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Elementary Textbooks to Classroom Teaching: A Situation Model Perspective

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Introduction

As the field of mathematics education continues to place an increasing emphasis on students' comprehension of mathematical concepts, arguably the fundamental goal of mathematics instruction remains learning for understanding (Hiebert & Carpenter, 1992; Hiebert et al., 1997; Stylianides & Stylianides, 2007; Silver, Mesa, Morris, Star & Benken, 2009). Yet, findings from recent international tests (TIMSS, 2013; PISA, 2013) revealed that U.S. students continue to exhibit a lack of mathematical understanding, especially when faced with cognitively high-demanding situations (PISA, 2013). Johnson-Laird (1983) claimed that when faced with these unknown quantitative situation, students have a better chance of increasing comprehension when they create coherent mental models through forming connections among various relationships. Indeed, current mathematics education research does indicate that comprehension improves when conceptually relevant connections to prior knowledge are formed (Sidney & Alibali, 2015). This is reflected by The Common Core State Standards (Common Core State Standards Initiatives [CCSSI], 2010) call for K-12 mathematics curricula to include "more coverage of higher levels of cognitive demand" (Polikoff, 2015, p. 1194). In contrast to previous standard-based curriculums (Porter et al., 2011), the CCSS suggest that students must be able to form better connected and conceptually grounded mathematical ideas in order to facilitate transfer of learning. The need to promote connection-making has therefore become increasingly more relevant.

In most U.S. mathematics classrooms however, "instructional tasks tend to emphasize low-level rather than high-level cognitive processes" (Silver et al., 2009, p. 503) and both instruction and curriculum materials generally lack connections within and across topics (Ding, in press; Schmidt, Wang, & McKnight, 2005). Specific to Common Core aligned curriculum, Polikoff (2015) found that although the elementary mathematics textbooks "cover most all the topics in the standards, they fail to reach the advanced levels of cognitive demand called for by the standards" (p. 1188). As a result, current learning environments (e.g., classroom teaching and textbooks) may not be promoting connection-making opportunities, which in turn may be prohibiting students from comprehending fundamental mathematical ideas. Research that indicates a U.S. preference for procedural focused learning (Baroody, 1999; DeSmedt et al., 2010; Torbeyns et al., 2009) with few references to tasks that assess targeted concepts (Crooks & Alibali, 2014) also suggests a lack of connection-making opportunities.

Although connection-making is a common theme across most current educational research on mathematical comprehension (Anthony & Walshaw, 2009; Barmby et. al, 2009; Blum, Galbraith, Henn & Niss, 2007; Businskas, 2008; Sidney & Alibali, 2015), few have explored a cognitive construct for which mathematical connection-making may be facilitated. As a result, little is understood about how curriculum materials and classroom instruction have helped to facilitate connection-making within current CCSS learning environments. According to Chingos and Whitehurst (2012), "the Common Core standards will only have a chance of raising student achievement if they are implemented with high-quality materials" (p. 1) and thus recent research has begun to explore methods for measuring the quality of Common Core curriculum materials. One of the first studies of this nature reveled that within four Common Core aligned fourth grade textbooks, there exists a "good deal of misalignment at the cognitive-demand level in textbooks—all of them systematically fail to cover the more conceptual skills called for by the standards" (Polikoff, 2015, p. 1203). Although further research across more grade levels and additional Common Core aligned textbooks is needed, it has also become "essential to move from the textbook into the classroom to understand how curriculum materials influence teachers' instructional responses to the standards" (p. 1208).

The purpose of this case study (Clark, 2005) is therefore to examine how one expert U.S. elementary teacher uses curriculum materials and classroom instruction in order to facilitate connection-making. From a situation model perspective, how the teacher transitions curriculum connections into classroom instruction will be analyzed in order to identify learning opportunities that may maximize students' mathematical comprehension surrounding multiplicative inverses. The findings from this study are expected to contribute to improving curriculum design and enhancing classroom teaching of elementary inverse relations. Although this study only explores comprehension of multiplicative inverse relations, methodologically, the coding framework developed for this study may be useful for future studies surrounding the comprehension of other fundamental mathematical concepts.

Literature Review

The Case: Multiplicative Inverse Relations

To investigate how students develop mathematical comprehension, this study focus on the case of multiplicative inverse relations. Inverse relations is a fundamental concept that transcends across various mathematical contexts and therefore the ability to use and reason with inverse operations serves as a fundamental building block for many quantitative concepts (Baroody, Torbeyns, & Verschaffel, 2009; Carpenter, Franke, & Levi, 2003; Nunes, Bryant, & Watson, 2009). The first formal teaching of inverse relationships occurs at the elementary level when exploring the connections between addition/subtraction (additive inverses) and multiplication/division (multiplicative inverses). These connections however appear to be insufficiently formed, since research reveals elementary school children generally lack a formal understanding of inverse relations (Baroody, Ginsburg & Waxman, 1983; De Smedt, Torbeyns, Stassens, Ghesquiere, & Verschaffel, 2010; Resnick, 1983). This presents a problem far beyond elementary classroom doors, since longitudinal empirical evidence (Baroody, 1987; Stern, 2005; Vergnaud, 1988) suggest that an elementary student's comprehension of inverse relations significantly predicts both algebraic and overall mathematical achievement in later years. Unfortunately, most prior research surrounding inverse relations has not focused on instructional practices that can be used to enhance these connections. Rather, research has primarily focused on *if* and *when* children show evidence of understanding inverse relations, not *how* and *why* this understanding occurs.

Although inverse operations have been identified as a critical piece of mathematical competency across all elementary grades levels (Common Core State Standards Initiatives [CCSSI], 2010), the majority of prior research on inverse relations has only focused on additive inverses (Cowan & Renton, 1996; Squire, Davies & Bryant, 2004) and thus there exists a large gap in literature involving how connection-making facilitates the comprehension of multiplicative inverses. The limited research that is available (Robinson & Dubé, 2009b; Thompson, 1994; Vergnaud, 1988) does however suggest that like the well-documented problems children have with comprehending additive inverses (Nunes et al., 2009a; Stern, 1992; Bryant, Christie, Rendu, 1999), multiplicative inverses are also a struggle for many elementary aged students (Robinson & Dubé, 2009b; Thompson, 1994; Vergnaud, 1988). For the purpose of this study multiplicative inverses refers to the complement principle, if $M \times N = P$, then $P \div M = N$. With regards to this principle, both Grossi (1985) (as cited in Vergnaud, 1988) and Thompson (1994) found that elementary students were unable to recognize the appropriateness of using either equation when solving application problems. Perhaps this indicates that the students had not yet developed a well-connected situation model for inverse relations.

Situation Model Perspective

In contrast to the act of simply doing mathematics, comprehension involves the ability to make connections between different aspects of mathematics in order to construct a coherent mental model for the purpose of transfer into other quantitative situations (Langer, 1984; Shepherd, Selden & Selden, 2012). As defined by Bruner (1990), a mental model is an internal representation of the thought process that someone has when attempting to create meaning from externally encountered experiences (Bruner, 1990). It is believed that this meaning occurs as a result of the human mind creating small-scale models of reality that are used to make connections between prior knowledge and current stimuli (Long, Seely, Oppy & Golding, 1996) in order to evaluate testable inferences about current and future situations (Johnson-Laird, 1983).

An important application of being able to construct meaning in an externally located situation is when a learner is trying to comprehend written language. Empirical evidence that the ability to comprehend is highly dependent on the ability to make connections to prior knowledge was arguably first noted by Barlett (1932), when he compared a text's surface representation to a reader's mental representation and found that "readers' memories for textual information were systematically distorted to fit their own factual and cultural knowledge" (Lorch & van den Broek, 1997, p. 214). Thus, cognitive and educational psychologists have extensively used the domain of reading comprehension to explore connection-making. Walter Kintsch's (1988) Construction-Integration theory of reading comprehension is among the many noteworthy knowledge acquisition theories that have been created as a result of advancements in linguistic research. This text processing theory is based on an inferential process of evaluating propositions in relationship to three types of mental representations that a learner forms while reading text: a surface component, a textbase representation, and a situation model (Kintsch, 1986).

According to Kintsch (1986; 1988), the process of forming these mental representations begins with the reader creating an initial list of propositions based solely on the words that they are reading. This is known as the surface component, or a verbatim representation of the text in which words and phrases themselves are encoded into memory. The second component, a textbase, represents the semantic structure of the text in that it captures the linguistic relationships among propositions represented in the text. As the textbase is created, entire sentences are read and the reader begins to make meaning of the text. Because the first two components only involve direct translation of what is explicitly written, limited connections to prior knowledge are needed and learners are therefore not required to make inferences. If however a reader draws on prior knowledge to create a more complete mental representation that can be used to make inferences between the situation the text represents and other contexts to which that text may be applied, then the final situation model component has been created. A situation model is therefore deeply connected to prior knowledge in such a way that allows for a learner to use new content knowledge in “novel environments and for unanticipated problem solving tasks” (McNamara et al., 1996, p.4).

Multiple studies (e.g., Kintsch, 1994; Osterholm, 2006; Weaver, Bryant & Burns, 1995) have shown the important role that situation models have in altering the definition of learning from not what is simply to be remembered, but rather what conclusions can be drawn based on an inference-making process. Kintch (1986) noted that in both a first grade and a college setting, once a situation model was formed for a mathematics based word problem, comprehension increased. This occurred because learners tended to make connections to prior knowledge and could reconstruct the problem using their situation model, as opposed to simply recalling the problem by use of the textbase component. The mental representations on which recall is based differs from the representation on which inference it based, and thus connection-making is especially important when learning a new mathematical concept (Sidney & Alibali, 2015).

How to Facilitate a Situation Model

To create an effective situation model, a learner must implement a deep level of inference making that demands connecting implicit and explicit information to one's prior knowledge (Zwaan & Madden, 2004). Although the amount and the ability to activate conceptually relevant prior knowledge has been shown to be a significant and reliable predictor of comprehension (Langer, 1984; McNamara et al., 1996; Pearson, Hansen & Gordon, 1979), novice learners often struggle to make connections to relevant prior knowledge (Novick, 1988). Therefore, in order to best facilitate understanding for learners with little prior knowledge, curriculum and instruction should be as coherent and explicit as possible (Kintsch, 1994; Reed, Dempster & Ettinger, 1985). In addition, analyzing experimental variables within learning opportunities that affect the ability for learners to make connections and draw inferences is essential in the pursuit of helping student enhance their ability to create situation models. According to *the Institute of Education Sciences (IES)*, these variables include the instructional tasks, types of representations, and the deep questions used during instruction (Pashler et al., 2007).

Instructional Tasks. A critical component in organizing instruction to improve student learning is to establish connections between instructional tasks and underlying principles (Pashler et al., 2007). Examples of instructional tasks include review tasks, instructional examples and practice problems. Just using a greater variability of tasks however, does not guarantee transfer benefits (Pas & Van Merrienboer, 1994; Atkinson, Derry, Renkl, Wortham, 2000). Instead, according to a situation model perspective, instruction should be designed to form connections within and between tasks in order to increase mathematical comprehension. Indeed, various instructional methods have been designed to develop these connections during mathematics instruction. They include interleaving instructional examples with practice problems (Pashler et al., 2007), using contrasting alternative solution methods (Rittle-Johnson & Star, 2007) and using both correct and incorrect examples during instruction (Booth et al., 2013). In addition, because the use of worked examples has been shown to increase initial comprehension within cognitively high demanding tasks (van Merriënboer, 1997; Renkl, 1997) they too have been extensively researched in mathematics education.

A worked example is "a step-by-step demonstration of how to perform a task or how to solve a problem" (Clark, Nguyen & Sweller, 2006, p. 190). The use of worked examples in mathematics instruction is supported by the belief that they serve as an expert's mental model and thus help to increase comprehension (Chi & VanLehn, 2012; Sweller, & Cooper, 1985). From the perspective of a situation model, the use of worked examples helps students develop a schema by facilitating connection-making between prior knowledge in order to increase the likelihood of transfer (Kirschner et al., 2006; Paas, Renkl & Sweller, 2003). Therefore, corresponding practice problems should have connections to the worked examples so as to practice the learned knowledge. Because worked examples and practice problems should be built on student's prior understanding, review tasks used during instruction should also provide opportunities to form connections to relevant prior knowledge. Unfortunately the amount of time allocated to review in U.S. mathematics classrooms is limited, maybe because an emphasis seems to be placed on allowing students enough time to work on practice problems (Jones, 2012; Stigler & Hiebert, 1999). In addition to lower amounts of instruction time devoted to worked examples, Ding and Carlson (2013) found that U.S. teacher lesson plans spend little time unpacking worked examples. It therefore seems as if there may be many opportunities to enhance connection-making within the instructional tasks used during mathematics instruction.

Representations. To allow students a hands-on exploration of mathematics, concrete manipulatives (e.g., blocks, rods, tiles) and concrete representations (e.g., story problems) are often used in elementary school classrooms (Clements, 1999). Martin and Schwartz (2005) believe that by interacting with concrete manipulatives, students form stronger connections to their internal mental models, which helps to increase mathematical comprehension. This has been empirically supported by Harrison & Harrison (1986), who provided descriptions of successful learning activities that utilized concrete objects such as rulers and place value cards. Although literature suggests that concrete representations are useful during initial learning (Resnick & Omanson, 1987), they also often contain irrelevant information that may prohibit students from making deep connections to the underlying principles (Kaminiski, Sloutsky, & Heckler, 2008). For instance, several studies (e.g., Gentner, Ratterman, & Forbus, 1993; Goldstone & Sakamoto, 2003; Son, Smith & Goldstone, 2011) have shown that using only concrete materials hinders transfer to unknown situations. It therefore is commonly believed that concrete representations alone do not guarantee comprehension (McNeil & Jarvin, 2007), and thus should not be the only representations used to facilitate situation models.

Problem solving by paper and pencil, without the use of manipulatives or drawings, is a common example of abstract representations in mathematics. Since abstract representations are purely symbolic, students who reason at the abstract level appear to do so as a result of interacting with a situation model. From the perspective of a situation model, abstract representations therefore need to be an integral part of instruction because they are essential in the inference making process of many advanced mathematical tasks (Fyfe, McNeil & Borjas, 2015). Novice learners however often struggle to attain mathematical comprehension when only abstract representations are used during instruction (McNeil & Alibali, 2000; Rittle-Johnson & Alibali, 1999). This was perhaps most famously noted when Carraher, Carraher and Schliemann (1985) found that the ability for Brazilian children street vendors to solve basic computational mathematics problems was dependent on the context and concrete representations of the problems. Therefore, there exists a need to facilitate connection-making between concrete and abstract representations.

Pashler et al. (2007) suggests that by integrating both concrete and abstract representations into instruction, students are better able to make connections to prior knowledge. In fact, instruction involving various representations has repetitively been shown to increase comprehension (Ainsworth, Bibby & Wood, 2002; Goldstone & Sakamoto 2003; Richland, Zur & Holyoak, 2007). Specifically, using concrete representations for initial learning and over time replacing parts of these representations with abstract representations, has been suggested by both theorists (e.g., Bruner, 1966) and researchers (Fyfe, McNeil, Son & Goldstone, 2014; Gravemeijer, 2002; Lehrer & Schauble, 2002). Known as concreteness fading (Goldstone & Son, 2005), empirical evidence supports the notion that students' transfer ability increases when a combination of representations is used during instruction (McNeil & Fyfe, 2012). Since transfer has been linked to the coherence of a situation model, it is important to analyze both the type and the sequence of representations found in current learning opportunities.

Deep Questions. Classroom discourse, the use of language within social contexts (Gee, 2010), helps to facilitate the development of student conceptual understanding (Chin, 2007; Mortimer & Scott 2003; Franke et al., 2009). Costa (2001) and Swartz (2008) provide empirical evidence that students attain deeper comprehension when they are provided with opportunities to converse within instructional settings, which Greeno (1991) agrees may contribute positively to the development of mental models. These opportunities include verbal interactions with teachers, which often involves the act of asking and answering questions**.** Questioning student understanding during classroom instruction is a critical learning opportunity that shapes student learning (van den Oord & Van Rossem, 2002) through eliciting students' explanations of underlying principles (Craig, Sullins, Witherspoon, & Gholson, 2006).

In order to help students build connections and improve learning, the *IES* recommends that teachers need to help students learn how to ask and answer deep questions (Pashler et al., 2007). Defined as a question that elicits deep explanations, deep questions include questions that target "causal relationships" (p. 29) and that are structurally connected to underlying principles. These include questions such as "why, why-not, how, and what-if" (p. 29). The inferential nature of these questions force students to distance themselves from the present in order to think about past or future events (Sigel & Saunders, 1979) and thus have been shown to have a direct impact on the cognitive process (Chapin & Anderson, 2003; Chin, 2006; Morge 2005). From the perspective of a situation model, focused and deliberate deep questions (Rubin, 2009) help students to facilitate connection-making between and within mathematical principles. Unfortunately, few deep questions are being asked in today's classrooms (Khan & Inamullah, 2011; Wimer, Ridenour, Thomas & Place, 2001) which may partially be why U.S. students continue to exhibit a lack of mathematical understanding,

The reviewed literature clearly supports the notion that a critical component of comprehension is the inference process that occurs as a result of making connections to prior knowledge (Pearson et al., 1979). I argue that students are best supported in this process when they are presented with learning opportunities useful for connection-making. Those opportunities that appear to be the most contributing factor in the creation of a situation model include: (a) the presentation of instructional tasks (b) the types of representations and (c) the use of deep questions. In response to Linn's (2006) call for future empirical research to explicitly search for ways to facilitate children's connections to prior knowledge, the following research questions have emerged: (1) How does a reformed elementary CCSS curriculum facilitate connection-making for the learning of multiplicative inverses? (2) How does an expert elementary mathematics teacher facilitate connection-making during classroom instruction on multiplicative inverses? (3) How does an elementary expert teachers' classroom instruction relate to the way in which curriculum materials facilitates connection-making?

Method

This case study (Clark, 2005) seeks to understand current learning opportunities that elementary students are exposed to with regards to connection-making. The focused case is multiplicative inverses, a critical topic that lends itself to form numerous connections (Baroody, Torbeyns, & Verschaffel, 2009; Nunes, Bryant, & Watson, 2009). The focused environment is one third grade U.S. expert teachers' classroom and the analysis involves examining how this teacher transitions textbook connections into classroom instruction.

Participant

This study involves one elementary school expert teacher who is a participant in a five-year National Science Foundation (NSF) funded project on early algebra in elementary schools. She was selected for this case study because she received the highest Algebraic Knowledge for Teaching (AKT) score among all participants in year one of this project. She teaches third grade for a large high-needs urban school district in Pennsylvania, and there were $n = 27$ students in her class at the time of this study. She was selected from grade 3 because according to the Common Core State Standards (CCSSI) this is where multiplicative inverses is first taught. She is considered an expert teacher based on criteria used to select participants for the above mentioned NSF project. This expertise includes 17 years of teaching experience and she is also a Nationally Board Certified Teacher (NBCT).

Data Sources

This study analyzes curriculum materials and classroom instruction for the following four lessons on multiplicative inverses: (1) Solving Division Problems (2) Multiply or Divide? (3) Writing Story Problems (4) Missing Factors (see Table 1).

(INSERT TABLE 1 ABOUT HERE)

Curriculum Materials. *Investigations* in Number, Data and Space (Investigations, 2013), is the curriculum that is used by the teacher in this study. *Investigations* is a K-5 elementary mathematics curriculum that according to Barshay (2013), is one of the most widely used mathematics curriculums in U.S. elementary schools. *Investigations*, claims to be based on extensive classroom testing with a focus on allowing time for students to develop a strong mathematical conceptual skill set (Investigations, 2013). Four lessons involving multiplicative inverses (see Table 1) were selected from the curriculum's teaching guides (*Curriculum Units*).

Videotaped Lessons. The teacher in this study agreed to be videotaped while instructing each of the four lessons involving multiplicative inverses. All four lessons were videotaped using two digital video cameras, one that followed the teacher throughout the lesson and one that was set up to capture student interactions. The teacher camera footage was used when coding the connection-making opportunities that occurred during instruction. The lessons were enacted during the 2014-2015 academic school year and were an average length of 52 minutes.

Teacher Interviews. Immediately following each videotaped lesson, a structured interview with the teacher was conducted and recorded. The purpose of this interview was to get immediate self-reporting feedback about the effectiveness of each lesson. The questions asked during this interview included specific questions about the effectiveness of the instructional tasks, representations and deep questioning techniques that were used during the lesson. The teacher interview will aid in understanding how and why this teacher made instructional decisions. It will also be used for triangulation with the curriculum and videotaped lessons. The interview protocol can be found in Appendix C.

Data Analysis

Based on a situation model perspective of comprehension, a researcher developed coding framework for connection-making (Table 2) was developed. This framework was adapted from a teacher lesson planning framework used by Ding and Carlson (2013). It includes three main categories (instructional tasks, representations and deep questions) which have all been shown to affect a student's ability to form connections to relevant prior knowledge. A scale of 0-2 was used to code the effectiveness for facilitating connection-making within each subcategory in the framework (See Table 2 for details). With regards to instructional tasks, these subcategories include review tasks, worked examples and practice problems. The subcategories for representations involve concrete, abstract, and the sequence of representations subcategories. Finally, the framework includes deep questions asked for prior, current and future knowledge.

To answer Research Question 1 – How does a reformed elementary CCSS curriculum facilitate connection-making for the learning of multiplicative inverses?– each of the curriculum materials for each of the four lessons were scored and analyzed based on the connection-making framework. This included scoring each subcategory based on a 0-2 scale for the level to which the curriculum materials appear to facilitate connection-making. All subcategory scores were summed, and a connection-making score per curriculum lesson was determined. Averaging the four curriculum lesson scores, yielded an overall curriculum connection-making score. In addition, averages across the four lessons were calculated for each category and each subcategory. Along with a qualitative analysis that includes typical ways in which the curriculum materials utilize instructional tasks, representations and deep questions, the curriculum connection-making score and subcategory averages will be used to determine the extent to which learning opportunities within curriculum materials establish and enhance students' situation models of multiplicative inverses. One curriculum lesson (25% of curriculum materials) was randomly selected and a second researcher determined a curriculum connection-making score to check reliability. Seven of the 9 subcategories (78%) were coded identically which resulted in only a 1 point difference in curriculum score (possible score range 0-18).

To answer Research Question 2 –How does an expert elementary mathematics teacher facilitate connection-making during classroom instruction on multiplicative inverses?– each of the four video-taped enacted lessons were analyzed and scored based on the same connection-making framework used to code the curriculum materials. Similar to the coding of the curriculum materials, a 0-2 scale was used to measure how effective the subcategories for instructional tasks, representations, and deep questions used during instruction were for facilitating connection-making. The subcategory scores within each enacted lesson were summed to determine a teacher connection-making score for each lesson. The average of these four scores yielded an overall teacher connection-making score. In addition, averages across the four lessons were also calculated for each category and each subcategory. Along with a qualitative analysis that includes typical ways in which the teacher facilitated connection-making, these scores and averages will be used to determine the extent to which learning opportunities found within classroom instruction promote the development of situation model. The enacted lesson (25% of the videotaped lessons) that corresponded to the curriculum reliability check, was used to check for reliability of the teacher connection-making score. Seven of the 9 subcategories (78%) were coded identically which resulted in only a 1 point difference in teacher score (possible score range 0-18).

To answer Research Question 3 –How does an elementary expert teachers' classroom instruction relate to the way in which curriculum materials facilitates connection-making?– the teacher connection-making scores for each enacted lesson was compared to the corresponding curriculum connection-making score. This comparison will determine if there is evidence that this expert teacher enhanced curriculum materials in order to increase connection-making learning opportunities. If the analysis suggests that she does enhance these opportunities via her classroom instruction, calculated subcategory scores will be analyzed to determine specific ways in which this occurs and qualitative analysis will provide examples of common enhancements. The teacher interviews will also be used to assess the enhancements of the instructional decisions. It should be noted that although slight differences existed in subcategory scores, the check for reliability indicated no variability in the difference between the curriculum and the teacher score (each researcher coded a 5 point higher teacher score).

Results

How does the curriculum facilitate connection-making for multiplicative inverses?

Averaging the four curriculum lesson scores ($L1 = 11,\ L2 = 11,\ L3 = 9,\ L4 = 13$) yielded an overall curriculum connection-making score of 11 (out of 18 total points), suggesting that curriculum materials provide at least some connection-making opportunities. Average curriculum category scores for instructional tasks, representations and deep questions are presented in Table 4.

(INSERT TABLE 4 ABOUT HERE)

Table 4 illustrates that curriculum instructional tasks and representations have a similar average connection-making score for the four lessons in this study, and are both higher than the score calculated for deep questions. Nonetheless, all three average category scores indicate that there exist opportunities to enhance connection-making across various aspects of the curriculum.

Curriculum Instructional Tasks. Connection-making within relevant review tasks seems to be almost non-existent in this curriculum as indicated by an average connection-making subcategory score of 0.75 out of 2 (see Table 4). Although two of the lessons do begin with tasks that implicitly connect to a previous lesson's targeted content, these connections are not made explicit. For example, one lesson begins with an activity in which students must decide which type of problem (multiplication or division) a question represents, but the curriculum only suggest reviewing one problem (multiplication) with the students during this review. As a result, no connection to multiplicative inverses is made during this review exercise (it is later made during a worked example task). Instead, the curriculum leaves students on their own to facilitate connections during the limited review tasks that are suggested. In contrast, most of the worked examples in the curriculum form explicit connections between multiplication and division (average connection-making score of 1.50 out of 2), and practice problems (average connection-making score of 1.75 out of 2) seem to be well connected to the worked examples. This might be partially due to the fact that the worked examples used during instruction are often the first few practice problems in the student activity book.

Curriculum Representations. Concrete and abstract representations are both used extensively in the curriculum for purposes of developing connections to prior and current content, as evident by the connection-making scores of 1.75 and 1.50 respectively (see Table 4). Many of the instructional tasks involve story problems that are situated in concrete contexts (e.g., frogs in pond, desks in classroom, flowers in bouquets) and various references for using concrete manipulatives (e.g., cubes, drawings, tally marks) to model these problems are made throughout the curriculum. An example of how the textbook uses a concrete representations to illustrate how 28 desks can be broken into groups of 4 desks is provided in Figure 1. This figure also provides a side-by-side example of how the curriculum often used abstract representations to form connections between multiplication and division.

(INSERT FIGURE 1 ABOUT HERE)

Although the subcategory average indicates that representations were not always presented as a linear progression from concrete to abstract (a sequence of representation connection-making score of 1 out of 2), Figure 1 provides evidence that this seems to be at least partially the goal. Nonetheless, the curriculum emphasizes the use of many different types of strategies (e.g., tallies, skip counting, symbolic), but only once during lesson four briefly mentions that teachers should question students so to "encourage them to develop more efficient strategies" (Investigations, 2013, p. 132). The failure to discuss efficiency of these strategies early in the learning of multiplicative inverses, may perhaps limit connection-making opportunities between concrete and abstract representations.

Curriculum Deep Questions. According to Table 4, deep questions appears to be the category in which there exists the greatest opportunity for improving connection-making (overall average category score of 0.56 out of 2). While a decent amount of deep questions in the curriculum are aimed at facilitating connections within current targeted content (subcategory connection-making score of 1.50 out of 2), few were found to involve prior content (subcategory connection-making score of 0.75 out of 2), and no deep questions existed to connect to future content. The deep questions that were included in these lessons were almost always related to facilitating connections within worked examples and often included "how many," "how did you," "can you," and "why did you" types of questions. These deep questions were aimed at developing explicit connections within the current content domain of multiplicative inverses.

How does an expert teacher facilitate connection-making for multiplicative inverses?

Averaging the four teacher lesson scores ($L1 = 17, L1 = 18, L3 = 17, L4 = 17$), yielded an overall teacher connection-making score of 17.25 (out of 18 total points). This suggests that the expert teachers' classroom is rich with connection-making opportunities. Accordingly, the instructional tasks, representations and deep questions used during her classroom instruction strongly facilitate the creation of a situation model for multiplicative inverses. Category and subcategory average teacher connection-making scores can be found in Table 5.

(INSERT TABLE 5 ABOUT HERE)

Table 5 indicates that across the four lessons, the teacher received a perfect connection-making score for instructional tasks, and nearly perfect scores for representations and deep questions.

Teacher Instructional Tasks. The highest possible average score (2.00) was assigned to each of the three instructional task subcategory scores for teacher connection-making, indicating that well connected review tasks, worked examples and practice problems were used during instruction. The teacher began each lesson with instructional review tasks that were explicitly connected to the targeted content of multiplicative inverses. These tasks were always connected to the content in the previous lesson and often dealt with previously learned problem solving strategies as opposed to only focusing on final solutions. During two of the lessons the teacher also facilitated deep connections to content that had been learned in the more distant past. One of these cases involved forming connections to additive inverses while the other linked prior geometry knowledge (area of rectangle) to the current targeted multiplicative inverse content. In addition, the teacher often made references to a book of additive inverse story problems that contained individual student problems. This seemed to provoke a personal connection for each student as they recalled the specifics of their own problems.

The worked examples used by the teacher were always deeply connected to the inverse relationship between multiplication and division. For example, after working through the multiplication problem “A robot has 4 hands. Each hand has 6 fingers. How many fingers does the robot have altogether?” the teacher facilitated a discussion about reversing the problem and together the class determined the division problem to be “There are 24 fingers from a robot. This robot hands. How many fingers in each hand?” Immediately following this worked example, the students spent time writing their own inverse story problems before sharing them in a whole-class discussion. This alternation between worked examples and practice problems was consistently used in her instruction in such a way that promoted connection-making between and within the various instructional tasks. During both the instruction of worked examples and the discussions involving corresponding practice problems, the teacher almost always promoted the use of different solution strategies. This caused a great deal of time to be spent on only a few individual instructional tasks, but through the use of representations and deep questions, these few tasks were unpacked in great detail. Overall, the teacher's instructional tasks were designed to deliberately facilitate connection-making within multiplicative inverses.

Teacher Representations. Both concrete and abstract representations were extensively used to develop connections to prior and targeted content during this teacher's instruction, as indicated by both subcategories receiving an average connection-making score of 2 (see Table 5). Every worked example used by the teacher was situated in a rich concrete context and on several occasions the teacher attempted to make these contexts personal. For instance, she made reference to her own classroom when discussing a practice problem about the grouping of desks and also used the names of her own children when creating a worked example involving the sharing of balloons. Both cases illustrate her attempt to facilitate connection-making within real life situations. Overall, the concrete representations used by the teacher throughout instruction, included tallies, arrays and drawings. Figure 2 shows her typical use of these concrete representations. On several occasions it was also mentioned that students could use blocks if needed, but the teacher never actually included blocks in her instruction.

(INSERT FIGURE 2 ABOUT HERE)

The teacher used these concrete representations to create connections within the target content, instead of just using them as a means by which to show a visual picture of the situated problem. This became evident during the robot example when the targeted content involved distinguishing between the two types of division problems (grouping or sharing). Although the problem asked to find out how many fingers the robot had on each hand, the teacher began the problem with writing out 24 tallies (the number of total fingers given in the problem). Quickly, she realized that the problem did not provide her with the number of fingers in each group and thus she did not know how many tallies to circle at one time. Instead, she erased the 24 tallies and drew four circles (to represent the four hands) and began to place fingers into each circle until there were equal amounts on each hand and all fingers were accounted for. It was obvious that her mistake was not planned, however, she used this instance to facilitate a discussion about the importance of determining what pieces of information are provided in the statement of the problem. The use of the side-by-side balloon illustrations (Figure 2) was another effective instance of where the teacher used concrete representations to facilitate connection-making. During this example, one student could even be heard explaining “I see it…they are different!”

This third grade teacher also effectively used abstract representations to form connections within the domain of multiplicative inverses (see Table 5). This most often occurred when students answered a question in the form of an equation, provided an explanation based on the missing factor approach when asked how they arrived at the equation. In these instances, the teacher ended up writing down both the solution equation and the inverse equation (often with an empty box as seen in Figure 3). In doing so, the teacher provided explicit representations of students’ abstract thinking, which later facilitated the introduction of a multiplication/division fact family (Figure 3).

(INSERT FIGURE 3 ABOUT HERE)

The teacher in this study also used the “house” notation and the fraction notation of division as seen in Figure 3. Although connections to these abstract representations were not made explicit during instruction, one is left to assume that these were introduced as a means of making connection to future targeted content (i.e., long division and fraction).

Although individual instructional tasks and even whole day lessons did not always follow a progressively abstract sequences of representations, the overall average sequence of representations teacher subcategory score (1.50 out of 2) suggests this to be the case when looking across the four lessons. During initial learning of multiplicative inverses, the teacher emphasized the importance of multiple solution strategies, with no preference given to any one type of representations. As indicated by the instructional tasks in which the teacher asked students to solve a story problem involving a product of 24 and a factor of 4, the use of tallies, repeated addition, skip counting and the equation were all considered to be acceptable solution methods (Figure 4). In general, this teacher emphasized the use of representations as a means to finding the solution.

(INSERT FIGURE 4 ABOUT HERE)

Interestingly, even if a students' first strategy involved the abstract representations, the teacher encouraged the student to think about other strategies that could be used to solve the problem. Although this may suggest that the teacher had no regard for developing efficient strategies, according to the classroom discourse, her main goal at this point in the instruction was to use multiple strategies to promote connection-making between various representations and across worked examples. It is worth noting here that the only time efficiency was mentioned was when students had incorrectly attempted to use their own hands as concrete representations for robots that had six fingers. After a quick discussion of why this representations would not work, the teacher stated that "we want more efficient…quicker ways" and then discounted the effectiveness of a student's repeated addition solution.

Several other indications in the enacted lessons point towards the teachers attempt to use a progressively more abstract sequences of representations. For example, the multiplication and division chart (Figure 4) used during instruction facilitates the task of turning a concrete story problem into a solvable equation. Filling this chart in from left to right indicates that student's encounter the concrete questions "how many groups" and "how many in each group" before having to reason with the abstract principles of product and equations. The general form of a multiplication problem is also listed at the top of the chart and the teacher uses this when facilitating connections between concrete quantitates and the words product and factor. Another instance of abstract progression involved using array cards (Figure 4) to play a game called "the missing factor." The students had previously used the array cards to determine the product of two numbers but when doing so could rely on the concrete representations of rows and columns illustrated on the multiplication side of the cards. With the missing factor game however, no such concrete representations were available because the division side of the cards contained no arrays. The teacher facilitated a discussion about the similarities and difference between finding the product versus finding the missing factor and stressed the importance of using concrete thoughts (arrays) to reason abstractly.

Teacher Deep Questions. In contrast to the literature that suggests that few deep questions are being asked in today's classrooms (Khan & Inamullah, 2011; Wimer, Ridenour, Thomas & Place, 2001), the classroom discourse in this elementary teachers' mathematics instruction is predominately facilitated by the asking and answering of deep questions. The teachers uses deep questions in order to promote inferential thinking by forcing students to search for connections between current content and prior knowledge. It is obvious that when asked deep questions, students in this classroom are given many opportunities to express their levels of understanding in both whole class and small group settings. As evident by an average connection-making category score of 1.92 out of 2 (see Table 5), deep questions posed by this teacher help to facilitate conversations that are rich with connection-making opportunities.

To facilitate connections to prior knowledge, the teacher asked two main types of deep questions. The first type of deep questions elicited students' previous understanding of inverse relations by asking how current problems were similar or different to previous applications. To draw connections to prior knowledge of additive inverses, this type of question took the form of "Do you think multiplication and division are related like addition and subtraction?" However, the purpose of this type of question shifted to drawing connections to previously learned multiplication knowledge once students were presented with division story problems. These questions often then took form of "What is different about this problem compared to problems that we had been working with before?" Illustrating the importance behind both purposes of this type of deep questions, one student deduced "multiplication is like addition, you are adding them all up and division is separating them." This statement provides a strong indication of the students well-connected situation model for inverse relations. The other type of deep questions that the teacher asked in order to facilitate connections to prior knowledge, involved questions related to the quantities inherent in multiplicative inverses (e.g., groups, factor, product). Examples of these questions include: (1) "Does this problem involve the number of groups?" (2) "Does anyone know another name for dimension?" (3) "What other words do we use in multiplication?" (4) "Can there be a factor in division?" These deep questions stressed the importance of previously learned vocabulary and concepts and also helped to facilitate connection-making between concrete (groups) and abstract (factor) quantities found within multiplicative inverses.

Many of the deep questions that were used to facilitate connections within the current to-be-learned content revolved around problem comprehension or strategy selection. These problem comprehension questions focused on helping students extract necessary information in order to determine what was being asked in a given story problem. On several occasions the teacher asked deep questions to determine if a given problem represented a multiplication or a division scenario and suggested that students examine the known and unknown information provided in the question. This comprehension skill included having students fill in the multiplication and division chart (Figure 4) by identifying the number of groups, determining how many were in each group, and/or pinpointing the product that was given in a specific story problem. When it appeared as if some students were struggling to determine these quantities, the teacher provided contextual support for the deep questions. For instance, instead of just asking what the number 20 represented in a worked example that involved placing 20 muffins into bags of 4 muffins, the teacher asked the following questions: (1) “Is 20 muffins the number of groups?” (2) “Would 20 be the number of muffins in each group?” (3) “Is 20 the total number of muffins? The product?” Similar to analysis of another expert teacher’s questioning abilities (Chen & Ding, 2016), this teacher used specific questions to unpack a broad question for purpose of helping to orient students’ attention. This was evident by the almost instantaneous reactions of those who had been struggling but now appeared to understand that 20 represented the product given in the problem. This mimics research (McNeil & Alibali, 2000; Rittle-Johnson & Alibali, 1999) that shows novice learners often struggle to attain mathematical comprehension when only abstract representations are used during instruction. These deep problem comprehension questions not only helped students recognize the difference between multiplication and division, but were also used to help students form connections between the two types (grouping and sharing) of division story problems.

As indicated previously, the teacher in this study encouraged multiple solutions and thus many deep questions were asked with regards to strategy selection. These questions often went beyond the "what," to include deep questions such as "how" and "why," in order to facilitate connections within and across various solution strategies. For example, when working with a story problem involving the factors of 4 and 6, one student suggested using tallies to determine the product. After the teacher wrote 4 groups of 6 tallies each on the board, the student stated the correct answer of 24. Next, in order to help students form connections between solutions strategies, the teacher proceeded to ask the deep questions "How did you do it? Did you count by 1's or 6's?" After a bit of hesitation, the student explained that she knew "two groups of 6 is 12 and so 12 and 12 is 24." The teacher used this opportunity to help the class form the explicit connection that the use of tallies involves counting by 1's however, the girl was counting by 6's which meant that she was employing the thought process of repeated addition. In order to create further connections between strategies, the teacher asked students "how could I skip count the whole way?" This allowed students to form a better connection between the numbers 4 and 6 and therefore one might view these deep questions as an attempt to facilitate the progression from concrete to abstract representations.

With regards to future content, the teacher used deep questions to form connections to two key future mathematical topics. The first topic involved using multiplicative inverses in order to rewrite a multiplication story problem into a division story problem. Even though this "reversal" task was the main focus point for the third lesson in this study, the teacher actually modeled this task when presenting worked examples in each of the first two lessons. After the reversal in the first lesson, the teacher asked several deep questions about why and how she had "changed the problem," and informed students that they would be doing this task in the near future. This connection to future knowledge was also made during the second lesson when students were asked if they thought they too could "turn [their] own multiplication problem into a division problem?" By proposing deep questions for the purpose of challenging students to think about how they could use current knowledge to perform a task in a future situation, the teacher has allowed more time for inference-making and thus has helped to facilitate the creation of a situation model.

Deep questions were also used by this teacher for the purpose of forming connections between division and the future content of fractions. Once students were shown how division could be written in fraction form, on several occasions the teacher posed questions involving the use of this notation. One of these deep questions was "how come you didn't say 2 divided by 6" when writing the fraction corresponding to splitting six into two equal parts. Although students responded to this question by simply saying "because you couldn't do it" or "it wouldn't make sense," it is clear that the deep question prompted students to use their current knowledge in order to make inferences about what would happen in a future unknown situation (dividend < divisor). The teacher also asked several deep questions for the purpose of forming connections to the future topic of improper fractions (dividend > divisor). For example, when discussing a practice problem that involved sharing 18 cards evenly among four friends, the teacher asked "what will happen if it was not even – 19 cards- who gets the last card?" Student responses included the words "extra" and "remainder" which indicates students inference-making within this new unknown situation. Taken together, these two examples suggest that the use of deep questions appear to have facilitated the creation of a situation model for future exploration of fractions.

How does teacher instruction relate to curriculum opportunities for connection-making?

Comparing coded scores for facilitating connections, reveals a teacher connection-making score that is on average 5.5 points higher than the curriculum score (Table 3). Teacher scores in all subcategories for every lesson were greater or equal to the curriculum score. From the perspective of a situation model, this suggests that an expert teacher enhances curriculum materials in order to facilitate connections during mathematics instruction for the purpose of increasing student comprehension. The subcategories in which the teacher seemed to enhance curriculum connections the most, included review tasks and both prior and future knowledge questions. That is, it was found that this teacher enriched her instruction the most by creating connections to prior knowledge that were not always found in the curriculum, often by asking deep questions.

Instructional Tasks. In terms of overall instructional tasks, the worked examples and practice problems that were used by the teacher during instruction seemed to pretty consistently match those that were given in the curriculum. In fact, every example provided in the curriculum materials for these four lessons was used during instruction except for one. The example problem that was provided in the curriculum as a model for writing story problems included six children attending the movies with the task of determining the product if each ticket cost $4.00. Instead, the teacher used the aforementioned balloon problem (Figure 2), involving a product of 6 and factors of 3 and 2, in order to distinguish the difference between a division problem involving sharing with one that involves grouping. Making the distinction between these two types of division was not mentioned anywhere in the curriculum, which illustrates this expert teachers' ability to adapt curriculum instructional tasks in order to enhance connection-making. In addition, it is interesting to note that even though the movie problem only dealt with a multiplication example, the curriculum instructed students to write both multiplication and division problems. The teacher however used her balloon example to show the two different types of division problems and then also used techniques learned from multiplication (e.g., tallies) to make explicit connections between multiplication and division. This example represents a clear instance when the teacher enhanced the curriculum materials to provide connection-making opportunity and is reflected in the teacher's higher worked example subcategory score.

Across lessons, the teacher enhanced connection-making opportunities within instructional tasks mainly as a means of review. The average review instructional tasks subcategory score for the curriculum was 0.75 in comparison with the teacher's 2.0 score. The review that did appear within the curriculum materials were not explicitly connected to multiplicative inverses. For example, each curriculum lesson included "ten-minute math" tasks for which students had to determine time from an analog clock and also suggested student activity book problems included basic addition expression and addition story problems. In contrast, the teacher's review instructional tasks always facilitated connection-making within multiplicative inverses. One teacher instructional review task that this was evident for was a "math warm up" that involved analyzing quadrilaterals. During this review, the teacher had the students use prior knowledge to determine the perimeter and area of two different rectangles. After explicitly connecting area to finding a product, the teacher also had students write division expressions that could represent the two rectangles. Interestingly, the two rectangles had the same area, and a discussion followed that involved the concept that multiple factors can lead to the same product, further strengthening the connection to multiplicative inverses. Interestingly, since the one rectangle was a square, the teacher also made a connection to the future concept of square numbers.

Representations. With the exception of using fraction notation to represent division, both the concrete and abstract representations used by the teacher during instruction were similar to what was found in the curriculum. Throughout all four lessons the curriculum materials and the teacher promoted the use of multiple solution strategies. During each of the first two lessons, the curriculum illustrated a worked example with 3 different solution methods and suggested that "students should be able to solve the problem in several ways" (Investigations, 2013, p. 117). This was very similar to the trend that was seen in the teacher's instruction, as the worked examples often involved 4 different representations. Further, during the first lesson the curriculum provides example questions of how a teacher could facilitate connections between worked examples and the targeted content which included "Why did he do that?—How many times did he count by 4?—How many groups did she make" (p. 119)? The teacher seemed to embrace these questions as she used them throughout all of her instruction. This illustrates her ability to carry over effective curriculum materials from one lesson, in order to better facilitate connection-making across other lessons. The slightly larger teacher connection-making scores found within the abstract and concrete representations subcategories are a result of the teacher going beyond simply using various strategies to make connections to the targeted content. In addition, the teacher did not view strategies in isolation, rather she made connections between strategies as shown in the multiple solution strategies of Figure 4.

Although the teacher tended to offer more explicit connections involving representations, like the curriculum these representations did not always progress from concrete to abstract. For example, there were several occasions that the curriculum used number sentences to represent a story problem prior to suggesting that students might use other concrete representations. On the other hand, as seen in Figure 1, the curriculum materials suggest using a chart that clearly shows a sequence of representations progressing from concrete to abstract. Both of these instances were also found during the teacher's instruction. Perhaps the mixed messages in the curriculum may have influenced the teacher to not stress the desire for students to progress towards abstract reasoning. However, unlike the curriculum, the teacher did use abstract number sentences involving division during the very first lesson to initiate connection-making between multiplication and division. The teacher also very briefly discussed the inefficiency of tallies when dealing with large quantities and in one of her post-instructional interviews she explained, "I like the fact that we learn that every strategy is important and we share out and as long as you get to the answer it's okay to use that. Now of course I want those students who are drawing it out to be more efficient and get to where the other students are and they will." As a result, the teacher average sequence of representations score is slightly higher than the corresponding curriculum score.

Deep Questions. As indicated by the overall category scores (Tables 4 and 5), the major difference in connection-making between the teacher and the curriculum was found to be the inclusion of deep questions. This difference was the greatest when analyzing deep questions that attempted to elicit students to form connections to prior knowledge and when deep questions were used for the purpose of forming connections to future knowledge. In fact, both of these uses for deep questions were almost extinct from the curriculum, whereas they were the predominant instructional tool that the teacher used in order to both review and preview content knowledge. Not only were the deep questions extinct, but very few indications in the curriculum suggested any facilitation of connection-making to either prior or future content. According to the teacher however, this was an important part of her instruction because as she indicated in a post lesson interview that "obviously whatever we have done previously I want them to relate that to division. Especially in this unit because it is so much related." These review questions were often comparison type questions similar to "what is different from this array game than before?" In addition, she stressed the importance of using deep questions to help facilitate connections to future knowledge because she know that her instructional lesson "dealt with a lot of equal groups so I tried to throw in there like what if this was 19 just to see if it was left over could we still share out, could we still divide, is it possible? That will come up a lot with fractions too…though it might still not be in the exact curriculum investigations, I would like…to challenge students."

Interestingly, the curriculum included many deep questions that could be asked for the purpose of forming connections within the current to-be-learned content of multiplicative inverses. Because the teacher incorporated many of these curriculum deep questions into her instruction, no large difference was found in this subcategory of deep questions. The teacher did however enhance the curriculum with additional deep questions for current knowledge, most of which solicited students to explain their conceptual understanding. Further, when students made mistakes in their reasoning, the teacher posed deep questions in order to force students to analyze their inference process. For example, when a student divided 24 by 6 however conceptually it should have been divided by 4, she asked the student "why" and then followed this up with the questions "where is the 6 in the problem?" Instead of just explaining why a student was incorrect, this teacher gave students opportunities to correct their reasoning and hence allow them to refiner their inference process.

The teacher also enhanced the curriculum's deep questions by including many references to the contextual meanings inherent within division story problems. She emphasized that the students should "box or circle" their answers and often reminded them to provide a context for their solutions by asking them questions such as "what does the 5 mean?" Moreover, although not suggested by the curriculum, the teacher often asked students to self-reflect. Typical deep questions involving this task included "What did you learn about division today?"—and—"How do you feel about division so far?"

Discussion

Instruction of elementary inverse relations often lacks connections made to underlying principles (De Smedt et al., 2010), which often prohibits understanding (Torbeyns et al., 2009). A lack of connection-making opportunities within curriculum and instruction may therefore be contributing to U.S. students' continual lack of understanding fundamental mathematical ideas. Although past research on inverse relations has focused on if and when children show evidence of understanding inverse relations, by examining factors that facilitate connection-making, this study has begun an exploration into why and how this understanding occurs. Based on the fact that many researchers agree that the most influential factor of comprehension is a learner's ability to construct a coherent situation model (Glenberg, Kruley, & Langston, 1994; Graesser, Millis & Zwaan, 1997; Perfetti, 1989; Zwaan, Magliano, & Graesser, 1995), analyzing experimental variables within learning opportunities that affect the ability for learners to make connections and draw inferences is essential in the pursuit of increased mathematical comprehension. This study has examined how both curriculum and instruction can help students on this pursuit of transforming propositions into understanding that can be applied in future situations.

Research shows that students who have little prior knowledge benefit from curriculum and instruction that is coherent and explicit as possible (Kintsch, 1994; Reed, Dempster & Ettinger, 1985). Since the relationship between multiplication and division is one of the first instances in which students are faced with inverse reasoning, the prior knowledge students had of the targeted concept in this study was limited. However, whereas the expert teacher in this study provided very explicit opportunities to form connections between prior knowledge and current or future situations, curriculum materials did not always do so even though they are Common-Core aligned. In response to Polikoff's (2015) notion that "it would be worthwhile to investigate the extent to which textbook content may be associated with effectiveness" (p. 1207), the findings in this study suggest that more than current reformed curriculum materials may be needed in order to answer the CCSS call for students to form connections in order to facilitate transfer of learning.

The teacher in this study did not seem to change the structure or sequence of the curriculum's instructional tasks and representations, which may indicate that elementary mathematics teachers tend to see their role as the deliverer of an already existing curriculum. Many teachers might therefore believe that merely enacting a reformed curriculum provides enough connection-making opportunities. Further, due to the fact that very few U.S. elementary teachers specialize in mathematics, some teachers may be uncomfortable or simply might not know how to better facilitate connection-making and thus have very little influence on a students' situation models. However, contrary to past research (Ding & Carlson, 2013), this expert teacher spent a large portion of time unpacking worked examples for the purpose of showing explicit relationships between and within the instructional tasks and representations, whereby providing necessary deep connections needed for the creation of a situation model.

The expert teacher in this study enhanced curriculum connection-making opportunities the most by asking deep questions about previous and future content which often were not found in the curriculum. By doing so, a learning environment that was rich with conversation and infused with connection-making was established. The deep questions posed by this expert teacher forced students to make inferences between the current content and other contexts to which that content could be applied, the very foundation of a situation model. The ability for this expert teacher to create this type of learning environment, is unlike many of today's classrooms in which deep questions are not as common (Khan & Inamullah, 2011; Ridenour, Thomas & Place, 2001). Although it is unknown whether the ability to facilitate connection-making is due to the teacher's expertise, it appears as if curriculum alone does not equip teachers with the tools necessary for helping students create the deepest levels of comprehension.

Although their still exists a large gap in current knowledge surrounding how best to facilitate connection-making in mathematics education, this study suggest that a teachers classroom instruction can greatly enhance a curriculum's connection-making opportunities. Because teachers use textbooks to guide instruction (Ball & Cohen, 1996), this finding is important in that it may inform curriculum designers about opportunities to better support teachers in facilitating children's connections to prior knowledge and targeted content. By illustrating how an expert teacher enhances curriculum materials to help students develop situation models, this study provides empirical research on how to strengthen conceptual understanding. Future research should continue to examine curriculum materials and classroom instruction for the purpose of further identifying how best to structure learning environments in order to uimize students' mathematical comprehension.

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Table 1. *Targeted Content: Math Focus Points for Each Multiplicative Inverse Lessons*

	Lesson 1	Lesson 2	Lesson 3	Lesson 4
Title	Solving Division Problems	Multiply or Divide?	Writing Story Problems	Missing Factors
Math Focus Points	Understanding division as the splitting of a quantity into equal groups. Using the inverse relationship between multiplication and division to solve problems.	Using the inverse relationship between multiplication and division to solve problems. Using multiplication combinations to solve division problems. Using an understanding division notation.	Understanding division as the splitting of a quantity into equal groups. Writing and solving multiplication problems in context. Writing and solving division problems in context. Using and understanding multiplication notation. Using and understanding division notation.	Using multiplication combinations to solve division problems. Using the inverse relationship between multiplication and division to solve problems. Using and understanding multiplication notation. Using and understanding division notation.

Table 2. *Coding Framework for Connection-Making: Facilitating a Situation Model*

Category	Subcategory	0	1	2
Instructional Tasks	Review	The task was a routine review of prior content but no connections to the targeted content was made.	An implicit connection to the targeted content was made, but not well developed.	An explicit connection to the targeted content was established and well developed.
	Worked Examples	No connections to prior or the targeted content was made.	Implicit connections to prior or the targeted content were made, but not well established or discussed. Clear opportunities to make connections are also missed.	Explicit connections to prior or the targeted content were made. No clear opportunities to make connections are missed.
	Practice Problems	Practice problems have no connection to the worked examples.	Practice problems have an implicit connection to the worked examples.	Practice problems have an explicit connection to the worked examples.
Representations	Concrete	No concrete representations (ie. manipulatives, pictures, or story situations) are used to form connections to prior or the targeted content within instructional tasks.	Concrete representations are used to form connections to prior or the targeted content within instructional tasks, but the connections are not well developed.	Instructional tasks are situated in rich concrete contexts (i.e. story problems) and are used to form well developed connections to prior or the targeted content within instructional tasks.
	Abstract	No abstract representations (ie. numbers, mathematical symbols, equations) are used to form connections to prior or the targeted content within instructional tasks.	Abstract representations are used to form connections to prior or the targeted content within instructional tasks, but the connections are not well developed.	Abstract representations (i.e. equations) are used to form well developed connections to prior or the targeted content within instructional tasks.
	Sequence of Representations	No connections between concrete and abstract representations are made between instructional tasks.	A connection between concrete and abstract representations is established between instructional tasks, but it does not progress from concrete to abstract.	A clear connection between concrete and abstract representations is established that indicates a progression (concrete to abstract) of worked examples for the purpose of forming connections to the target concept.

Questions	Prior	No deep questions are asked for the purpose of forming connections to prior knowledge.	Some deep questions are asked for the purpose of forming connections to prior knowledge; but there remain important missing connections to prior knowledge.	Deep Questions are posed to elicit students to form connections between prior knowledge and the targeted concept and there are no important missing connections to prior knowledge.
	Current	No deep questions are asked for the purpose of forming connections within the current to-be-learned content knowledge (ie. between examples)	Some deep questions are asked for the purpose of forming connections within the current to-be-learned content knowledge but the connections remain at the surface level.	Deep questions are posed to elicit students to form connections within the current to-be-learned content knowledge.
	Future	No deep questions are asked for the purpose of forming connections to future knowledge.	Some deep questions are asked for the purpose of forming connections to future knowledge; but the connections are implicit.	Deep questions are asked for the purpose of forming connections to future knowledge and the connections are explicit.

(Table 2 continued).

Table 3. *Textbook and Enacted Lesson Scores for Facilitating Situation Models* (Paper 2)

Lessons	Curriculum Score	Teacher Score	Difference
#1	11	17	+ 6
#2	11	18	+ 7
#3	9	17	+ 8
#4	13	17	+4
Average	11	17.25	+ 6.25

Table 4. *Curriculum Connection-Making Scores Across Lessons*

Categories	Averages	Subcategories	Averages
Instructional Tasks	1.00	Review	0.75
		Worked Examples	1.50
		Practice Problems	1.75
Representations	1.06	Concrete	1.75
		Abstract	1.50
		Sequence of Representations	1.00
Deep Questions	0.56	Prior	0.75
		Current	1.50
		Future	0.00

Table 5. *Teacher Connection-Making Scores Across Lessons*

Categories	Averages	Subcategories	Averages
Instructional Tasks	2.00	Review	2.00
		Worked Examples	2.00
		Practice Problems	2.00
Representations	1.83	Concrete	2.00
		Abstract	2.00
		Sequence of Representations	1.50
Deep Questions	1.92	Prior	2.00
		Current	2.00
		Future	1.75

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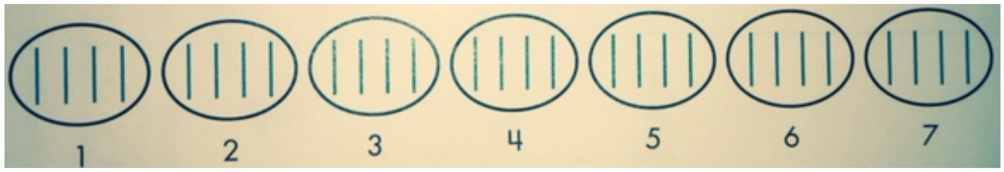
<tr><td>Concrete Representation</td><td>
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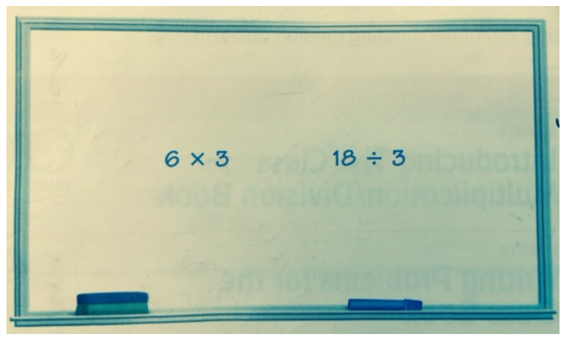
<tr><td>Abstract Representation</td><td>
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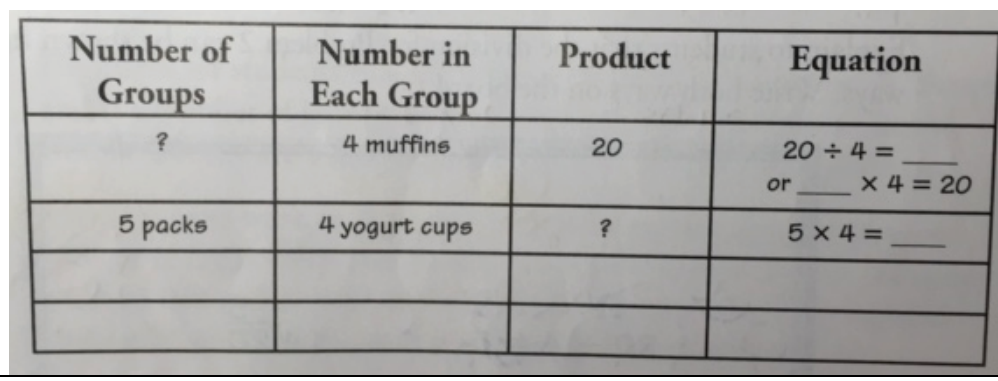
<tr><td>Sequence of Representation</td><td>
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or ____ × 4 = 20</td></tr>
<tr><td>5 packs</td><td>4 yogurt cups</td><td>?</td><td>5 × 4 = ____</td></tr>
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Figure 1. Representations illustrated in the curriculum.

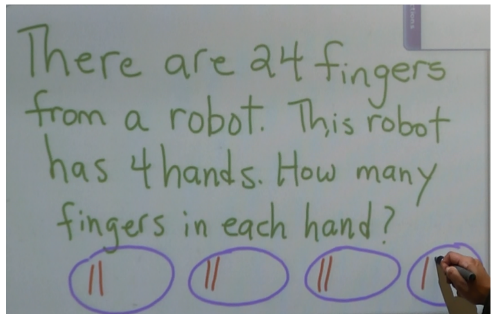
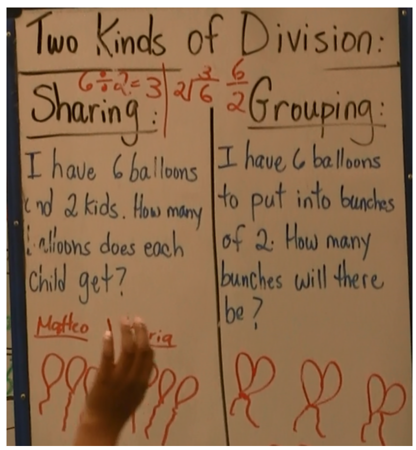
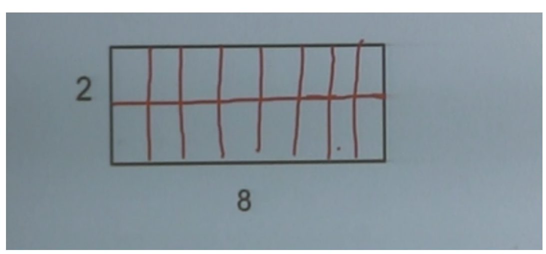
Tallies	
Pictures	
Arrays	

Figure 2. Concrete representations illustrated during instruction.

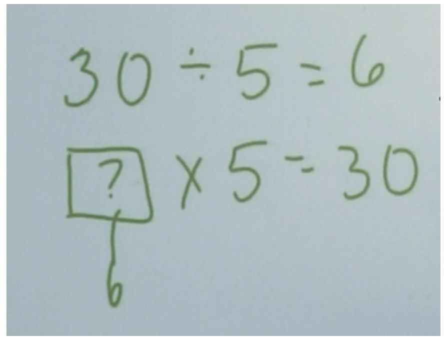
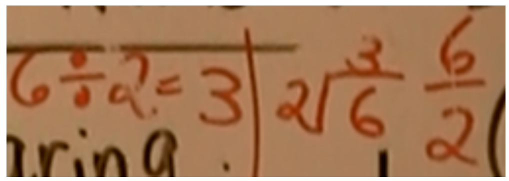
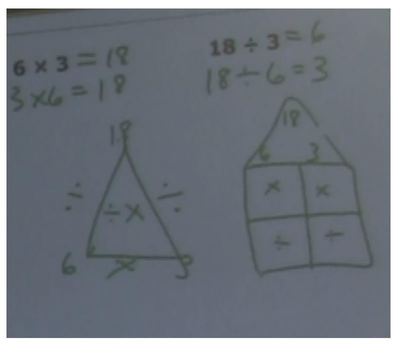
Multiplicative Inverse Equations	
Different Abstract Representations of Division	
Multiplicative Fact Family	

Figure 3. Abstract representations illustrated during instruction.

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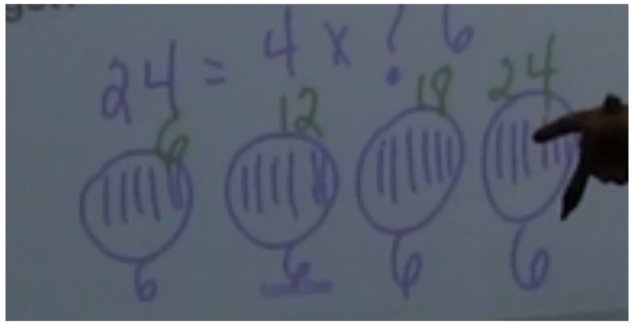
<tr><td>Multiple Solution Strategies</td><td>
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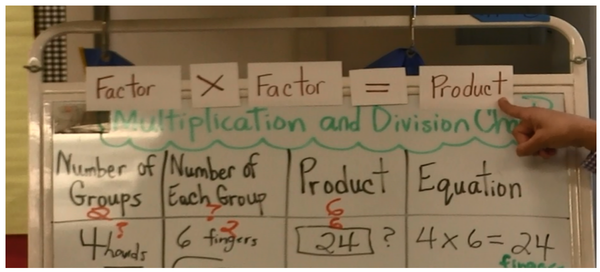
<tr><td>Multiplication & Division Chart</td><td>
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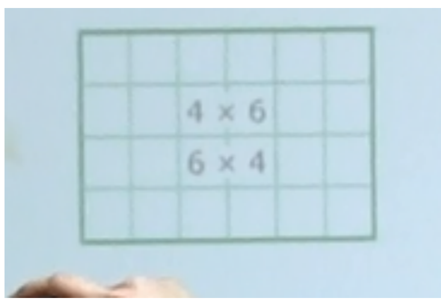
<tr><td>Array Cards</td><td>

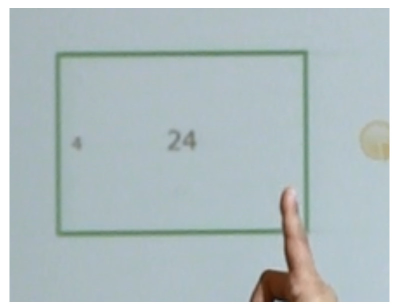

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Figure 4. Sequence of representations illustrated during instruction.