This five-year project (2014-2019) aims to identify, from a cross-cultural perspective, essential algebraic knowledge for teaching (AKT) that fosters students’ algebraic thinking in elementary school. Focusing on two fundamental mathematical ideas that are early algebra topics – inverse relations and properties of operations (Common Core State Standards Initiative, 2010) – this study explores AKT based on integrated insights of U.S. and Chinese expert teachers’ classroom performance. This study is innovative because it is among the very first to seek AKT focusing on fundamental mathematical ideas from a cross-cultural perspective. The conceptual framework for identifying AKT is aligned with high-quality cognitive research recommendations on worked examples, representations, and deep questions (Pashler et al., 2007). It is expected that the identified AKT along with these aspects will contribute to students’ deep understanding of fundamental mathematical ideas and thus algebraic readiness.

Objective #1: Identify AKT that facilitates algebraic thinking and develop preliminary findings into teaching materials (Ys1-3).

Objective #2: Disseminate preliminary findings and refine research-based teaching materials based on evaluative data (Ys3-4).

Objective #3: Integrate research with education through course development at Temple and teacher outreach in Philadelphia (Ys3-5).

Participants

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<tr>
<td>Inverse relations</td>
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<tr>
<td>Properties of Operations</td>
<td>5</td>
<td>200</td>
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<tr>
<td>Decomposition topics</td>
<td>15</td>
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Materials and Instruments

Each teacher taught 4 lessons on either inverse relations (additive or multiplicative) or the basic properties of operations (commutative, associative, and distributive) based on teachers’ existing textbooks. The US textbooks included: Investigations, Go Math! My Math, which are all aligned with the Common Core. The Chinese textbooks are one of the three main textbook series used in China, which was published by the Jiaogu Educational Press (JSEP).

Working examples: Interleaving worked examples with problem solving exercises.

Representations: Making connections between concrete and abstract representations.

Deep questions: Asking deep questions to elicit student self-explanations.

Research

<table>
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<tr>
<th>Year</th>
<th>Data collection</th>
<th>Inverse Relations</th>
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<tr>
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<td>Year 2</td>
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<td>Year 3</td>
<td>Data analysis &amp; material development</td>
<td>US-China online teacher forum</td>
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<td>Year 4</td>
<td>Data analysis &amp; material development</td>
<td>US-China online teacher forum</td>
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<td>Year 5</td>
<td>Data analysis &amp; material development</td>
<td>US-China online teacher forum</td>
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To link AKT to student learning gains, we conducted pre and post tests with students. These instruments (inverse relations, the basic properties of operations) were developed based on the literature and were reviewed by the advisory board members. Validity and reliability was ensured through pilot tests.

Video Coding

Partial Results based on Y1 US videos

Our video studies (Ding, Hassler, & Chen; 2016) show that U.S. expert teachers spent sufficient time on worked examples. However, some teachers taught 3-4 repetitive examples in a quick pace which led to little depth.

With regard to representations, US teachers with higher video scores used representations to model quantitative relationships (as opposed to finding answers). Some teachers used the “bar models,” which is similar to the tape diagram emphasized by the Common Core.

Deep questions appeared to be challenging for many teachers. Teachers with higher scores (e.g., T5) asked comparison questions to facilitate structural connections (e.g., Can you change this problem to its reverse?).

Partial Results based on Y1 Chinese video

Chen & Ding (2016) reported a case study where a Chinese expert teacher in all four lessons spent 30%-40% class time unpacking just one example. The teacher always situated the worked examples in concrete word problem contexts (consistent with the textbook presentation). The quantitative relationships were modeled through varied representations including tape diagrams. The solutions were always numerical. This sequence shows concreteness fading (Goldstone & Son, 2005).

The Chinese teachers asked deep questions among which, comparison questions are most impressive.

References


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