connection-making that occurred as a result of asking these prior knowledge DQ, one student in T1's class deduced "multiplication is like addition, you are adding them all up and division is separating them." Likewise, after begin asked a similar DQ by T2—"Do we know two other operations that are opposites?"—a student formed the connection that "Inverse. When you do division, you are subtracting groups. When you do multiplication, you are adding groups." These statements provide strong indication that well-connected situation models for inverse relations were being formed as a result of both teacher's DQ. This connection-making opportunity was also enhanced each teacher reviewing the previously learned concept of fact families. DQ were also used during review in order to stress the importance of previously learned vocabulary involving the inherent relationships between various quantities within multiplicative inverses (e.g., groups, factor, and product).

Although DQ were asked by both teachers for the purpose of forming connections within the current to-be-learned content of multiplicative inverses, many of T1's DQ were asked for the purpose of guiding instruction whereas T2's DQ were mainly for evaluation purposes. This appeared to be mainly due to the fact that T1 created a learning environment which was very inference oriented and which was based on continuous conversations related to children's mathematical reasoning. T2's instruction was less investigative and relied more on a show-and-tell format. As a result, T2 often missed opportunities to ask DQ in order to facilitate connection-making. Specifically, T2 asked mainly procedural type questions such as –"How much will 3 rows of 6 be?"—or—"How did you get 3 from $18 \div 6$?". T1 on the other hand asked questions such as—"Can you describe?"—or—"What makes you think it is multiplication?"—or—"Can you compare these two strategies?"—to elicit deep conversation and deep conceptual understanding. It is interesting to note that T2 seemed to only ask conceptual questions when students provided incorrect solutions.

DQ were used only by T1 for the purpose of forming connections to future knowledge. These DQ revolved around forming connections between division and the future content of fractions. On several occasions T1 posed DQ such as "how come you didn't say 2 divided by 6" when writing the fraction corresponding to splitting six into two equal parts. Although students responded to this question by simply saying "because you couldn't do it" or "it wouldn't make sense," it is clear that the DQ prompted students to use their current knowledge in order to make inferences about what would happen in a future unknown situation (dividend < divisor). In addition, T1 also asked several DQ for the purpose of forming connections to the future topic of improper fractions (dividend > divisor). For example, when discussing an IT for sharing 18 cards evenly among four friends, T1 asked “What will happen if it was not even – 19 cards—Who gets the last card?” Student responses included the words “extra” and “remainder” which indicates Taken together, these two examples suggest that T1’s use of DQ appear to have facilitated the creation of a situation model for future exploration of fractions. It should be noted that T2 made a few references to the future discipline of Algebra, but these references were not in the form of DQ.

Significance

Current research reveals that instruction of inverse relations primarily involves procedural techniques (Baroody, 1999; DeSmedt et al., 2010; Torbeyns et al., 2009), with few references to conceptual understanding (Crooks & Alibali, 2014). Past research on inverse relations has focused on *if* and *when* children show evidence of understanding inverse relations, whereas this study lays a foundation for investigating *why* and *how* this understanding occurs. From a situation model perspective, how expert elementary mathematics teachers facilitate connection-making through the use of instructional tasks, representations, and deep questions has never been done. Although their still exists a large gap in current knowledge surrounding how best to facilitate children’s connections to prior knowledge, this lays a foundation for future empirical research.

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Table 1. *Targeted Content: Teacher Provided Goal*

	T1 Lesson 1	T1 Lesson 2	T2 Lesson 1	T2 Lesson 2
Lesson Titles	Solving Division Problems	Multiply or Divide?	Solve Division Story Problems	Relate Multiplication and Division
Math Focus Points (T1) & Objectives (T2)	Understanding division as the splitting of a quantity into equal groups. Using the inverse relationship between multiplication and division to solve problems.	Using the inverse relationship between multiplication and division to solve problems. Using multiplication combinations to solve division problems. Using and understanding division notation.	Students will be able to use a model, in order to write related multiplication and division facts.	Students will be able to use a model, in order to write related multiplication and division facts.

Table 2. *Coding Framework for Connection-Making: Facilitating a Situation Model*

Category	Subcategory	0	1	2
Instructional Tasks	Review	The task was a routine review of prior content but no connections to the targeted content was made.	An implicit connection to the targeted content was made, but not well developed.	An explicit connection to the targeted content was established and well developed.
	Worked Examples	No connections to prior or the targeted content was made.	Implicit connections to prior or the targeted content were made, but not well established or discussed. Clear opportunities to make connections are also missed.	Explicit connections to prior or the targeted content were made. No clear opportunities to make connections are missed.
	Practice Problems	Practice problems have no connection to the worked examples.	Practice problems have an implicit connection to the worked examples.	Practice problems have an explicit connection to the worked examples.
Representations	Concrete	No concrete representations (ie. manipulatives, pictures, or story situations) are used to form connections to prior or the targeted content within instructional tasks.	Concrete representations are used to form connections to prior or the targeted content within instructional tasks, but the connections are not well developed.	Instructional tasks are situated in rich concrete contexts (i.e. story problems) and are used to form well developed connections to prior or the targeted content within instructional tasks.
	Abstract	No abstract representations (ie. numbers, mathematical symbols, equations) are used to form connections to prior or the targeted content within instructional tasks.	Abstract representations are used to form connections to prior or the targeted content within instructional tasks, but the connections are not well developed.	Abstract representations (i.e. equations) are used to form well developed connections to prior or the targeted content within instructional tasks.
	Sequence of Representations	No connections between concrete and abstract representations are made between instructional tasks.	A connection between concrete and abstract representations is established between instructional tasks, but it does not progress from concrete to abstract.	A clear connection between concrete and abstract representations is established that indicates a progression (concrete to abstract) of worked examples for the purpose of forming connections to the target concept.

Deep Questions	Prior	No deep questions are asked for the purpose of forming connections to prior knowledge.	Some deep questions are asked for the purpose of forming connections to prior knowledge; but there remain important missing connections to prior knowledge.	Deep Questions are posed to elicit students to form connections between prior knowledge and the targeted concept and there are no important missing connections to prior knowledge.
	Current	No deep questions are asked for the purpose of forming connections within the current to-be-learned content knowledge (ie. between examples)	Some deep questions are asked for the purpose of forming connections within the current to-be-learned content knowledge but the connections remain at the surface level.	Deep questions are posed to elicit students to form connections within the current to-be-learned content knowledge.
	Future	No deep questions are asked for the purpose of forming connections to future knowledge.	Some deep questions are asked for the purpose of forming connections to future knowledge; but the connections are implicit.	Deep questions are asked for the purpose of forming connections to future knowledge and the connections are explicit.

(Table 2 continued).

Table 3. *Connection-Making Score for Facilitating Situation Models*

Lessons	T1 Score	T2 Score
(1) Multiplication and Division	17	12
(2) Solving Inverse Story Problems	18	11
Average	17.5	12.5

Table 4. *Teacher Connection-Making Subcategory Scores*

Categories	Subcategories	T1 L1	T1 L2	T2 L1	T2 L2
	Review	2	2	1	1
Instructional Tasks	Worked Examples	2	2	1	1
	Practice Problems	2	2	2	2
	Concrete	2	2	1	1
Representations	Abstract	2	2	2	2
	Seq. of Rep.	1	2	2	2
	Prior	2	2	2	1
Deep Questions	Current	2	2	1	1
	Future	2	2	0	0
Total Score		17	18	12	11