CAREER: Algebraic Knowledge for Teaching: A Cross-cultural Perspective

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Overview of the Project

This five-year project (2014-2019) aims to identify, from a cross-cultural perspective, essential *algebraic knowledge for teaching* (AKT) that fosters students' algebraic thinking in elementary school. Focusing on two fundamental mathematical ideas that are early algebra topics – *inverse relations* and *properties of operations* (Common Core State Standards Initiative, 2010) – this study explores AKT based on integrated insights of U.S. and Chinese expert teachers' classroom performance. This study is innovative because it is among the very first to seek AKT focusing on fundamental mathematical ideas from a cross-cultural perspective. The conceptual framework for identifying AKT is aligned with high-quality cognitive research recommendations on worked examples, representations, and deep questions (Pashler et al., 2007). It is expected that the identified AKT along with these aspects will contribute to students' deep understanding of fundamental mathematical ideas and thus algebraic readiness.

Objectives of the Project

Objective #1: Identify AKT that facilitates algebraic thinking and develop preliminary findings into teaching materials (Ys1-3).

Objective #2: Disseminate preliminary findings and refine research-based teaching materials based on evaluative data (Ys3-4)

Objective #3: Integrate research with education through course development at Temple and teacher outreach in Philadelphia (Ys3-5)

	Research	Education
Year 1	Data collection: Inverse Relations	
Year 2	Data collection: Properties of Operations	
Year 3	Data analysis & material development Workshops for teachers in both countries	US-China online teacher forum
Year 4	Reteach lessons & material refinement	US-China video conference Chinese classroom visit Workshops for SDP novice teachers
Year 5		SDP teacher conference Temple course revision and development

Participants

US teachers: Philadelphia; Chinese teachers: Nanjing

Y	Topic	US Classroom		Chinese Classroom		# of Lessons/Teacher	# of taped lessons
		# of teachers	# of students	# of teachers	# of students		
1	Inverse Relations	8	200	8	400	4	64
2	Properties of Operations	8	200	8	400	4	64
4	Reteach both topics	16	400	16	800	4	128

Materials and Instruments

Each teacher taught 4 lessons on either inverse relations (additive or multiplicative) or the basic properties of operations (commutative, associative, and distributive) based on teachers' existing textbooks. The US textbooks included: Investigations, Go Math & My Math, which are all common core-based. The Chinese textbooks are one of the three main textbook series used in China, which was published by the *Jiang Su Educational Press* (JSEP).

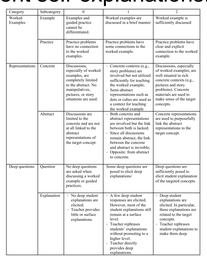
To link AKT to student learning gains, we conducted pre and post tests with students. These instruments (inverse relations, the basic properties of operations) were developed based on the literature and were reviewed by the advisory board members. Validity and reliability was ensured through pilot tests.

Video Coding

Worked examples: Interleaving worked examples with problem solving exercises.

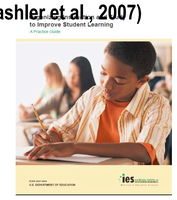
Representations: Making connections between concrete and abstract representations.

Deep questions: Asking deep questions to elicit student self-explanations.

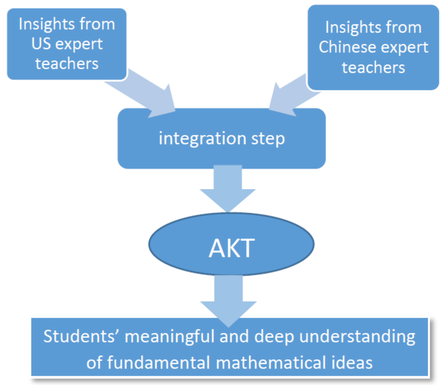


Category	Subcategory	0	1	2
Worked Examples	Example	Examples and practice exercises cannot be differentiated.	Worked examples are discussed in a brief manner.	Worked example is sufficiently discussed.
	Practice	Practice problems have no connection to the worked examples.	Practice problems have some connections to the worked example.	Practice problems have clear and explicit connections to the worked example.
Representations	Concrete	Discussions, especially of worked examples, are well situated in the abstract. Not manipulatives, pictures and story situations are used.	Concrete contexts (e.g., story problems) are involved but not utilized to make sense of the worked example. Semi-abstract representations such as dots or cubes are used as a context for teaching the target concept.	Discussions, especially of worked examples, are well situated in rich concrete contexts (e.g., pictures and story problems). Concrete contexts are used to make sense of the target concepts.
	Abstract	Discussions are limited to the abstract level and are not at all linked to the abstract representations of the target concept.	Both concrete and abstract representations are used but the link between both is lacked. Since all discussions remain abstract, the link between the concrete and abstract is invisible. Opposite: from abstract to concrete.	Concrete representations are used to purposefully link the abstract representations to the target concept.
Deep questions	Question	No deep questions are asked when teaching worked example or guided practice.	Some deep questions are posed to elicit deep explanations.	Deep questions are sufficiently posed to elicit self-explanations of the targeted concepts.
	Explanation	No deep student explanations are elicited. Teacher provides little or surface explanations.	A few deep student responses are elicited. Teacher follows up on student explanations still remain at a surface level. Teacher rephrases students' explanations without attempting to a higher level. Teacher directly provides deep explanations.	Deep student explanations are elicited. Teacher uses these explanations are related to the target concepts. Teacher rephrases student explanations to make them deep.

The IES recommendations (Pashler et al., 2007)



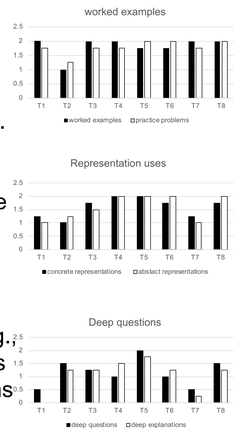
Data Analysis



Partial Results based on Y1 US videos

Our video studies (Ding, Hassler, & Chen, 2016; Hassler & Ding, 2016) show that U.S. expert teachers spent sufficient time on worked examples. However, some teachers taught 3-4 repetitive examples in a quick pace which led to little depth.

With regard to representations, US teachers with higher video scores used representations to model quantitative relationships (as opposed to finding answers). Some teachers used the "bar models," which is similar to the tape diagram emphasized by the Common Core.



Deep questions appeared to be challenging for many teachers. Teachers with higher scores (e.g., T5) asked comparison questions to facilitate structural connections (e.g., Can you change this problem to its reverse?).

Partial Results based on Y1 Chinese video

Chen & Ding (2016) reported a case study where a Chinese expert teacher in all four lessons spent 30%-40% class time unpacking just one example.

The Chinese teacher always situated the worked examples in concrete word problem contexts (consistent with the textbook presentation). The quantitative relationships were modeled through varied representations including tape diagrams. The solutions were always numerical. This sequence shows concreteness fading (Goldstone & Son, 2005).

The Chinese teachers asked deep questions among which, comparison questions are most impressive.

Partial Results based on Y1 Student data

Based on Chinese and US first and second graders' pre- and posttest, Li, Hassler, and Ding (2016) analyzed students' correctness and explanations of inverse-based problems. It was found that students in both countries improved their computation skills and understanding of inverse relations. However, Chinese students demonstrated quicker and larger progress. Given that US students came from a more diverse population with low-income families, US students' achievement is also encouraging. This may suggests a possible connection between AKT and student achievement gains.

In fact, Stull, Ding, Hassler, Li, and George (2016) reported that there is a significant relationship between AKT scores and students' learning of inverse relations, controlling various non-instructional factors.

Conclusions based on Y1 Data

To date, our cross-cultural findings suggest three necessary components of AKT: (1) unpacking worked examples (2) using contextual support to illustrate quantitative relationships, and (3) asking comparison questions to promote connection making.

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