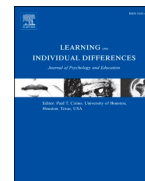




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Profiles of fraction knowledge in first grade and their relation to cognitive and mathematical skills^{☆,☆☆}

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ABSTRACT

Latent profile analysis was used to examine variation in early fraction knowledge among first graders ($N = 204$; $M_{age} = 6.68$ years) across nonsymbolic and symbolic representations. Three distinct profiles of fraction understanding emerged. Profile 1 exhibited strong nonsymbolic knowledge, Profile 2 demonstrated similarly strong knowledge, except for weaker performance in nonsymbolic equivalence, and Profile 3 showed generally limited understanding across most areas. However, by the end of first grade, children initially identified as Profile 2 performed comparably to those in Profile 1 on measures of equivalence. Additionally, children in both Profiles 1 and 2 showed stronger cognitive skills and higher year-end mathematics achievement than those in Profile 3. These findings suggest that understanding of nonsymbolic equivalence may develop over the course of first grade in children who otherwise demonstrate strong foundational fraction knowledge, so that an initial relative weakness in that area is not concerning.

Educational relevance statement

The study reveals three distinct profiles of first graders' early fraction understanding, which predicted end-of-year mathematics outcomes. Findings suggest that early understandings, such as equal sharing and adding equal parts, prepare students for later mathematics success. Lower scores in nonsymbolic tasks assessing whether children can judge the equivalence of fractional quantities seem to be made up naturally. Children with uniformly lower levels of fraction knowledge showed weaker understanding of later math concepts, as well.

1. Introduction

Difficulties with fractions forecast problems with more advanced

mathematics, such as algebra (Booth et al., 2014). Unfortunately, fractions are challenging for many students, some of whom continue to struggle throughout their schooling (e.g., Braithwaite et al., 2018; Jordan et al., 2017; Resnick et al., 2016). The gap between students with low versus high fraction knowledge in fourth grade widens significantly over time (Resnick et al., 2016).

Even before formal fraction instruction in third grade (in the U.S.), there is substantial individual variation in informal fraction knowledge (Viegut et al., 2023). These informal understandings may emerge through mathematical interactions within or outside of school, which can vary widely (e.g., Daucourt et al., 2021; Jordan et al., 2009). For example, informal fraction understandings can be developed through equal sharing during play or iterating smaller blocks to match the size of a larger block (Eason & Ramani, 2020; Schmitt et al., 2018). Students

^{*} Data and analysis code for this project are available on OSF: https://osf.io/d4wf5/?view_only=5dfab65619d042c38f33a8624942353f and https://osf.io/e27fp/?view_only=493d57641eff4777be012d54f7815fc6.

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may then integrate and map individual insights into knowledge across their understanding of number, number relations, and numerical operations (Jordan et al., 2022).

Potential heterogeneity in informal fraction knowledge remains unexplored, though it is a possible leverage point for fostering formal fraction knowledge. Thus, we expanded on our prior work in which we found that first graders' overall informal fraction knowledge predicted math achievement at the end of first grade (Viegut et al., 2023), in a two-part exploration. First, we acquired a new cohort of students from public schools in mixed income communities to determine if we could replicate our prior findings with students from independent schools in a more diverse setting. Second, after combining the prior sample with the new cohort to increase our statistical power, we examined heterogeneity in patterns of informal fraction knowledge using latent profile analysis (LPA) and determined whether these patterns or profiles relate to later math achievement.

1.1. Theoretical underpinnings

Knowledge of numbers—including fractions—requires both nonsymbolic and symbolic understandings, which may develop at different rates (e.g., Viegut et al., 2023). Jordan et al. (2022) posit that young children's number sense involves three strands: knowledge of number (e.g., subitizing, cardinality), number relations (i.e., numerical magnitude), and number operations (i.e., decomposing and composing quantities). However, the level of representation (i.e., nonsymbolic vs. symbolic) influences children's competencies and developmental trajectories within and across these strands. According to this framework, understanding nonsymbolic representations—such as those shown with concrete objects or pictures—is foundational for understanding symbolic representations, such as number words and written numerals. For example, children can often determine the answer to $2 + 1$ in a nonsymbolic context before they can solve the same problem when it is presented verbally or in written form (Huttenlocher et al., 1994). Thus, children may be able to reason about nonsymbolic fraction representations before they can reason about symbolic ones, although competencies in both formats may mutually reinforce one another over time (e.g., Siegler et al., 2010).

Consistent with this framework, we identified facets of informal fraction understanding that are associated with later, more formal fraction knowledge, including both nonsymbolic components and symbolic components. There are several facets of nonsymbolic fraction understanding, such as equal sharing, adding equal parts to make a whole, and fraction equivalence in concrete or pictorial contexts. *Equal sharing* involves partitioning a quantity into equal groups to share with others (e.g., sharing six cookies among two students) and is viewed as important for early fraction learning (Eason & Ramani, 2020; Empson, 1999; Empson & Levi, 2011). *Adding equal parts* involves iterating fractional parts to create one whole (e.g., counting how many one-third pieces add up to 1 whole), which is foundational to formal understandings of larger fractional units (e.g., Hurst et al., 2022). *Fraction equivalence* involves discerning that the same amount can be represented in multiple ways (e.g., two halves of a candy bar and four fourths of the same bar both make one whole), a critical feature of fraction magnitude knowledge (Viegut et al., 2023).

Symbolic fraction understanding involves knowledge of fraction words (*word mapping*) and notation (*fraction notation*). Even though formal symbolic fraction instruction comes later, some students enter school comprehending common fraction words, such as “one-half” (Empson, 1999) and simple fraction notation (e.g., $\frac{1}{2}$; Brizuela, 2006; Starling-Alves et al., 2023). Notably, students' understanding of equal sharing (e.g., Yoshida & Sawano, 2002), adding (or iterating) equal parts (e.g., Hurst et al., 2022), fraction equivalence (e.g., Kalra et al., 2020), and symbolic fraction words and notation (e.g., Booth et al., 2014; Torbeyns et al., 2015) are all related to students' development of

formal fraction knowledge and mathematical competence.

Thus, there are five related but potentially distinct components of early fraction knowledge that may vary at the start of formal schooling due to varying informal experiences with fractions such as equal sharing situations (e.g., Empson & Levi, 2011; Viegut et al., 2023), exposure to words like “half” or “quarter” in playful situations and everyday conversations (Eason & Ramani, 2020), or spatial experiences like iterating a small block to match the size of a bigger block (e.g., Schmitt et al., 2018). Moreover, some children may be exposed to simple fraction notation through everyday activities, such as cooking. These experiences may help explain early variability in fraction knowledge and may also lead to different profiles of informal knowledge by skill type. For example, some children may show relative strengths in nonsymbolic equal sharing but less knowledge of equivalence concepts or fraction symbols.

1.2. The role of whole number knowledge and general cognitive skills

Fraction knowledge develops in a wider context, not in isolation. Beginning with mathematical knowledge, understanding whole numbers supports fraction knowledge, as both rely on students' understanding of numerical magnitude (e.g., Bailey et al., 2014; Möhring et al., 2018). Knowledge of whole number magnitude and arithmetic in first grade predicts later fraction magnitude skills, even when accounting for other cognitive skills (Bailey et al., 2014). Whole number knowledge at the start of first grade predicts end-of-year informal fraction knowledge, over and above start-of-year fraction knowledge and other cognitive skills (Viegut et al., 2023).

Cognitive skills, including executive functioning and spatial skills, are also associated with fraction knowledge (e.g., Vukovic et al., 2014). Executive function skills, including cognitive flexibility (shifting attention appropriately to successfully switch tasks), working memory (manipulating, integrating, and operating on information in short and long-term memory), and inhibitory control (inhibiting irrelevant responses; Miyake et al., 2000), are related to success in fractions and mathematics more broadly (e.g., Jordan et al., 2013; Zhang et al., 2023).

Spatial scaling, or mentally shrinking and enlarging shapes proportionally, and vocabulary knowledge are associated with math skills and formal fraction knowledge (e.g., Boyer & Levine, 2012; Chow & Jacobs, 2016). We consider proportional reasoning a general cognitive skill in the present study. Although children's ability to judge proportional amounts in spatial representations is conceptually similar to early nonsymbolic fraction reasoning and is correlated with formal fraction knowledge (Möhring et al., 2016), reasoning with proportions requires a broader understanding of how to compare two quantities. This skill is conceptually similar to, but not the same as, reasoning within an approximate ratio processing system (e.g., Kalra et al., 2020; Park & Matthews, 2021). For example, whereas Park and Matthews (2021) asked students to compare which quantity was larger, relying on an innate approximate number system, in the current study we ask students to reason with quantities on a continuous rating scale.

Some literature suggests that certain cognitive skills may be more strongly related to specific components and patterns of students' early fraction knowledge than others. Inhibition may be important in the earlier stages of skill acquisition as students are learning to work with fractions differently than they do whole numbers. For example, when identifying equivalent nonsymbolic fractions students may need to inhibit potential impulses to count the absolute number of parts shaded, instead attending to the proportion or relative amount shaded. Cognitive flexibility may be important when navigating “rules” related to whole numbers (e.g., 4 is bigger than 2) and understanding if and how they apply to fraction understanding (e.g., $\frac{1}{4}$ is smaller than $\frac{1}{2}$). In contrast, executive functioning may be less important, for example, when fraction and whole number principles align (e.g., needing to add equal parts to make a whole). Other studies show that some aspects of executive function (e.g., inhibitory control) are related to symbolic judgments (e.

g., Leib et al., 2023) and nonsymbolic fraction magnitude judgments (e.g., (Abreu-Mendoza et al., 2020). Visual-spatial working memory and skills like spatial scaling may more closely relate to iterating problems. Having stronger visual-spatial working memory or better spatial scaling skills may provide an advantage when solving such problems, which require a child to hold multiple pieces of spatial information in mind simultaneously and transform or mentally manipulate those fractional parts. Lastly, children's success with mapping fraction word items to pictures (e.g., "which picture shows one-fourth of a whole bar?") may be more closely related to receptive vocabulary.

1.3. The present study

The main aim of the current study was to characterize profiles of informal fraction knowledge in the fall of first grade. Because children's informal knowledge of fractions predicts their mathematics achievement (Viegut et al., 2023), and because the various nonsymbolic and symbolic components of informal fraction knowledge develop at different rates (Jordan et al., 2022), it is important to identify which aspects of informal fraction knowledge are most strongly related to later fraction and math learning. This determination could guide the development of educational strategies that lay the groundwork for formal learning about fractions.

We address this aim using latent profile analysis (LPA), a form of mixture modeling that detects homogeneous subgroups within a larger sample that are maximally distinct from each other in their performance patterns (Oberski, 2016). LPA is a useful tool for investigating within-person variability, or interpersonal differences in understanding of different constructs, that could be displayed by differential levels of performance across a set of related measures (Hickendorff et al., 2018). Researchers have used LPA to capture variation in early mathematical understanding such as preschoolers' early math skills and decimal understanding (e.g., Gray & Reeve, 2016; Resnick et al., 2019; Scalise et al., 2021).

Although LPAs are inherently an exploratory data analytic approach, we posit there are distinct profiles of early fraction understanding. Specifically, we hypothesize that there will be variability across aspects of children's early fraction understanding: nonsymbolic understanding (which we further divide into equal sharing, adding equal parts, and fraction equivalence), fraction word mapping, and fraction notation. We focus on these five aspects based on our prior work suggesting these aspects are foundational for fraction understanding (Viegut et al., 2023). Because nonsymbolic and symbolic fraction understanding are distinct, the development of these understandings might vary depending on individual differences such as cognitive factors and educational opportunities (Dehaene, 1992; Jordan et al., 2022). Indeed, previous research using LPA has found distinct profiles of understanding proportional reasoning (Vanluydt et al., 2022) and multiplicative reasoning during equal sharing (Ching & Wu, 2021), both of which are key components of early fraction knowledge. In addition, LPAs have uncovered distinct profiles of formal fraction knowledge in later years (McMullen et al., 2018; Resnick et al., 2019). Taken together with the high variability in early fraction understanding (Viegut et al., 2023) and individual differences in performance prior to formal instruction (Resnick et al., 2016), we expect these younger children may also exhibit distinct profiles of early fraction understanding.

Examining whether there are profiles that help explain this variability, and whether these patterns are related to number-specific and/or domain-general cognitive markers, would deepen our understanding of students' informal fraction knowledge. This understanding could then be used as a leverage point for future instruction. Indeed, customizing instruction to profile membership based on domain specific and general cognitive skills may result in greater improvements in student comprehension skills rather than using a one-size-fits-all approach (Kulesz et al., 2024).

Using longitudinal data collected across the school year, we also

explore the extent to which students' profile membership (in fall) is differentially related to general cognitive skills and whole number knowledge (in winter), and mathematics achievement (in spring). Although we expect whole number knowledge and general cognitive skills to relate to mathematical outcomes, we did not have a specific hypothesis on which components of early fraction knowledge will be most related to cognitive skills or to later math achievement.

2. Method

2.1. Transparency and openness

We followed Journal Article Reporting Standards (JARS) for transparency in our scientific processes (Kazak, 2018). All deidentified data and analysis code are available on OSF (https://osf.io/d4wf5/?view_only=5dfab65619d042c38f33a8624942353f). Non-proprietary research materials designed for this previous study can be found on OSF as well (https://osf.io/e27fp/?view_only=493d57641eff4777be012d54f7815fc6). A portion of the data were collected as part of an initial study investigating correlates of early fraction knowledge as part of a larger grant (see Viegut et al., 2023). Data were cleaned and merged in R (version 4.1.1; R Core Team, 2021) using R Studio Version 1.4.1717 (R Studio, 2020) and the *tidyverse* package (Wickham et al., 2019). Replication analyses were also conducted in R. All subsequent analyses were conducted in MPlus Version 8.3 (Muthén & Muthén, 2019). This study was not pre-registered.

2.2. Participants

We combined the original sample (Viegut et al., 2023) with a replenished second cohort of first graders intentionally recruited from a wider range of schools, including schools serving a greater proportion of students from lower socioeconomic backgrounds. This combined sample is larger and more racially, ethnically, and socioeconomically diverse, allowing for greater statistical power to analyze latent subgroups of students (Geiser, 2013).

Participants were first graders from five elementary schools (two public, three private/parochial) in the Mid-Atlantic region of the United States. During the 2021–2022 school year we had $n = 109$ participants; an additional $n = 101$ children participated in the 2022–2023 school year, bringing our total study sample to $N = 210$ students. Two participants unenrolled in a participating school during the study period, and an additional four participants missed at least one assessment session due to illness. These six participants (less than 3 % of the sample) were dropped from analyses.

The final analytic sample for the present study included 204 first-grade participants (103 from the 2021–22 school year, and 101 from the 2022–23 school year; $M_{\text{age}} = 6.69$ years old, $SD = 0.35$). According to the caregivers' reports, 47 % of the children were female, 52 % male, and 1 % other or unreported; the racial/ethnic breakdown was 46 % White, 17 % Asian, 12 % Black, 5 % Hispanic, and 12 % multiracial, with 8 % unreported. Although we were not able to collect information on socioeconomic backgrounds of individual participants due to privacy concerns, participating schools served communities that ranged from low- to middle-income. One of the five participating schools was a Title I school, with 51–60 % of students eligible for free or reduced-price lunch. Though many of the curricula used by the schools in our sample address limited early fraction concepts by the end of first grade (i.e., partitioning shapes into two or four equal parts), these concepts were not addressed at the beginning of the school year or during the fall study period.

We distributed informed consent letters to families of all first-grade children at each participating school at the beginning of the school year. Seventy-six percent of enrolled first graders were given consent by parents or guardians to participate. This study was approved by the University of Delaware Institutional Review Board.

2.3. Measures

We administered measures to students at three timepoints: fall, winter, and spring. We detail measures administered at these timepoints below.

2.3.1. Indicators of early fraction knowledge

We assessed early fraction knowledge with a 43-item researcher-designed measure (Viegut et al., 2023) that captures variability in first-grade children's early fraction knowledge. Items measured children's understanding of (1) *equal sharing* in a nonsymbolic context; (2) *fraction equivalence* without symbols; (3) *adding equal nonsymbolic parts* to complete a whole; (4) *mapping fraction words* to nonsymbolic representations; and (5) *fraction notation* or written fraction symbols. This measure was developed as a robust assessment of early fraction knowledge and items were carefully selected to avoid ceiling or floor effects. We included both discrete and continuous (21 items; 12 nonsymbolic, 9 symbolic) representations. Items were not selected to have an equal representation of each across the measure (see Viegut et al., 2023 for details on scale creation).

Equal sharing items (11 items) required children to reason with fractional parts in equal sharing situations. Students were presented with varying numbers of rectangles and were asked to evenly distribute them between groups. *Fraction equivalence* items (3 items) assessed whether children could identify equivalent proportions of rectangular models. Students were presented with rectangles that had discrete portions shaded and were asked to select which rectangle had the same proportional amount shaded as a target. *Adding equal parts* items (6 items) involved iterating or repeating equal fractional parts to produce a given whole. In some of these problems, children had to choose which size tile to use to cover a whole path using 2, 3, or 4 equal parts. In other problems students were given a tile of a specific size and asked how many equal tiles were needed to make a given whole. *Word mapping* items (20 items) involved mapping fraction words to visual representations, or vice versa. On some items, students were asked to point to a

picture that matched the fractional word (i.e., “one-half,” “one-third,” “one-fourth,” and “one-quarter”) said by the experimenter. On other items, students were shown a single picture and asked to choose the fractional word it represented (see Fig. 1). *Fraction notation* items (3 items) involved mapping symbolic unit fractions ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$) to visual representations. Students were asked to point to a picture that matched a specific fraction.

The assessment took approximately 15 minutes to administer. Internal consistency for the analytic sample in fall was $\alpha = 0.84$ and $\alpha = 0.87$ in spring. Fig. 1 displays internal reliabilities for each subscale and example items.

2.3.2. Whole number knowledge

We administered the first-grade form of the *Screener for Early Number Sense* (SENS) as a measure of whole number knowledge (Jordan et al., 2026). Children were given the number and number relations subtests (17 items), covering symbolic competencies such as counting, magnitude comparison, and identification. Performance was assessed by the number of items answered correctly. The SENS took approximately 9 minutes to administer, and internal consistency for the analytic sample was $\alpha = 0.88$.

We also measured children's whole number estimation on a number line, following the procedure described in Viegut et al. (2023). Each child's performance was scored by their average percent absolute error (PAE), with lower PAE indicating more accurate estimates. This measure was included in the regression analyses for the purposes of replicating our previous paper, but we did not include it in the subsequent LPA analyses due to limited variability in children's performance and overlap with the more comprehensive SENS measure of whole number knowledge.

2.3.3. Math achievement

We administered the Numeration subtest of the *KeyMath-3 Diagnostic Assessment* (Connolly, 2007), a multi-grade standardized

Component	Question	Stimulus	Answer Choices	Internal Reliability
Equal Sharing	“Two hungry bunnies want to share these 6 carrot sticks so that they each get the same amount. How much should each bunny get? This much (point to choice #1), this much (point to choice #2), or this much (point to choice #3)?”			Fall: $\alpha = .66$ Spring: $\alpha = .71$
Equivalence	“Which of these rectangles (point to options) has the same amount shaded as this rectangle (point to stimulus)?”			Fall: $\alpha = .83$ Spring: $\alpha = .85$
Adding Equal Parts	“This monster wants to use 2 pieces that are the same size to build a road between the house (point) and the school (point). Should the monster use two of these pieces (point to choice #1), two of these (point to choice #2), two of these (point to choice #3), or two of these (point to choice #4)?”			Fall: $\alpha = .79$ Spring: $\alpha = .74$
Word Mapping	“Is this ghost more than half, less than half, or half of the length from 0 (point to 0) to 1 (point to 1)?”		“More than half” “Less than half” “Half”	Fall: $\alpha = .66$ Spring: $\alpha = .74$
Fraction Notation	“Which picture (circle choices) shows this much (point to stimulus)?”	$\frac{1}{2}$		Fall: $\alpha = .59$ Spring: $\alpha = .64$

Fig. 1. Examples of fraction subgroup items, including equal sharing, equivalence, adding equal parts, word mapping, and fraction notation.

measure of broader math achievement. The Numeration subtest covers increasingly more difficult items on number quantity, ordering, and place value of whole and rational numbers. Internal consistency from the norming sample of first-grade children is $\alpha = 0.86$. Total assessment times varied, as each participant started with the first item and continued until they made four consecutive errors. Performance was measured by converting total correct responses into normed scale scores relative to a national sample of students. The average national scale score is $M = 10$ ($SD = 3$).

2.3.4. General cognitive skills

We measured executive function (which included a composite score of inhibitory control and cognitive flexibility) and visual-spatial working memory. We also measured two spatial reasoning skills – spatial scaling and spatial proportional reasoning – and receptive vocabulary. Tasks took approximately 30 minutes combined to administer (Fig. 2).

2.3.4.1. Executive function. We used a digital touch-screen version of the Hearts & Flowers task (Davidson et al., 2006) to assess executive function. The task was administered in three blocks; in the first two blocks, children pressed on the left or right side of the screen depending on a specific rule based on the stimuli they saw (i.e., press the same side for hearts, press the opposite side for flowers). The third block required students to remember and apply the previous blocks' rules. Because the students in this sample had high accuracy on all blocks, including the challenging third block ($M = 95.1\%$), reaction time is the most appropriate measure of executive function (Camerota et al., 2019). Subsequently, we calculated a change in response time on correct trials from the control block to the second and third blocks (which measure inhibitory control and cognitive flexibility, respectively), and added the standardized scores from both blocks together (these scores were significantly correlated, $r = 0.496$, $p < .001$). Internal consistency in the analytic sample was $\alpha = 0.76$.

2.3.4.2. Visual-spatial working memory. We used a touch-screen app version of the Corsi block test, PathSpan, to assess visual-spatial working memory (LeFevre et al., 2010). Children were instructed to copy the pattern of a sequence of lighted dots in forward and reverse orders. We measured performance as a proportion of correctly matched sequences averaged across both forward and reverse span tasks. Internal consistency in the analytic sample was $\alpha = 0.74$.

2.3.4.3. Spatial scaling. We used a touch-screen adaptation of the spatial scaling task from Frick and Newcombe (2012) to assess the ability to map distances in one space to distances in another digital, referent space. Children located a hidden egg by estimating its placement, using physical maps of various scales and shapes as a guide. For each of the 24 trials, responses were scored as 0 (worst), 1, 2, or 3 points (best), depending on the absolute deviation of the child's response from the correct position. Task performance was measured as a percentage of correct points out of 72. Internal consistency for the analytic sample was $\alpha = 0.64$.

2.3.4.4. Spatial proportional reasoning. We used a digital adaptation of the proportional reasoning task (Möhrling et al., 2018) to assess acuity in estimating proportions with continuous representations. Children estimated the proportion of juice in various cups. We measured performance as the average PAE between children's estimates and the target proportion across all trials, with a lower PAE indicating higher accuracy. Internal consistency in the analytic sample was $\alpha = 0.78$.

2.3.4.5. Receptive vocabulary. We used the Picture Vocabulary test from the NIH Toolbox for Assessment of Neurological and Behavioral Function to assess receptive vocabulary (Slotkin et al., 2012). Children selected one of the four presented images that best matched the single word played out loud from the iPad. Test-retest reliability based on the test's norming sample was 0.84.

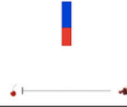
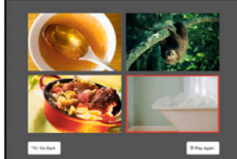
Cognitive Skill	Task	Description of Task	Example of Visual Stimuli
Cognitive Flexibility and Inhibitory Control	Hearts & Flowers (Davidson et al., 2006)	Children had to press on the left or right side of the screen depending on rule and stimuli. In the third block, students had to remember and apply the previous block's rules.	
Visual-spatial Working Memory	Corsi Block test (LeFevre et al., 2010)	Children copied a pattern of lighted dots in forward and reverse order.	
Spatial Scaling	Egg Hunt (Frick & Newcombe, 2012)	Children had to locate a hidden egg by estimating where it is on a map.	
Spatial Proportional Reasoning	Harry's Juice (Möhrling et al., 2018)	Children estimated the proportion of juice in various cups.	
Receptive Vocabulary	Picture Vocabulary Test (Slotkin et al., 2012)	Children selected which image best matched spoken word.	

Fig. 2. Examples of general cognitive skills measures assessing cognitive flexibility and inhibitory control, visual-spatial working memory, spatial scaling, spatial proportional reasoning, and receptive vocabulary.

2.4. Procedure

Each child participated in three 30-minute sessions that were conducted one-on-one with a trained researcher. Sessions were distributed across the school year, allowing us to track early fraction knowledge from the fall (Session 1) to the spring (Session 3). Session 1 took place in the fall when students completed the assessment of early fractions and the SENS as well as the whole number line estimation task. Session 2 occurred in the winter, during which children completed the general cognitive assessments. Session 3 took place in the spring, when children were once again given the assessment of early fractions task and additionally assessed on math achievement. For each time point, data collection within each school took place in one to three weeks.

2.5. Data analytic plan

Prior to conducting the LPA, we examined whether our prior findings held with a more diverse sample. We conducted two multiple regressions predicting spring fraction knowledge and general math knowledge (KeyMath-3) by students' fall informal fraction knowledge, SENS scores, and cognitive skills. All variables were approximately normally distributed except whole number estimation. We adjusted the positive skew using log transformation in the regression analyses (e.g., Bailey et al., 2012). All other assumptions of regression were met. Residuals were homoscedastic and there were no problems with multicollinearity (VIF for all variables was below 3). The combined sample (Cohorts 1 and 2) was used in subsequent analyses.

We then used latent profile analysis (LPA) to identify distinct profiles of early fraction understandings prior to formal fraction instruction. We formed the latent construct of *early fraction knowledge* using the five skills measured in our assessment of early fractions (Viegut et al., 2023) as indicators. Performance on each of these skills was operationalized as the proportion of problems students solved correctly (scale: 0–1).

We used the standard approach to LPA modeling by first defining a two-profile solution and then increasing the number of profiles by one until a well-fitting and theoretically meaningful set of profiles emerged (Nylund et al., 2007). We determined comparative model fit by examining a range of fit indices. These included the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC), and the sample-size adjusted BIC (aBIC). Smaller AIC, BIC, and aBIC indicate better model fit. We considered a variety of likelihood ratio tests (i.e., the Lo-Mendell-Rubin Likelihood Ratio Test (LMR-LRT), the Vuong-Lo-Mendell-Rubin Likelihood Ratio Test (VLMR-LRT), and the bootstrap likelihood ratio test (BLRT); Lo et al., 2001; McLachlan & Peel, 2000). These tests assess whether the number of profiles in the current solution better fit the data than the prior more parsimonious model (Nylund et al., 2007). We also evaluated the model's entropy, or classification quality, considering an entropy of 0.80 or more as good classification accuracy (Geiser, 2013). We considered fit indices along with conceptual interpretation of profiles as well as whether there were a meaningful number of participants in each profile (i.e., at least 5 % or 25 participants; Weller et al., 2020). We examined the conditional probabilities to ensure that participants had a high probability (>90 %) of being in one profile and a low probability (<10 %) of being in other profiles (Geiser, 2013; Weller et al., 2020). Although all fit indices and ratio tests were considered, we put particular emphasis on the aBIC and the BLRT test, both considered some of the most accurate indicators (Nylund et al., 2007; Spurk et al., 2020) as well as the conceptual viability and interpretability of the profiles, the latter being an important consideration for the proposed subsequent analyses.

To identify the extent to which profile membership distinguishes between children's numerical and cognitive skills, end-of-year fraction knowledge, and math performance, we used the Bolck-Croon-Hagenaars method (BCH; Asparouhov & Muthén, 2021; Bolck et al., 2004). The BCH method allows for the comparison of profiles on continuous outcome measures (e.g., cognitive skills; Asparouhov & Muthén, 2021).

The BCH method uses a weighted multiple group analysis within which groups correspond to the original latent classes. We used raw scores of all cognitive measures, with the exception of the executive function measure; for this variable, we used z-score values for the reported analyses for ease of interpretation.

2.5.1. Data missingness

There were four students (2 %) missing data from Sessions 1 and 2 and three students (1.5 %) missing data from Session 3. Given the low missingness rate, we used listwise deletion to account for missing data in the regression analyses (as in our prior study) and full information maximum likelihood (FIML) in the LPA and BCH analyses. Mplus uses FIML as a default estimator for missing data. FIML uses all available information, including auxiliary variables, to estimate parameters that are minimally biased and is a gold standard approach for handling data (Acock, 2012; Lee & Shi, 2021). Imputation is not required for low levels of missingness (Acock, 2012).

3. Results

We report descriptive statistics from our previous study (Cohort 1), the additional students recruited to obtain a larger and more racially and socioeconomically diverse sample (Cohort 2) and the combined sample (Cohort 1 + Cohort 2) in Supplemental File A. All the key relations among variables found in Viegut et al. (2023) held in Cohort 2 alone and in the combined sample. First, although Cohort 2 showed lower average informal fraction knowledge than Cohort 1, children in Cohort 2 showed significant growth in informal fraction knowledge from fall ($M = 50.4\%$, $SD = 16.6$) to spring ($M = 59.8\%$, $SD = 19.5$), $t(97) = 9.05$, $p < .001$. The combined sample's informal fraction knowledge also improved from fall ($M = 54.7\%$, $SD = 16.4$) to spring ($M = 63.7\%$, $SD = 17.3$), $t(199) = 12.4$, $p < .001$, even though fractions are not a focus of instruction. Second, the regression results were similar, as shown in Table 1. Shapiro-Wilk tests of normality showed that the residuals were approximately normally distributed for all regression tests. Fall whole number knowledge, as measured by the SENS, significantly predicted spring informal fraction knowledge when controlling for fall fraction knowledge and other cognitive skills in Cohort 2 alone ($\beta = 0.22$, $p = .010$) and in the combined sample ($\beta = 0.25$, $p < .001$). Early fraction knowledge in fall of first grade explained significant variance in spring math achievement when accounting for fall whole number knowledge and cognitive skills in both Cohort 2 alone ($\beta = 0.29$, $p < .001$) and in the combined sample ($\beta = 0.23$, $p < .001$). Lastly, fall whole number knowledge also significantly predicted spring math achievement in Cohort 2 alone ($\beta = 0.38$, $p < .001$) and in the combined sample ($\beta = 0.52$, $p < .001$). All betas are standardized.

Some new findings also appeared. In the previous analyses with only Cohort 1 (Viegut et al., 2023), none of the general cognitive skills were significantly related to spring fraction knowledge or math achievement. In the Cohort 2 replication, spring fraction knowledge was significantly predicted by vocabulary ($\beta = 0.17$, $p = .003$), and spring math achievement was predicted by visual-spatial working memory ($\beta = 0.30$, $p < .001$) and negatively predicted by spatial scaling ($\beta = -0.12$, $p = .035$). In our combined sample, spring fraction knowledge was still significantly predicted by vocabulary ($\beta = 0.11$, $p = .010$) and also by spatial proportional reasoning ($\beta = -0.13$, $p = .002$), and spring math achievement was predicted by visual-spatial working memory ($\beta = 0.14$, $p = .004$), though these standardized beta weights show small effect sizes and are comparatively weaker than the mathematics predictors. It is important to note that Cohort 2 had significantly lower vocabulary and spatial scaling scores and were equivalent on the EF measure and visual-spatial working memory (see Supplemental File A). Although there were some inconsistent patterns across the cohorts, the main findings regarding the predictive power of whole number knowledge and informal fraction knowledge held across two samples with differing educational and demographic characteristics. We use the combined

Table 1

Regression predicting spring fraction knowledge and math achievement scores.

Predictor	Early Fraction Knowledge (Spring)			Math Achievement (Spring)		
	Cohort 1 ^a	Cohort 2	Combined	Cohort 1 ^a	Cohort 2	Combined
	N = 103 β (SE)	N = 97 β (SE)	N = 200 β (SE)	N = 103 β (SE)	N = 97 β (SE)	N = 200 β (SE)
SENS	0.211* (0.056)	0.217** (0.054)	0.248*** (0.040)	0.621*** (0.011)	0.377*** (0.012)	0.512*** (0.008)
WNL	−0.064 (2.206)	−0.117 (2.173)	−0.080 (1.596)	−0.118 (0.436)	−0.013 (0.473)	−0.078 (0.329)
Frac (fall)	0.631*** (0.080)	0.431*** (0.087)	0.514*** (0.060)	0.185* (0.012)	0.292*** (0.019)	0.221*** (0.013)
EF	−0.106 (0.524)	−0.022 (0.481)	−0.046 (0.361)	−0.085 (0.104)	0.022 (0.105)	−0.038 (0.076)
VSWM	−0.105 (0.455)	0.099 (0.532)	−0.001 (0.352)	0.008 (0.090)	0.295*** (0.116)	0.138** (0.074)
Scaling	−0.072 (0.082)	−0.002 (0.082)	−0.042 (0.059)	0.033 (0.016)	−0.122* (0.018)	−0.053 (0.012)
PR ^b	−0.086 (0.163)	−0.086 (0.108)	−0.125** (0.090)	−0.003 (0.032)	−0.041 (0.023)	−0.026 (0.019)
Vocabulary	−0.006 (0.035)	0.166** (0.035)	0.109* (0.025)	0.010 (0.007)	0.086 (0.008)	0.060 (0.005)
R ²	64.3 %	82.8 %	73.5 %	74.4 %	77.2 %	73.0 %

Note. SENS = Screener for Early Number Sense. WNL = Whole Number Line Estimation. Frac = Early Fraction Knowledge. EF = Executive Functioning. VSWM = Visual-Spatial Working Memory. Scaling = Spatial Scaling. PR = Spatial Proportional Reasoning. All betas are standardized. Participant numbers refer to the sample with complete data (i.e., after listwise deletion).

^a Results slightly differ from those reported in Viegut et al. (2023). In prior analyses, we controlled for age and gender; however, we did not in the analyses reported in this study. The patterns found in Viegut et al. (2023) still hold.

^b Lower score indicates higher accuracy.

* $p < .05$.

** $p < .01$.

*** $p < .001$.

sample in subsequent analyses.

3.1. Latent profiles of early fraction knowledge

3.1.1. Identification of a latent profile structure

We fit four models (with 2, 3, 4, and 5 profiles) before encountering a non-significant LRT which indicated that a 5 profile solution was no better than a 4 profile solution (see Table 2 for fit statistics). We

determined that the 5-profile solution was not the best fitting model, as it had suboptimal fit indices and two profiles smaller than our threshold of 25 students per profile (Weller et al., 2020). The 4-profile model also had one profile below this threshold, making it a suboptimal solution. However, some fit indices supported this solution; we report it in supplemental analyses (Supplemental File B).

We interpreted the 3-profile solution based on optimal fit indices and qualitatively distinct profiles when compared to the 2-profile solution.

Table 2

Model comparisons of 2-, 3-, 4, and 5-profile models at fall.

Model and Profile	Count based on most likely latent class	Proportion based on most likely latent class	Entropy	AIC/BIC	aBIC	BLRT	LMR-LRT	VLMR-LRT
Two Profile			0.929	70.538/ 123.628	72.935	−133.418**	221.360**	−131.418**
1	136	0.67						
2	68	0.33						
Three Profile			0.914	28.831/ 101.830	32.127	−19.269**	52.075*	−19.269*
1	108	0.53						
2	43	0.21						
3	53	0.26						
Four Profile			0.966	−70.231/ 22.676	−66.036	7.585**	107.687**	7.585**
1	48	0.24						
2	90	0.44						
3	48	0.24						
4	18	0.09						
Five Profile			0.895	−83.619/ 29.197	−78.525	63.116**	24.617	63.116
1	18	0.09						
2	48	0.23						
3	48	0.23						
4	55	0.27						
5	35	0.17						

Note. AIC = Akaike information criterion; (a) BIC = (Adjusted) Bayesian information criterion; BLRT = parametric bootstrapped likelihood ratio test; LMR-LRT = Lo-Mendell-Rubin adjusted likelihood ratio test; VLMR-LRT = Vuong-Lo-Mendell-Rubin likelihood ratio test.

* $p < .05$.

** $p < .001$.

That is, the 2-profile solution yielded a “high performing” and “low performing” profile, masking some of the heterogeneity found in the 3-profile solution. The 3-profile solution yielded one profile of students with generally lower scores than students in the other two profiles who were higher performing but in varying ways. Thus, the 3-profile solution did not simply yield a low, medium, and high performing group. Fig. 3 and Table 3 display the 3-profile solution and the classification probabilities.

3.1.2. Characterizing the three latent profiles

Among all students, performance did not differ in understanding of equal sharing, word mapping, or fraction notation. However, we saw variation in students' performance on two skills across all three profiles: fraction equivalence and adding equal parts. We therefore used performance on these two tasks to inform our naming and characterization of the profiles.

Across the five indicators of informal fraction knowledge, children displaying Profile 1 (P1; $n \sim 108$, 59.2 %) demonstrated a range of understanding, from high (equal sharing, equivalence, adding equal parts) to moderate (word mapping) to at chance (fraction notation). This profile suggests that, although some children have broad informal fraction knowledge in first grade, there is variability in how they understand different aspects of informal fraction knowledge. The finding supports our choice of LPA for our data analytic approach. Because of the comparatively strong performance on both fraction equivalence and adding equal parts measures, we labeled P1 as *Strong Demonstrated Understanding*. This profile was the largest in the current sample.

Children in Profile 2 (P2; $n \sim 43$, 21.1 %) had a similar performance to children in P1 on equal sharing, adding equal parts, word mapping, and fraction notation, suggesting some consistency in relative

performance. However, there was a stark contrast between P1 and P2's performance on equivalence items, with children in P1 scoring extremely high and children in P2 performing below chance (<33 %). We therefore called this profile *Partially Demonstrated Understanding* because of the weaker demonstrated understanding of equivalence compared to P1.

The key characteristic of children in Profile 3 (P3; $n \sim 53$, 26.0 %) is that they performed below chance on equivalence and at chance on adding equal parts. Recall that children in P1 performed high on both sets of items, and children in P2 scored low on equivalence and high on adding equal parts. We named P3 *Limited Demonstrated Understanding* because of the comparatively weaker performance on equivalence and adding equal parts measures. Children in P3 performed similarly to children in P1 and P2 on the other indicators of fraction knowledge (equal sharing, word mapping, and fraction notation). Taken together, these results highlight the heterogeneity in patterns of informal fraction knowledge.

3.2. Relations between early fraction understanding and numerical and cognitive skills

Differences in whole number knowledge and general cognitive skills by profile are in Table 4. Here we highlight key profile comparisons. Students characterized by P1 and P2 both showed higher concurrent start-of-year performance on the SENS, receptive vocabulary, visual-spatial working memory, and spatial proportional reasoning and both outperformed P3 on these measures. As these cognitive skills differentiated P1 and P2 from P3, which also differed in students' understanding of equal sharing and adding equal parts, it is possible that they are associated with developing these particular components of fraction

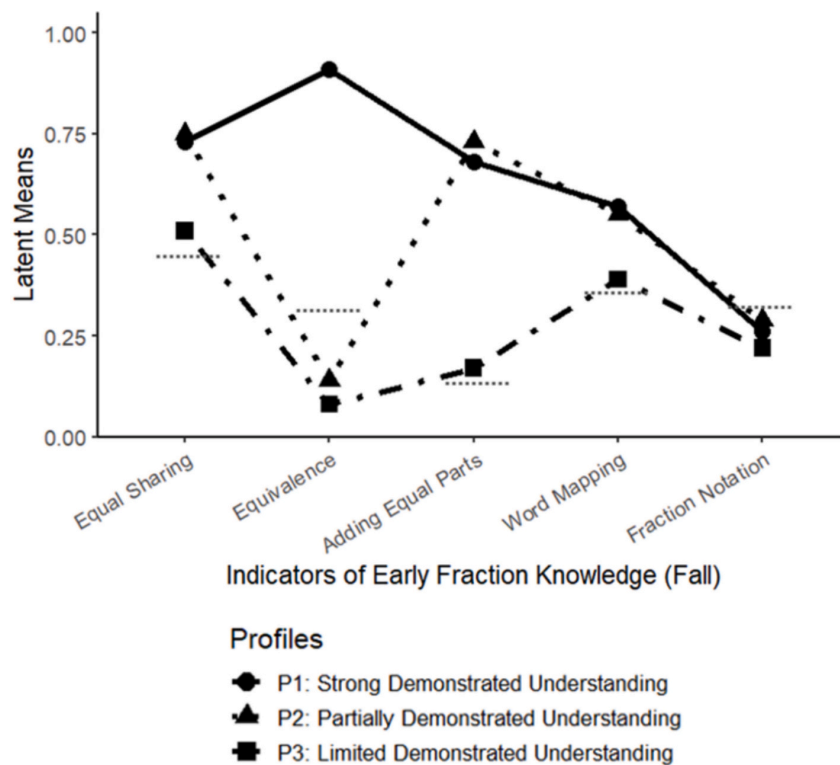


Fig. 3. 3-Profile solution of early fractions knowledge at fall.

Note. P = Profile. Profiles are named based on the two skills where there are significantly different variations: equivalence and adding equal parts. P1: Strong Demonstrated Understanding ($n \sim 108$, 52.9 %), P2: Partially Demonstrated Understanding ($n \sim 43$, 21.1 %), and P3: Limited Demonstrated Understanding ($n \sim 53$, 26.0 %). Dotted horizontal lines indicate chance performance for each subscale.

Table 3
Means and classification probabilities for 3-profile solution at fall.

Indicator Value	Profile		
	P1: Strong Demonstrated Understanding (<i>n</i> ~ 108)	P2: Partially Demonstrated Understanding (<i>n</i> ~ 43)	P3: Limited Demonstrated Understanding (<i>n</i> ~ 53)
	<i>M</i> (<i>SE</i>)	<i>M</i> (<i>SE</i>)	<i>M</i> (<i>SE</i>)
Equal sharing	0.73 (0.02) ^a	0.75 (0.02) ^a	0.51 (0.03) ^b
Equivalence	0.91 (0.01) ^a	0.14 (0.03) ^b	0.08 (0.03) ^b
Adding equal parts	0.68 (0.03) ^a	0.73 (0.04) ^a	0.17 (0.04) ^b
Word mapping	0.57 (0.02) ^a	0.55 (0.03) ^a	0.39 (0.02) ^b
Fraction notation	0.26 (0.03) ^a	0.29 (0.06) ^a	0.22 (0.05) ^a

Profile	Classification Probabilities		
P1	0.999	0.001	0.000
P2	0.002	0.910	0.088
P3	0.007	0.055	0.938

Note. Means are proportionally correct. Different superscript letters indicate significant differences ($p < .05$) between profiles among components of early fraction knowledge, as calculated using an ANOVA with Tukey's HSD. *M* = Mean, *SE* = Standard Error; P = Profile. *N* = 204 students. The subscripts ^a and ^b indicate significant differences; means with different subscripts are significantly different from one another. For example, the latent values of equal sharing for P1 and P3 and for P2 and P3 are significantly different from one another; the latent values for P1 and P2 are not significantly different from one another.

understanding. Students characterized by P1 demonstrated stronger performance on the spatial scaling task than students in P3. There was no difference between students in P2 and either of the other two profiles

on this measure. There was no significant difference between the three profiles in performance on executive functioning.

3.3. Predicting end-of-year math performance and fractions knowledge

Table 4 presents differences in end-of-year math performance and fraction knowledge by profile. P1 and P2 performed similarly on the end-of-year math achievement measure and overall fraction measure (i. e., composite score) and outperformed P3 (see Table 4).

3.3.1. Exploratory post hoc analyses: examining early fraction knowledge at spring

Although our primary research interests were examining the relation between fall early fraction knowledge and overall spring fraction knowledge as well as math performance, we also were interested in how these spring scores differed by fall profile. That is, did patterns of spring fraction understanding look different depending on students' most likely fall profile membership? We were particularly interested in children's spring performance on equivalence items, as this component was one of the differentiating indicators in the fall LPA analysis.

3.3.1.1. BCH analysis of spring early fraction knowledge by fall profile membership. We did not have the power to examine the relationship between fall and spring performance using latent transition analysis; this type of longitudinal analysis requires a larger sample size of at least 300 students (e.g., Nylund-Gibson et al., 2023). Instead, we conducted a BCH analysis with fall profile membership as the predictor and the five components of spring early fraction knowledge as the outcomes to examine if the three profiles differentially improved on specific components (see Fig. 4), an approach used in prior research (e.g., Vanluydt et al., 2022). Descriptively, performance on equivalence measures appeared to improve from fall to spring for students in P2 ($M_{Fall} = 0.138$, $M_{Spring} = 0.709$) and P3 ($M_{Fall} = 0.080$, $M_{Spring} = 0.288$) with P2 means suggesting more improvement between fall and spring than P3. Children in P1 appeared to perform the same on this measure in fall ($M_{Fall} =$

Table 4
Mean differences in students' whole number knowledge, general cognitive skills, and later math knowledge in relation to early fraction knowledge by profile.

Profiles	Whole Number Knowledge	General Cognitive Skills					Math Knowledge (Spring)	
	SENS	Executive Function	Picture Vocabulary	Visual-Spatial Working Memory	Spatial Scaling	Spatial Proportional Reasoning ^a	Math Achievement ^b	Spring Early Fractions Knowledge
	<i>M</i> (<i>SE</i>)	<i>M</i> (<i>SE</i>)	<i>M</i> (<i>SE</i>)	<i>M</i> (<i>SE</i>)	<i>M</i> (<i>SE</i>)	<i>M</i> (<i>SE</i>)	<i>M</i> (<i>SE</i>)	<i>M</i> (<i>SE</i>)
P1 (Strong Demonstrated Understanding, <i>n</i> ~ 108)	76.98 (2.05)	729.82 (34.22)	64.67 (2.84)	4.42 (0.20)	59.00 (1.09)	21.14 (0.78)	12.18 (0.31)	70.99 (1.33)
P2 (Partially Demonstrated Understanding, <i>n</i> ~ 43)	79.29 (3.38)	648.44 (60.72)	60.07 (4.75)	4.98 (0.33)	56.23 (1.69)	21.22 (1.20)	12.83 (0.47)	69.91 (2.07)
P3 (Limited Demonstrated Understanding, <i>n</i> ~ 53)	37.32 (3.80)	731.26 (82.69)	37.36 (4.29)	2.52 (0.29)	50.91 (1.94)	26.72 (1.18)	7.52 (0.49)	42.50 (2.04)

Comparisons of Profile Means	χ^2	χ^2	χ^2	χ^2	χ^2	χ^2	χ^2	χ^2
P1 v P2	0.34	0.57	0.69	2.05	1.90	0.003	1.34	0.20
P1 v P3	61.24**	0.00	11.02*	27.83**	13.15**	15.49**	65.06**	81.05**
P2 v P3	83.88**	1.36	28.06**	29.28**	3.73	9.33*	54.83**	137.05**

Note. *M* = Mean, *SE* = Standard Error, P = Profile. Bolded numbers are statistically significant chi-square tests from BCH analyses.

^a Lower score indicates higher accuracy.

^b Standard score with *M* = 10, *SD* = 3.

* $p < .01$.

** $p < .001$.

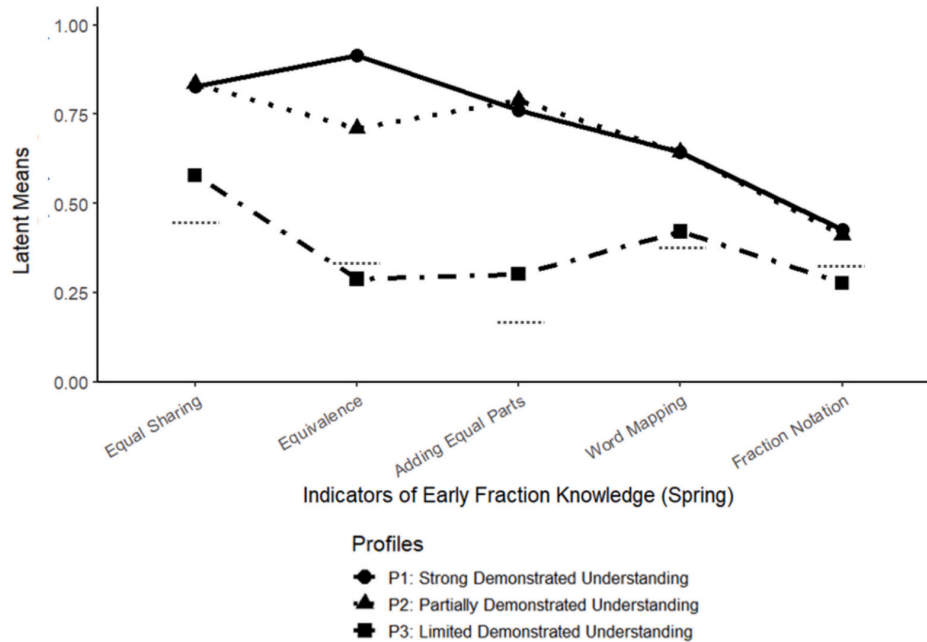


Fig. 4. Mean performance on spring fraction knowledge components by fall profile membership.

Note. P = Profile. All means are latent means for correctness on spring fraction knowledge components estimated through BCH analyses. Latent means displayed are not a result of LPA on Spring indicators. Profiles 1, 2, and 3 performed significantly differently on the equivalence indicator in spring ($p < .01$). Latent means for equivalence performance and means and significance tests for other indicators are reported in Supplemental File C.

0.914) and spring ($M_{\text{Spring}} = 0.915$), though, notably, this score was nearly at ceiling in the fall. Based on the BCH analysis, although P2 and P3 showed equally low performance on equivalence items in the fall, they significantly differed from each other in the spring ($M_{\text{Profile 2}} = 0.709$; $M_{\text{Profile 3}} = 0.288$, $p < .001$). Specifically, children who displayed P2 in the fall showed much higher accuracy on equivalence than P3, who continued to perform near chance on equivalence items (e.g., 33 % accuracy) in the spring. P2 approached the high accuracy of children in P1 on equivalence items ($M_{\text{Profile 1}} = 0.915$). However, children in P1 still outperformed those P2 on equivalence items ($p = .004$). Full results of this BCH analysis are included in Supplemental File C.

3.3.1.2. Fraction equivalence errors. Because equivalence was both one of the most important indicators of fall profile membership and the fraction component for which performance patterns across profiles seemed to change most from fall to spring, we wanted to understand more about children's responses on these items. Notably, that children in both P2 and P3 performed *below* chance on equivalence items in the fall raises the possibility of systematic rather than random error. These children may be consistently making a whole number bias error by using whole number properties, such as counting, to determine equivalence (Ni & Zhou, 2005). Whole number bias is most commonly evident when children begin to learn fractions, although it persists throughout learning (Alibali & Sidney, 2015; Braithwaite & Siegler, 2018).

To explore this issue, we conducted an error analysis of children's performance on the equivalence items. We coded children's errors on these items as *whole number bias errors* or *other errors*. Recall that on equivalence items, children were asked to choose which of three rectangles had an equal proportion shaded as the target rectangle (e.g., one out of three parts shaded). One answer choice was correct (e.g., two out of six parts shaded), one answer choice matched the absolute number of parts shaded (e.g., one out of six parts shaded), and one answer choice showed some other fraction (e.g., two out of three parts shaded). Children who selected the choice that matched the absolute number of parts shaded, ignoring the proportional size of the part, were coded as making

a *whole number bias error*. We coded responses selecting the other incorrect answer choice as *other error*.

Fig. 5 displays results of the error analysis. Children in P2 and P3 did not significantly differ in the proportion of their errors that were whole number bias errors in the fall ($\chi^2 = 0.005$, $p = .945$) or spring ($\chi^2 = 0.050$, $p = .823$). Both P2 and (to a lesser extent) P3 increased in correctness on equivalence items overall, resulting in a lower number (but similar proportion) of whole number bias errors.

4. Discussion

We uncovered three profiles of informal fraction knowledge at the start of first grade. Children in Profile 1 (*Strong Demonstrated Understanding*) showed strong nonsymbolic fraction knowledge compared to the other two profiles. Children in Profile 2 (*Partially Demonstrated Understanding*) showed equivalent understanding except for relatively weaker fraction equivalence understanding. Children in Profile 3 (*Limited Demonstrated Understanding*) showed comparatively weaker knowledge. Note that these three profiles varied on level of understanding of nonsymbolic fraction knowledge, because, as expected, all children demonstrated minimal symbolic fraction knowledge, reflecting limited exposure to these more formal skills (e.g., Jordan et al., 2022). Formal fraction instruction typically begins in third grade or later in the US (Common Core State Standards Initiative, 2024). Notably, the curricula in participating schools did not include formal fractions instruction during the study period.

4.1. Understanding variability across profiles

Aside from a significant difference in initial understanding of fraction equivalence, P1 and P2 largely looked the same, which did not confirm our hypothesis of durable differing patterns of informal fraction knowledge. Instead, we found an interesting pattern: although P1 and P2 initially differed in their understanding of fraction equivalence, students in both profiles demonstrated similar cognitive skills and end-

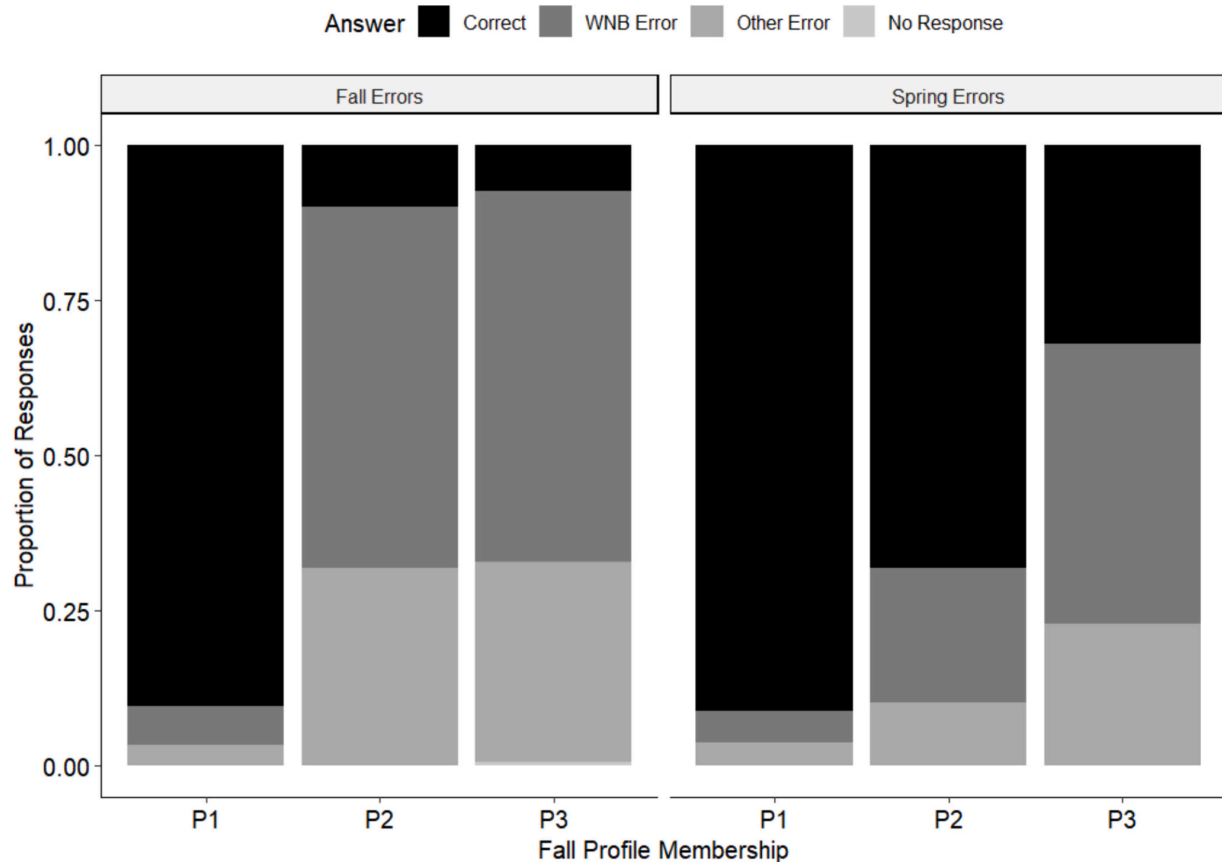


Fig. 5. Error analysis of students' responses on equivalence problems.

Note. Figure represents students' responses on all three equivalence questions. P = Profile. WNB Error = Whole Number Bias Error.

of-year math performance. These findings suggest that knowledge of fraction equivalence might develop later for some children, aligning with the number sense framework, which suggests that different formal and informal experiences may facilitate the development of different aspects of number knowledge (Jordan et al., 2022). Although students in P2 at the start of first grade were characterized by lower levels of fraction equivalence understanding, they appeared to develop this skill over the course of the year. Future longitudinal research using multiple data points over first-grade could clarify this learning trajectory.

Students in both P1 and P2 excelled at adding equal parts, suggesting these are foundational components of early fraction knowledge that may be related to later mathematical performance. Both profiles also showed stronger visual-spatial working memory and spatial proportional reasoning skills than P3 (in addition to other cognitive skills). Iterating fractional parts requires children to mentally hold, transform, and manipulate fractional pieces to visualize a whole. Perhaps then the comparatively stronger performance of P1 and P2 on adding equal parts can be in part explained by stronger visual-spatial skills. More research is needed to explore this relation.

That equivalence understanding in early first grade, as assessed on our measure, may not be as related to cognitive skills and later fraction knowledge was interesting. Understanding fraction equivalence requires knowledge of the proportional relation between the absolute number of parts shaded and the proportion of shaded parts relative to the denominator. It is important to note that our equivalence tasks used discrete markings (e.g., to show two fourths vs. one half of a rectangle), which could prompt answers based on counting. Exploratory analyses, though, revealed that this improvement was not a product of fewer

misconceptions, such as those related to whole number counting bias. Perhaps children in P2, who showed an advantage over P3 in the fall in all components except equivalence, leveraged their other informal fraction knowledge to think about nonsymbolic fraction equivalence and reduce systematic whole number bias errors as well as other errors by the end of the school year.

These differing profiles of fraction knowledge may be related to differential mathematical input. Students have varied experiences in mathematics from their home environment (e.g., Daucourt et al., 2021) and from their lived experiences more broadly (e.g., Turner et al., 2009). Teachers may provide fraction-related instruction in different components of informal fraction knowledge even though they are not explicitly included in the curriculum (e.g., instruction during geometry and measurement). The findings align with an asset-based perspective, which posits that there are different ways of knowing and doing mathematics that students learn from schooling, from their home life, and from their interactions with the world around them (Celedón-Pattichis et al., 2018; Civil, 2016; Swirbul & Melzi, 2024).

This perspective is important when considering P3. Fifty-three students in our sample (26 %) were characterized by comparatively low nonsymbolic fraction skills, which predicted lower fractions and general mathematics performance at the end of the year. However, P3 showed relative strength in equal sharing compared to their performance on other measures, with accuracy above chance in fall and spring. Because nonsymbolic equal sharing was a strength for students across all three profiles, it may serve as an effective foundation for early fraction instruction, even for those with relatively limited informal fraction knowledge. Supporting this idea, an intervention in a first-grade

classroom that promoted equal sharing led to improvements in recognizing fraction representations (Empson, 1999). Because of children's everyday experiences with equal sharing, experts have recommended using equal sharing activities to support beginning fraction learning (Empson & Levi, 2011; Siegler et al., 2010). For example, children might use their equal sharing knowledge to understand unit fractions or the idea that equal sharing with more children results in smaller pieces, that is, one half of a piece is more than one fourth of the same piece. Our findings offer empirical support to these recommendations by showing that most students have relative strengths in equal sharing, but they also suggest that students may benefit from building understanding of all informal fraction concepts, especially adding equal parts and equivalence.

4.2. Relations of profiles to cognitive skills and later math performance

Profiles differed in their whole number knowledge, with P1 and P2 outperforming P3 on number sense. Whole number knowledge provides a critical foundation related to learning fractions, including proportional reasoning (Szkudlarek & Brannon, 2021), fraction number line estimation (Yu et al., 2022), and general conceptual and procedural knowledge of fractions (Lin & Powell, 2021). However, it also could be the case that early fraction understanding helps foster whole number knowledge development, or that these skills co-develop (e.g., Bailey et al., 2017; Kahl et al., 2022; Zhang et al., 2023). Future research should work to disentangle this relationship.

P1 and P2 displayed stronger visual-spatial working memory than P3. P1 displayed stronger spatial scaling than P3. P1 and P2 also outperformed P3 on receptive vocabulary skills. Vocabulary and visual-spatial skills are related to children's understanding of mathematical concepts (e.g., Boyer & Levine, 2012; Jordan et al., 2013; LeFevre et al., 2010; Purpura et al., 2011). These findings suggest that multiple cognitive factors may relate to the development of early fraction understanding. Indeed, we see multiple factors (e.g., visual-spatial working memory, receptive vocabulary) appear related to greater math performance and fraction knowledge. As we do not have multiple data points measuring children's cognitive skills, examining the developmental trajectories of these factors, or whether they develop in conjunction with components of early fraction knowledge, is beyond the scope of our study. Future research should examine these relations through a multi-timepoint longitudinal study that measures each of these skills repeatedly across timepoint. However, our study provides correlational evidence that these cognitive skills are related to the development of different components of informal fraction understanding. We also note that it is also possible that these associations represent a broader pattern of overall achievement. That is, perhaps these students entered schooling with stronger foundational skills overall.

Surprisingly, we did not find a significant difference between levels of executive functioning (as measured by inhibitory control and cognitive flexibility tasks) by profile membership. This finding contrasts with prior literature, which shows that kindergartners and second graders with stronger inhibitory control perform better on nonsymbolic proportion judgments (Abreu-Mendoza et al., 2020; Park et al., 2021). These findings are partially because children can attend to the important components of the problems without getting distracted by conflicting whole number information (Emslander & Scherer, 2022). We may not have detected significant differences in students' executive functioning due to our fraction measures. Conflicting whole number information that may activate whole number bias is only present in some discrete items (for an example, see the equivalence question in Fig. 1). Executive function scholars have also begun to more critically evaluate the usefulness of decontextualized executive function measures that may be deficient in measuring executive function skills for students with varying cultural backgrounds (e.g., Miller-Cotto et al., 2022; Niebaum & Munakata, 2023).

4.3. Differences between Cohort 1 vs. Cohort 2

In the first step of this investigation, we added a larger sample to our previous work and generally replicated our prior findings (Viegut et al., 2023) with this new group. The main findings that whole number knowledge and informal fraction knowledge predicted later math knowledge held across both samples. Analyses with Cohort 2, however, revealed differential relations between cognitive skills and later math knowledge, findings that were not evident in Cohort 1. These inconsistent patterns across Cohort 1, Cohort 2, and the combined sample have several potential explanations. One possibility is that different groups of students may recruit different cognitive skills for building early fraction and math skills. Another possibility is that expanding the range at the lower end of the scale, especially in relation to vocabulary and spatial scaling findings, allowed us to detect relations not previously detectable in a more restricted range. It is also possible that lower levels of vocabulary and spatial scaling (as seen in Cohort 2) limit students' ability to achieve in mathematics but that levels in the mid to high range may be enough to support but not differentiate mathematics achievement.

4.4. Implications for instruction

Although exploratory, our findings illuminate potential avenues for personalizing instruction based on profile membership. For example, students who exhibit characteristics of P3 (i.e., relatively weak understanding of equivalence and adding equal parts) may benefit from targeted instruction in these areas to build their early fraction knowledge, whereas students who already have relatively strong understanding of adding equal parts (i.e., P1 and P2) may improve through typical math instruction. Given over half the sample already possessed knowledge of many aspects of fraction reasoning (P1), our findings also suggest that many children are ready for more challenging fraction instruction in first grade (e.g., understanding fraction notation). Toward this end, a preliminary randomized controlled study (Jordan et al., 2025) showed that first-graders' symbolic fraction skills can be developed in meaningful ways through targeted training activities.

We speculate that focusing on adding equal parts may be beneficial for helping children understand unit fractions and their symbols (e.g., $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1$ or one whole; see Braithwaite & Siegler, 2021; Jordan et al., 2024 for discussions of this concept with older students). Adding equal parts was a relative strength of both P1 and P2, both of which outperformed P3 on both the early fractions and overall math measures. Relatedly, instruction should leverage first graders' understanding of equal sharing to promote an understanding of fraction equivalence. For example, because our data suggest that children seem to understand how to divide a candy bar into equal amounts, instruction could leverage this to help students understand fraction equivalence by showing that one-half of a candy bar is the same as two-fourths of a candy bar.

4.5. Strengths, limitations, and future directions

The present study leverages the statistical advantages of LPA to examine individual differences in children's early fraction knowledge and how different patterns relate to cognitive and mathematical skills. Because LPA allows for heterogeneity of responses, we were able to look at subgroups within this broader population without imposing pre-ordained categories. This is the first study to our knowledge that used LPA to examine variability in students' early fraction knowledge and how this variability relates to later math skills. These findings add to the body of literature examining heterogeneity in student understanding of mathematics.

Despite its strengths, we acknowledge several limitations of the study. The early fraction knowledge assessment was designed to characterize first graders' informal fraction knowledge. Items were not

balanced among the subscales to ensure the assessment was brief enough for first graders and to avoid ceiling or floor effects (see Viegut et al., 2023 for further explanation), resulting in uneven subscales of informal fraction knowledge. Although we attempted to mitigate this issue by using the proportion of correctly solved problems on each subscale as indicators (ensuring indicators were on the same scale), these uneven subscales resulted in variability in how many items students could get correct on each subscale, impacting the variability of the indicator. Importantly, the standard errors of the subscales did not considerably vary, suggesting that this is a potential conceptual issue, rather than an issue leading to heterogeneous variances. Further, we remind readers that this is not a causal study and therefore causal claims cannot be made between fraction knowledge and cognitive and whole number skills.

Prior research has highlighted the importance of overall informal fraction knowledge (e.g., Viegut et al., 2023) but has yet to highlight specific components of informal knowledge that may lay the foundation for learning formal fraction concepts. The current findings suggest that early understanding of concepts may help children benefit from math instruction and lead to stronger math achievement. Should this relation be the case, bolstering student understanding of these concepts could promote later mathematical understanding. We suggest future experimental work that manipulates and intervenes on the potential leverage points studied to determine causality and support students' early fraction learning.

4.6. Conclusion

Students enter first grade with variable profiles of early fraction knowledge. These different profiles are related to whole number knowledge and cognitive skills as well as formal math understanding at the end of first grade. Specifically, our findings highlight the potential importance of students' understanding of adding equal parts and add nuance to the importance of nonsymbolic fraction equivalence knowledge. These findings provide grounds for future work to examine whether bolstering students' early and informal fraction understanding paves the way for formal fraction understanding and mathematics knowledge.

CRedit authorship contribution statement

Elena M. Silla: Writing – review & editing, Writing – original draft, Visualization, Investigation, Formal analysis, Conceptualization. **Alexandria A. Viegut:** Writing – review & editing, Writing – original draft, Formal analysis, Conceptualization. **Eva Redican:** Visualization, Project administration, Data curation. **Christina Areizaga Barbieri:** Writing – review & editing, Supervision, Funding acquisition, Data curation, Conceptualization. **Ilyse Resnick:** Writing – review & editing, Supervision, Conceptualization. **Nora S. Newcombe:** Writing – review & editing, Supervision, Funding acquisition, Conceptualization. **Nancy C. Jordan:** Writing – review & editing, Supervision, Funding acquisition.

Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work the author(s) used Chat GPT in order to proofread the manuscript and references for typographical errors. After using this tool/service, the author(s) reviewed and edited the content as needed and take(s) full responsibility for the content of the published article.

Appendix A. Supplementary data

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.lindif.2025.102866>.

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