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Transitioning Textbooks into Classroom Teaching: An Action Research on Chinese Elementary Mathematics Lessons

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Introduction

Elementary students around the world have been reported to have difficulties in learning mathematics especially fundamental mathematical ideas such as inverse relations (Baroody, 1999; Carpenter, Franke, & Levi, 2003; Torbeyns, De Smedt, Ghesquière, & Verschaffel, 2009). International comparison studies, however, indicate that Chinese students generally possess superior mathematical achievement (Li, Ding, Capraro, & Capraro, 2008; Li, Hassler, & Ding, 2016; PISA, 2006, 2009), which may be partially attributed to Chinese outstanding textbook presentations (Ding, 2016; Ding & Li, 2010, 2014) and classroom teaching (Cai, 1995; Li & Huang, 2013; Stevenson & Stigler, 1992). Prior studies also have reported that Chinese teachers use textbooks with loyalty (Ma, 1999). However, how Chinese teachers actually transition textbooks in classrooms to support student learning has rarely been documented and analyzed. The action research study takes a step further examining how an expert Chinese teacher transitioned elementary mathematics textbooks into classroom instruction to support student learning of ideas related to addition, subtraction, and the inverse relations between them.

Textbooks play an important role in learning and teaching, often through presentation frequency and sequence (Haman & Ashcraft, 1986), representation uses (Ding & Li, 2014, Murata, 2008), and other contributing factors. In particular, findings from international textbook studies have shed light on the improvement of students learning. Consider, for example, the aforementioned inverse relations between addition and subtraction. Ding (2016) reported cross-cultural differences in textbook presentations of this idea between U.S. and Chinese textbooks. For instance, Chinese textbooks initially introduced this concept through concrete story contexts while U.S. textbooks often use manipulatives (e.g., cubes and dominos) without contextual support. More importantly, Chinese textbooks stressed structural relationships (e.g., original –

lent out = leftover; lent out + leftover = original) while U.S. textbooks tended to emphasize symbol manipulations (e.g., 3 + 5 = 8 and 8 – 3 = 5). This textbook difference may have contributed to Chinese and U.S. students' overall learning differences in inverse relations (Li et al., 2016). In fact, Ding's study is also aligned with previous findings on other mathematical ideas such as the concept of equivalence. While prior studies over years reported students' robust learning difficulties with the "=," Li et al. (2008) depicted a sharply contrasting picture between the U.S. and Chinese students' understanding, which were further attributed to the textbook differences. Indeed, the Chinese textbook approach (initially presenting the "=' as a relational sign in the context of comparison) has been recently proven to be most effective by classroom experiments (Hattikudur & Alibali, 2010).

Textbooks, however, only present intended knowledge, which needs to be transitioned by teachers into enacted curricula in order to make an impact on student learning (Remillard, 2005). Prior studies reported that Chinese teachers uniformly acknowledged the important role of textbooks and they generally use textbooks with loyalty (Ma, 1999), which is different from U.S. teachers' multiple textbook uses including following, or incorporating, or interpreting textbook presentations (Remillard, 2005). Chinese teachers' loyal textbook uses, however, is different from blindly following textbook presentations. Indeed, Chinese teachers frequently "study" textbooks to enhance students' learning (Ma, 1999). Ding et al. (2013) surveyed/interviewed a total of 36 Chinese teachers who self-reported various approaches to study textbooks during planning. These included identifying the import and difficult teaching points, understanding the purposes of each worked example and practice problem, exploring the reasons behind certain textbook information, and exploring the best approaches to present examples from the perspective of students. Interestingly, many teachers stressed going beyond textual information to promote students' deep thinking. Others suggested a consideration of student learning interests and real-life experiences when presenting the targeted concepts. Likely due to Chinese teachers' frequent study of elementary textbooks (Ding, Li, Li, & Gu, 2013; Ma, 1999) in which the fundamental mathematical ideas are meaningfully presented (Cai & Moyer, 2008), Chinese teachers have demonstrated strong knowledge of and attention to fundamental mathematical ideas such as inverse relations (Ma, 1999), which may further contribute to their classroom teaching.

Prior findings have shed light on Chinese teachers' textbook uses. However, these findings were mainly based on interviews rather than actual classroom observations. Very few studies have explored Chinese teachers' enacted teaching in comparison with textbooks. This may have limited the contribution that Chinese textbooks and classroom instructions could have potentially made to the field. To narrow this gap, this study explores how a Chinese expert teacher transitions textbook presentations into classroom teaching.

To explore the Chinese teacher's transition process from textbook to classroom teaching, this study will use a three dimension cognitive construct that serves as a conceptual framework to guide this study. This construct includes the use of worked examples, representations, and deep questions, which were recommended by the Institute of Educational Sciences (IES) for teachers to organize classroom instruction to support student learning (Pashler et al., 2007). In fact, components of this cognitive construct were also involved in prior studies on textbook examinations (Ding, 2016; Ding & Li, 2010, 2014) and teachers' lesson planning (Ding & Carlson, 2013). According to the literature, worked examples can effectively constrain the way students interpret abstract principles (Colhoun, Gentner, & Loewenstein, 2005). In particular, worked examples (problems with solutions given) help students acquire necessary schemas to solve new problems (Sweller & Cooper, 1985). Classroom experiments have reported that the use of worked examples is more effective than simply asking students to solve problems (Zhu & Simon, 1987).. Recent studies also suggested interweaving worked examples and practice problems (Pashler et al., 2007).

The second dimension of the cognitive construct is representation uses. Concrete representations, such as graphs or word problems, support initial learning because they provide familiar situations that facilitate students' sense-making (Resnick, Cauzinille-Marmeche, & Mathieu, 1987). However, overexposing students to concrete representations may hinder their transfer of the learned knowledge because these representations contain irrelevant and distracting information (Kaminski, Sloutsky, & Heckler, 2008; Uttal, Liu, & Deloache, 1999). Thus, some researchers suggest fading the concreteness into abstract representations to promote generalization and transfer in new contexts (Goldstone & Son, 2005).

The last dimension of the construct is about deep questions. Students can effectively learn new concepts and ideas through self-explanations (Chi, 2000; Chi et al., 1989). However, they themselves usually have little motivation or ability to generate high-quality explanations. It is necessary for teachers to ask deep questions to elicit students' explanations. The deep questions refer to those questions that can elicit the underlying principles, causal relationships, and structural knowledge (Craig, Sullins, Witherspoon, & Gholson, 2006).

The above IES recommendations have been gleaned from numerous high quality cognitive and classroom research and provided general directions for teachers in all domains to organize instruction to improve learning (Pashler et al., 2007). However, without explicit illustrations with content-specific examples, teachers appeared to have challenges in incorporating them into actual practices (Ding & Carlson, 2013). For example, even if a textbook provides a worked example to teach inverse relations, instead of simply asking students to study the example provided by the textbook, how can a teacher unpack this worked example through appropriate use of representations and deep questions to help students make sense of inverse relations? In this action research study, we examine a Chinese expert teacher's transition from textbook lesson to enacted classroom teaching. In particular, we will ask for three questions: (1) How does the Chinese expert teacher transition the worked examples from the textbook to the enacted lessons? (2) How does the Chinese expert teacher transition the representations from the textbook to the enacted lessons? And (3) How does the Chinese expert teacher transition the deep questions from the textbook to the enacted lessons?

Methods

This study employs an action research method (Kemmis & McTaggart, 1982; Mills, 2011). Action research is a form of systematic and reflective inquiry conducted by participants to improve their understanding of the social or educational practices and the situation in which these practices are carried out (Kemmis & McTaggart, 1982). In this study, action research is undertaken by the first author who is a Chinese participating teacher involved in a large five-year cross-cultural project. Given that the purpose of the current study is to understand Chinese expert teachers' transition process from textbook to enacted teaching, it is appropriate to employ an action research method, which will allow the teacher to have a voice on the reasons behind the transition process. Indeed, action research is important for teachers themselves because the systematic analysis and reflections can enable teachers to achieve a deep understanding of their own practice (Adelman, 1975; Corey, 1953).

The Project and the Participant

The teacher participated in a large project supported by The National Science Foundation (NSF). This project aims to identify useful knowledge for teaching fundamental mathematical ideas based on expert teachers' classroom performance. In the first year, 8 US and 8 Chinese expert teachers were involved. The participating teacher of this action research is one of the eight Chinese teachers and is female. By the time of her involvement, she had 16-years of teaching experience with elementary mathematics looping from Grades 1-3. She also has received numerous teaching awards from teaching competitions at both local and national levels. While working as a full time mathematics teacher and serving as the director of the Teaching and Research Group (Ma, 1999) in an elementary school, she also is a first year doctoral student who is pursuing a PhD degree in a top tier normal University in China. With this NSF project, this teacher taught four lessons that directly or indirectly involved inverse relations, a critical early algebra topic that has been emphasized in the field (Carpenter et al., 2003).

Instructional Tasks

The four lessons were selected from the existing Chinese second grade textbook, *Jiang Su Educational Press* textbook (JSEP, Su & Wang, 2011), which was developed based on the new Chinese curriculum standards (Ministry of Education, 2011). The second author, as the project investigator, selected these lessons based on the literature assertion on inverse relations and the actual textbook opportunities. All four lessons were related to addition, subtraction, and explictly or implictly involved the inverse relations. According to the textbook, Lessons 1 and 2 were both additive comparison problems (Carpenter et al., 1999), which together indicated inverse quantitative relationship. Specifically, Lesson 1 was about how to equalize the "difference" between two given quantities (Xiaojun's 8 beads and Fangfang's 12 beads (see Figure 1a). Students were taught three methods: adding to the less, removing from the more, and moving from the more to the less. Lesson 2 was to find the large quantity (using addition) or small quantity (using subtraction) with the difference known. This lesson was situated in a story context about making flowers (see Figure 1b). Lesson 3 was to solve a two-step word problem which involved getting on and off a bus (see Figure 1c). To find the balance, students were presented with three methods: first add and then subtract, first subtract and then add, and add the difference. Finally, Lesson 4 was to check subtraction using addition through a story context of borrowing books (see Figure 1d).

Figure 1. Instructional tasks involved in four lessons

Procedures and Data collection

These four enacted lessons were designed and taught by the first author based on the textbooks. All of these lessons were changed to certain degrees to unpack the worked examples to teach the underlying concepts. In particular, except for lesson 4, three of the four lessons utilized student pre-learning through the previous days' homework. Note that students' pre-learning products were collected and examined by the teacher, allowing her to choose the representative work for class discussion. In fact, students' responses to the pre-learning tasks also provided the teacher with a sense about students' existing knowledge when they came to learn the new concepts.

For each enacted lesson, the teacher prepared corresponding electronic courseware that contains the designed example tasks and practice problems in a virtual learning environment. All of the lessons were videotaped by the NSF project with one camera focusing on the teacher and the other camera focusing on students. After each lesson, the teachers were interviewed based on a structured interview sheet that contained questions on worked examples, representations, and deep questions. All of this data was collected with a copy shared with the first author, who is a teacher researcher in this study.

Coding and Data Analysis

The four lessons were analyzed and coded based on a coding framework (see Table 1) that aims to identify changes occurred during the transition process from the textbook to the enacted lessons. In particular, the teacher as the first author analyzed and reflected upon the changes in terms of worked examples, representations, and deep questions, respectively. The second author as the project investigator watched all of the videos and discussed the codes and coding process with the teacher.

Table 1. *Coding Framework to Identify the Changes from the Textbook to Enacted Lessons*

		In the Textbook	In the Enacted Lesson	Rationale of Changes
Worked Examples	Lesson 1			

	Lesson 2			
	Lesson 3			
	Lesson 4			
Representations	Lesson 1			
	Lesson 2			
	Lesson 3			
	Lesson 4			
Deep Questions	Lesson 1			
	Lesson 2			
	Lesson 3			
	Lesson 4			

Worked examples. We coded four aspects for the use of worked examples in both the textbook and the corresponding lessons: context, space, frequency, and sequence. For context, we compared the story situation (if any) where the worked example was situated. For Space, we measured how much a worked example covered the textbook pages and computed the portion it took in the whole lesson. We also tracked time spent on a worked example in a lesson and computed its portion. Although the measurement units were not the same, they both reflected the proportion of "space" it occupied a lesson regardless of length or time. With regard to frequency, we identified the number of worked example as well as the associated subproblems. If an example task contained subproblems, it was still considered as one problem. For sequence, we coded the order that the worked example and practice problems appeared in each lesson. Next, we coded the detailed sequential activities presented in the textbook lesson and the enacted teaching.

Representations. For each textbook lesson and enacted teaching, we coded the types of representations involved in the worked example and practice problems, respectively (e.g., word problem, circle, stick, tape diagram, number line diagram, table, flow chat, and number sentence). In addition, the representational sequence in each worked example (from concrete to abstract, from abstract to concrete) were identified.

Deep questions. We first collected all the questions during the worked example discussion in the textbook and the enacted lesson. Next, we removed all questions that had yes-or-no or one-word answers (e.g., “Can you do it?”, ”Right or wrong?”). The rest of the questions were classified into three categories based on the purpose of questions. We then counted for the frequency for each type of question. Given that one type of questions appeared closest to the deep questions as defined by the literature (Pashler et al., 2007), we further analyzed them and identified two subtypes.

After the coding processes for worked examples, representations, and deep questions were completed, we compared the codes for both textbook lessons and the enacted teaching to identify the similarities and differences. Changes that occurred to the transition process from the textbook and the enacted lessons were recorded. Rationales behind the changes were reflected and recorded by the first author who was the participating teacher. Furthermore, typical textbook presentations and screen shorts of enacted teaching were noted so as to illustrate typical changes during the the transition process.

Results

Worked Examples: Transition from the Textbook to Enacted Lessons

Table 2 summarizes the use of worked examples in terms of content, space, frequency, and sequence in the textbook and the enacted teaching across four lessons.

Table 2. *The Use of Worked Examples in The Textbook and Enacted Lessons*

		Lesson 1	Lesson 2	Lesson 3	Lesson 4
Context	Textbook	String Beads	Make flowers	Take bus	Borrow books
	Enacted teacing	String Beads	Guess numbers	Take bus	Borrow books
Space	Textbook	40%	40%	40%	50%
	Enacted teaching	33%	29%	35%	33%
Frequency	Textbook	1	1(2)	1	1
	Enacted teaching	1	2	1	1
Sequence	Textbook	Example-practice	Example-practice	Example-practice	Example-practice
	Enacted teaching	Example-practice	Example-practice	Example-practice	Example-practice

Context. As indicated by Table 2, in three of the four lessons, the teacher utilized the same worked example contexts, string beads, take bus, borrow books, as given (see Figure 1). This is basically consistent with prior findings that Chinese teachers use textbooks with fidelity (Ding et al., 2013). However, in Lesson 2, the teacher used a game to replace the textbook example. The textbook example, make flowers, is a comparison problem either finding the large or the small quantity (see Figure 1a, 2a). Instead of using this textbook example, the teacher conducted a game named “Guess my number” before class, which targeted the same teaching objectives. In this game, students were first told that the teacher’s favorite number was 45. They were then asked to write a statement that illustrates there relationship between the teacher’s and their favorite number. Typical statements (e.g, My favorite number is 3 more than 45; My favorite number is 35 less than 45) were selected and presented by the teacher in class so students could find out what a student's favorite number was (see Figure 1 b). According to the teacher, the rationale of this game was to better engage students because such a game was familiar to students who had always demonstrated great interests to solve mystery problems about their teachers.

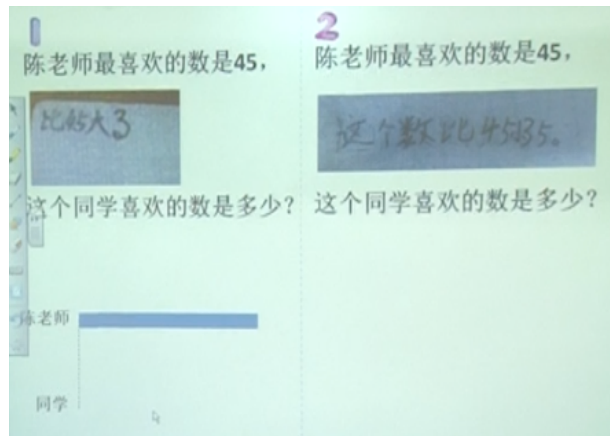
(a) Textbook: Make flowers	(b) Enacted teaching: Guess my number
(A pictorial story context, see Figure 1a) 	

Figure 2. A change of worked example context in Lesson 2.

Space. With regard to space, Table 2 indicates that the space of the worked examples in the textbook weighs more than in the enacted lessons. The worked example covers about 40% of space in each textbook lesson; yet, the portion in the enacted lesson ranged from 29% to 35% Why did the teacher spend less time on a worked example than what was suggested by the textbook? As mentioned in *Method* section, in three of the four lessons, students were asked to conduct a pre-learning of the worked example as part of their homework. This pre-learning task was the same as the worked example in the textbook. For example, in Lesson 1, students were asked to draw pictures to make the two given quantities (12 beads and 8 beads) equivalent. After the students submitted the pre-learning work next morning, the teacher examined each student work and selected the typical ones for class discussion. The teacher explained that her purpose of designing a pre-learning task was based on a consideration of students' different learning needs (e.g., length of time) and got students started thinking about the worked example in advance, which may have enabled less class time devoted to the discussion of the worked examples.

Frequency. Even though less time was spent on the worked examples in the enacted lessons, it seems that the substance of class discussion has not been decreased. As indicated by the "frequency" of worked examples in Table 2, the teacher discussed the same amount of worked example as the textbook but often with greater depth and richness of class discussion. For instance, in Lesson 2, the worked example in the textbook and the enacted lesson both contained two sub-problems (find the large or small quantity, see Figure 2). However, the enacted lesson involved a more abstract but powerful representation, the tape diagram (see Figure 1, elaborated upon in next section). In addition, in Lesson 1, the worked example's purpose was to make two quantities (8 beads and 12 beads) equivalent. The textbook's worked example presents three solutions: adding 4 to the small quantity; removing 4 from the large quantity, and moving 2 from the large to the small quantity (see Figure 3a). The enacted lessons not only discussed the same three solutions generated by students (see Figure 3b) but also a new method, that was not covered by the textbook. In this new method, a student suggested adding 7 to the small quantity and 3 to the large quantity, which can also equalize 8 and 12 beads (see Figure 3c). Most interestingly, one student used a table to sort out all kinds of possible ways (e.g., adding to both, removing from both, or adding to one while removing from the other, see Figure 3d), which has enabled the class to work on pattern-seeking, which could lead toward a higher-level of class discussion.

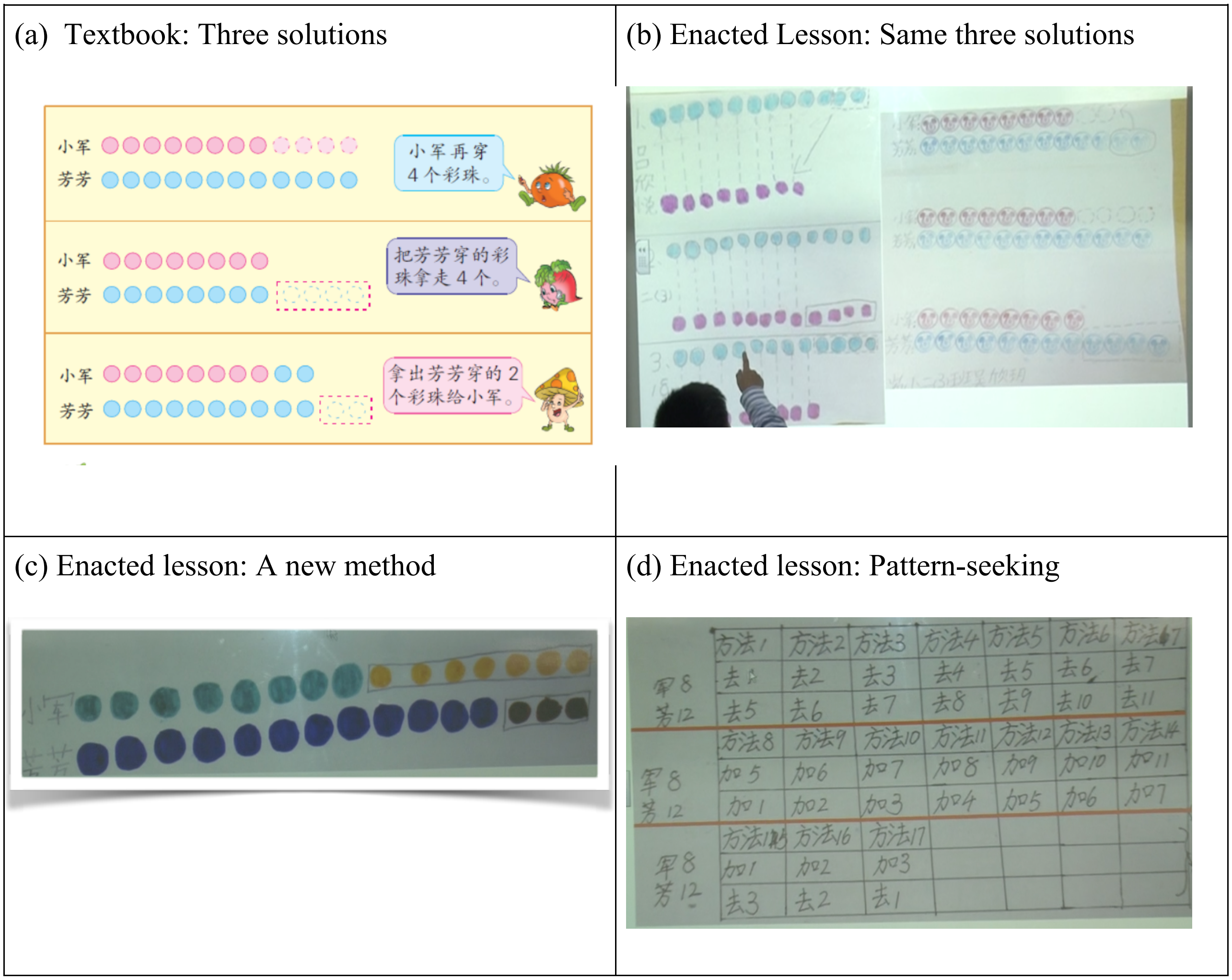


Figure 3. Types of solutions in the textbook and the enacted teaching in Lesson 2.

Sequence. Table 2 shows that the sequence of worked examples and practice problems were consistent between the textbook and the enacted lessons. All of the lessons only included one worked example followed by the practice problems. As such, within one 40 minute lesson, Chinese textbooks and teachers didn't interweave worked examples and practices problems, which was in sharp contrast with the research assertion (Pashler et al., 2007). A further inspection of the worked example in the textbook and enacted lesson shows different foci. In the textbook, the teaching sequence was *Guide-Solve-Discuss (GSD)*, suggesting a teacher first guide students to read and understand the known and unknown in the worked example problem.

Next, the teacher should ask students to solve the problem or sub-problem by themselves. Finally, the teacher will guide students to discuss their solutions or revisit the problem solving process (see Table 3). In the enacted teaching, however, pre-learning was widely used resulting in the first activity of student report, which occurred at the very beginning of the lesson. In addition, comparison was also frequently used in all lessons, which was not explicitly suggested by the textbook (see Table 3). As such, even though the textbook and the enacted lessons contained similar overall sequences (worked example to practice problems), the detailed teaching activities within the worked example were different. Consequently, the teacher guided reading suggested by the textbook was replaced by students' self pre-learning before class. Similarly, the teacher guided problem solving and revisiting process, suggested by the textbook, was taken place by student self-report of multiple solutions and comparisons among various solutions. Due to these changes, it appears that students in the enacted lessons had more opportunities to explain and demonstrate their individualized and diverse thinking.

Table 3: Compare the Teaching Sequence of the Textbook and Enacted Lesson

	In the textbook	In the enacted lesson
Lesson 1	• Identity known and unknown • Solve the problem • Revist the problem solving process	• Pre-learning • Report the PL results • Compare and discuss • Application
Lesson 2	• Identity known and unknown • Solve the problem	• Pre-learning • Solve the problem • Compare
Lesson 3	• Identity known and unknown • Solve the problem • Revist the problem solving process: Check and discuss	• Pre-learning • Report the PL results • Discuss • Conclusion and checking

Lesson 4	• Identity known and unknown • Solve the problem • Revist the problem solving process: Check and discuss	• Solve the problem • Report • Revisit the process • Compare • Check and Discuss

Representations: Transition from the Textbook to Enacted Lessons

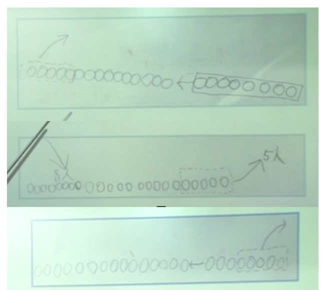
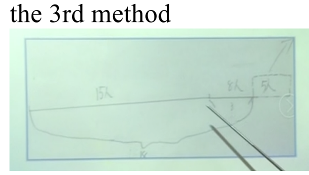
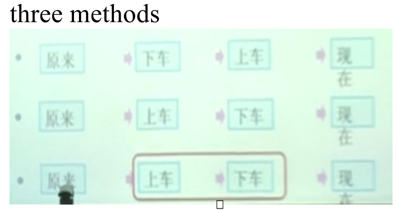
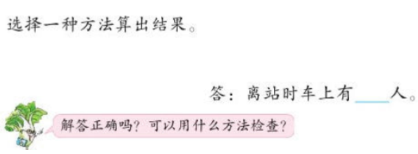
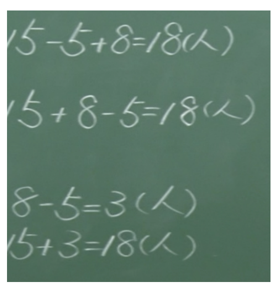
The use of representations in the textbook and enacted teaching is summarized in Table 4. In particular, the type of representations used in worked examples were listed, which also indicates the representational sequence involved in each worked example. Addition representations used in practice problems but not the worked examples were also listed.

Table 4. *The Use of Representations in The Textbooks and Enacted Lessons*

		Textbook	Enacted lesson
Lesson 1	Worked example	Word problem context Circle	Word problem context Circle **Table** **Number sentence**
	Practices (additional)	Stick Tape diagram	Stick Tape diagram
Lesson 2	Worked example	Word problem **Circle** Number sentence	Word problem **Tape diagram** Number sentence
	Practices (additional)	Tape diagram	Number line diagram
Lesson 3	Worked example	Word problem Number sentence	Word problem **Circle** **Number line diagram** **Flow chart** Number sentence
	Practices (additional)	Flow chart Tape diagram	Tape diagram
Lesson 4	Worked example	Word problem Number sentence	Word problem **Tape diagram** Number sentence
	Practices (additional)		Other types of schema diagrams

Representational Sequence. As indicated in Table 4, in both the textbook lesson and enacted teaching, the representational sequence in worked examples generally begins with concrete word problem situations and ends up with abstract number sentences. However, when comparing the detailed representational sequence, it was found that the enacted lesson always contained new types of representations that did not appear in the textbooks so as to sufficiently unpack a worked example. For instance, in lesson 3, both the textbook and the enacted lessons first presented the take bus problem. However, the processes of reasoning upon the story problem were quite different (see Table 5). The textbook guided students to think in three ways (adding the ones getting on the bus first; subtracting the ones getting off the bus first; or adding the balance). In contrast, the enacted lessons involved circle diagram, number line diagram, and flow chart representations to assist in students' thinking process. In fact, the circle representation and number line diagram were generated by students through pre-learning, which covered all three ways of thinking suggested by the textbook. In addition, the teacher also presented flow-charts, which was initially arranged by the textbook in the practice problem section; yet, the teacher moved it early so as to better teach the worked example. Consequently, when the textbook requested for one solution (number sentence), the enacted lesson came up with three different numerical solutions each matched the three ways of thinking.

Table 5. *Representation Sequence in the Worked Example in Lesson 3*

	The Textbook	The Enacted Lesson	
Pesent	Present the story problem (numbers were 34, 15, 18)  	Present the story problem (numbers were changed to 15, 5, and 8) 	
Guide	Guide students to think in three ways and suggest verbalizing them  	Student circle representation that show three methods 	Student's number line that show the 3rd method  Teacher's flow chart that shows three methods 
Solve	Ask students to select one method to find the answer:  	Generate number sentences in alignment with three methods discussed to find the answer:  	

Schematic Diagrams. One may also noticed that among the different types of representations, schematic diagram (e.g., type diagram, number line diagram) appeared frequently in both the textbook and the enacted lessons (see the bold in Table 4; also see examples in Figures 2 and 4). In the textbook, three out of four lessons used tape diagrams; in the enacted lessons, all four lessons used tape diagrams and (or) number line diagrams. In addition, tape diagrams only appeared in practice problems in the textbook; yet, this type of schematic diagram was moved to earlier in the lesson when teaching worked examples. In fact, the teacher in Lesson 4 added the tape diagram when teaching worked examples which was not included in the textbook presentation (see Figure 4).

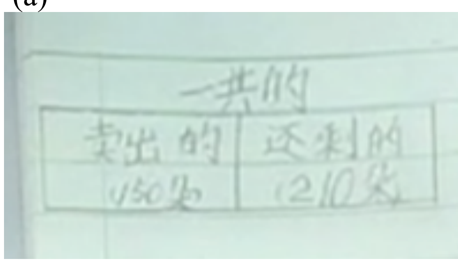
Why did the teacher heavily use schematic representations such as a tape diagram? According to the teacher's reflection, students' pre-learning work indicated their readiness for reasoning upon relatively more abstract representations. This was why she replaced the circle representation with the tape diagram in Lesson 2. In addition, the teacher viewed schematic diagrams as powerful tools to develop students' understanding of quantitative relationships. In the enacted teaching of Lesson 4, she used the tape diagram to stress the quantative relationship, "the total - the borrowed = the remained." Meanwhile, she expected students to understand the inverse quantitative relationship using the same tape diagram, "the borrowed + the remained = the total." Through such inverse reasoning process, she expected students to understand inverse relation - why one can use addition to check subtraction - at a structural level. This was similar to Ding's (2016) textbook exmaination.

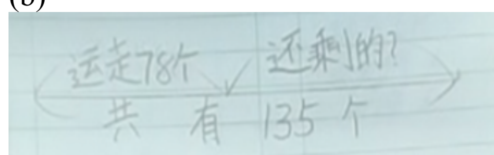
It appears that this teacher's consistent emphasis on schematic diagrams in enacted lessons facilitated students' structural understanding. As seen from her student in-class work on practice problems in Lesson 4, students were able to spontaneously draw schematics diagrams to represent structural relationships (see Figure 4). For instance, the exemplary student drawings presented in Figure 4 clearly resemble teacher's schematic diagrams with variations (e.g., a number line, a circle shape). All of these representations showed the part-whole relationships.

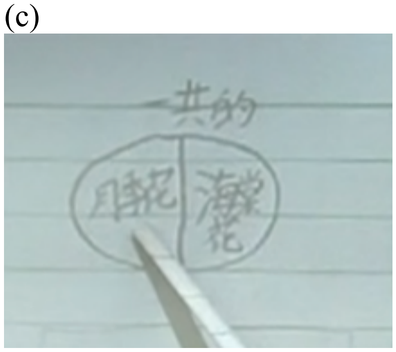
While the first three drawings (examples a-c) involved specific situations, example (d) illustrated the inverse relation among 91, 61, and 28 at an abstract level.

<table>
<tr>
<td>Worked Example</td>
<td>Textbook

</td>
<td>Enacted Lesson

</td>
</tr>
<tr>
<td>Practice Problems</td>
<td colspan="2">Enacted lesson: Students' spontaneous drawing of various schematic diagrams
(a)


(b)


(c)


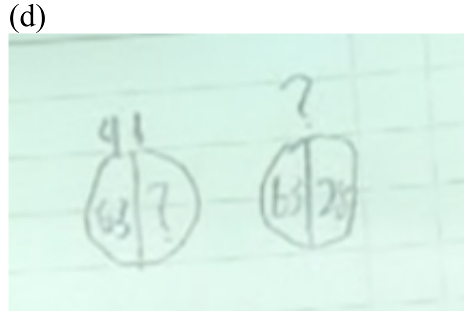
(d)

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Figure 4. Schematic diagrams in lesson 4.

Deep Questions: Transition from the Textbook to Enacted Lessons

As mentioned in Method section, questions that call for substantial answers (not the yes-or-no or single word answers) were coded for worked examples in both the textbook and enacted lessons. Based on the purpose of these questions – what kinds of students' responses they aimed to elicit - they fell into three categories: brainstorming, connection, and reflection. Example brainstorming questions were, “What information do you see from this picture? What methods may be used to solve this problem?” Example connection questions were, “How are they the same?” “How may we solve this problem using these circles?” and “Why may we use addition to check subtraction?” Finally, example reflection questions were, “What did you learn from this problem solving process? Why did we use subtraction?” Based on the definition of deep questions (Craig et al., 2006), connection questions had a clear goal for eliciting students’ understanding of relationships and were accurate depictions of deep questions. With regard to brainstorming and reflection questions, they were open-ended and could elicit students’ detailed responses. As such, it would be hard to anticipate what kinds of student responses they may provide. Table 6 summarizes the frequency of each type of question. A comparison of the number of questions in the textbook and enacted lesson indicates that the teacher asked a similar amount of brainstorming and reflection questions; yet, the enacted lessons contained many more connection questions than the textbook. This is most apparent in Lessons 1, 2, and 3 (see Table 6).

Table 6. *Types of Questions in the Worked Examples across Lessons*

	Lesson 1		Lesson 2		Lesson 3		Lesson 4	
	Textbook	Enacted Lesson	Textbook	Enacted Lesson	Textbook	Enacted Lesson	Textbook	Enacted Lesson
Brainstorming	1	2	2	4	2	1	2	1
Connections	0	3	1	5	1	6	1	2
Reflections	1	2	1	1	1	1	1	1
Total	**2**	**7**	**4**	**10**	**4**	**8**	**4**	**4**

Why did the teacher ask many connection questions? What was the nature of these questions? A closer inspection of all connection questions indicates two major subcategories: comparison questions and specific questions.

Comparison questions. In the enacted lessons, the teacher constantly asked students to do comparisons among different solutions. Example questions are "Compare these solutions. What are the similarities? What are the differences?" Excerpt 1 illustrates discussions of the pre-learning product about the stringing beads problem in Lesson 1, which illustrates the typical comparison questions and the corresponding student responses. In this excerpt, the teacher selected and shared with the class two similar pre-learning products, each illustrating three methods that were similar to the textbook solutions (see Figure 3b).

Excerpt 1

T: Which method in Student A's work is the same as which method in student B's work?

S1: The first methods in both student work are the same, because both moved some part.

T: What do you mean by "both moved some part?"

(At this time, another student pointed out that there were another pair of similar methods across both students' work)

T: Let's go back to what we just discussed. Why were they the same?

S2: Fangfang's beads in both students' first method are moved.

S3: They both move the more to the less.

In this excerpt, the teacher asked a comparison question expecting students to make connections of the detailed methods used by both students. Students' initial responses indicated partial understanding without clarity (both moved). With the teacher's continuing prompt, the students eventually noticed the essential similarity between these two pictures, that is, both moved the more to the less. As such, through comparison questions, the teacher were able to guide students to identify essential concept undergirding two individual cases.

In other occasions, comparison questions were used to make a connection between the different types or levels of representations. For instance, in lesson 3 (see table 5), when the number line diagram was shown, the teacher ask “what is the meaning of this part of the number line? Which circle diagram has the same meaning as this number line diagram?” When the flow chat were shown, the teacher asked “which one of these three flow chats is the same as which one of these circle diagrams?” Finally, when the number sentences were written, the teacher asked “what is meaning of this number? Can you point out which part of the diagram represent this number or quantity?” With ongoing comparison questions, students were engaged in co-constructing and connection-making between various representations.

Specific questions. The teacher in the enacted lessons also constantly asked specific questions to orient student thinking toward deep structures. Excerpt 2 illustrates a typical teaching episode during which the teacher asked specific questions. In this excerpt, the teacher showed a tape diagram to present 45 which was her favorite number. She then asked the students how they could draw another tape to represent the student’s favorite numbers (see Figure 2b), which included two subproblems: find out 3 more than 45 (Figure 2b, left) and find out 35 less than 45 (Figure 2b, right).

T: I will use the computer to draw this. If you say stop, I will stop drawing.

(The teacher moved the mouse on the screen without actually drawing it out)

T: (When the mouse reaches 45) Can I stop? (S: No) It has reached 45! Why can’t I stop?

(The teacher then asked students to draw the second tape for both problems. After students finished drawing, she guided the class discussion on two selected works).

T: (Subproblem 1) Why did you draw your second type longer?

S: Because the student's favorite number is 3 more than 45. So, this tape should be a little bit longer.

T: Why only a little bit?

S: Because 3 is a small number.

T: (Subproblem 2). Why did you draw you second tape shorter than the first one?

S: Because it is less than 45.

S: The number is far less than 45, so the tape will be far shorter than 45.

In Excerpt 2, the teacher showed the first tape and engaged students to draw the second tape based on the relationships involved. To ensure students' understanding of the involved relationships, she asked a series of questions drawing her students' attention to the length of the tape. Questions such as "It has reached 45! Why can't I stop?" and "Why only a little bit?" seemed to be trivial questions. Yet, when the purpose of these questions targeted the underlying structural relationships, they seemed to be quite deep.

Discussion

This study reports how an expert Chinese teacher transitions her textbook into actual classroom teaching in terms of the use of worked examples, representations, and deep questions. Quality textbooks are important resources but won't make an impact on student learning unless teachers transition the intended knowledge into enacted classrooms (Ramillard, 2005). However, the process of transition the intended knowledge from the static textbooks to complex classrooms is not simple. Our findings on a Chinese expert teacher's detailed transition process in terms of unpacking worked examples through representation and deep questions, therefore, enrich our understanding on how textbook materials may be better used to support student learning. More importantly, this study employed an action research method, which has allowed the teacher research to reflect upon whys she transitioned the worked examples, representations, and deep questions from the textbook into her classroom teaching in certain ways.

Worked examples have been studied for a long time due to its effect in supporting student learning (Renkl, Atkinson, & Grobe, 2004; Sweller & Cooper, 1985). In particular, it is suggested that interweaving worked examples and practice problems can reach a better effect. However, both the Chinese textbook and the enacted lessons do not show this pattern. Rather, Chinese math lessons usually target only at one worked example, which covers a significant space of a lesson (30%-40%). Given that a Chinese math lesson only takes 40 minutes, it makes sense that there is no "interweaving" pattern observed. To Chinese teachers, it appears that the most important teaching move is to discuss one worked example in depth so that students can grasp the underlying concept (Ding et al., 2013). This observation is in sharp contrast with many US teachers' beliefs and classroom behaviors. Many US teachers believe that the more examples the better. Regardless of whether these many examples were interweaved or not with practice problems, many US teachers spend only a few minutes on each example and move quickly to the next one (Ding & Carlson, 2013; Ding, Hassler, Li, & Chen, 2016). With a short period of instruction on worked examples, it is hard to say whether students can be supported to develop a schema for solving new problems.

In this study, the Chinese teacher has used almost all of the worked examples provided by the textbook. This finding echoes prior report on Chinese teachers' fidelity of textbook uses (Ding et al., 2013; Ma, 1999). However, the Chinese teacher made a great effort on unpacking the textbook example in depth, which integrated her careful design on representation uses and questioning. As reported, the textbook tends to provide verbal suggestions on multiple solutions, the teacher prompted students to discuss multiple concrete (or semi-concrete) representations. In particular, schematic diagrams have been widely used to teach every worked example, even though the textbook only included this diagram in practice problem sections. Given that the tape diagram as a type of semi-concrete representation can effectively show the problem structures, it is important to help students understand this representation. Ding and Li (2014) reported that Chinese textbooks provides various opportunities (e.g., ask students to draw the second tape and asking a questions about the tape) to engage student learning. In this study, we found that this Chinese teacher integrated all of these strategies. Indeed, she went further by asking a series of specific questions on the process of drawing a diagram, which certainly boosts students' understanding of this powerful but non-parent representation. Since schematic representations such as tape diagrams are emphasized by the common core and adopted by various new textbooks (Ding et al., 2016), our findings on how a Chinese expert teacher skillfully use schematic diagrams to unpack a worked example have practical importance. For instance, Ding et al. (2016) reported that US expert teachers need some support for better use of schematic diagrams in classrooms because these teachers tend to fully show the diagram or just use it to find answers.

To unpack the worked example, the Chinese teacher also frequently used deep questions. In particular, she helped students make various connections through asking comparison questions or specific questions both of which intended to elicit student explanation of structural relationships. These comparison questions focus not only mathematics content but also the underlying structures. As reported in Lesson 2, when the students found that both strings of beads were moved, it was just related to the surface features because the more important feature is related to "how" the beads were revmoved. Therefore, when students found that both strings of beads were moved from the less to the more, it was a structual finding which indicated a method to be used for solving other problems. This adds to Ding's (2016) finding that Chinese textbooks appear to stress structural relationships. Our findings further show how a Chinese teacher made these structural relationships visible and learnable for students. This finding also echos Ding et al., (2013) finding about Chinese teachers' stduy textbooks to maximize learning. In fact, the Chinese textbook lessons in this study appeared to ask opened ended questions that may elicit unpredictable student responses. The Chinese teacher went beyond what the textbook suggested and added detailed connection-making questions, which may have enabled successful transition from the textbook to the enacted lessons. It is also interesting to note that the teacher asked different types of comparison questions depending on different conditions. For instance, when representations were at the same level (e.g., circle and diagram), the teacher asked questions about the difference or similarities between these representations. In contrast, when the representations were at different levels (e.g., diagram and number sentence), the teacher tended to ask students to identify the relationship between them. Given that questioning is a critical but difficult pedagogy (Ding et al., 2007), our findings may shed light on how teachers may better use questioning to unpack a worked example.

Noticeably, the teacher has consistently applied pre-learning as part of worked example discussion. According to the teacher, this resolves the dilemma of students' different learning needs. Products generated from the pre-learning also gives the teacher assessment knowledge about students' existing concepts. Given that diversity and assessment are critical topics for US classrooms, the strategy of integrating pre-learning and worked example studies may be a path worthy of attention. In fact, worked examples in the literature are sometimes criticized due to the popular methods of telling and showing, which turns learning into a passive process. However, when pre-learning is integrated into the worked example study, students were able to generate various presentations and solutions, offering the follow-up class discussion rich space for deep learning of the intended concepts.

This study has limitations. We are fully cognizant that our findings are only based on one teacher's instructional findings of which may not be generalizable. We also acknowledge that our findings from Chinese classrooms may have obstacles to be applied to other countries due to possible cultural barriers. Nevertheless, our findings shed light on both practice and research. For instance, the IES recommendations serve as a practice guide for organizing instruction to improve learning (Pashler et al., 2007). However, most US teachers' lack the ability to utilize them in daily work (Ding & Carlson, 2013). The instructional insights of a Chinese expert teacher in alignment with the IES recommendations may shed light on classroom practices especially in areas of teaching fundamental mathematical ideas. In addition, the Chinese expert teachers' experiences may enrich cognitive research findings on the use of worked examples, representations, and deep questions, which may suggest new directions for future study.

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