



Contents lists available at ScienceDirect

Journal of Experimental Child Psychology

journal homepage: www.elsevier.com/locate/jecp

Early informal fraction knowledge matters: a longitudinal investigation between first and second grades

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ARTICLE INFO

Keywords:

Fractions

Numeracy

Mathematics achievement

Foundational mathematics skills

Number sense

ABSTRACT

Eighty children were studied from the start of first grade (mean age = 6.68, range 6.03–7.48) through to the end of second grade to characterize initial informal fraction knowledge and its relation to later informal fraction knowledge and mathematics achievement. Children's understanding of informal fractions grew across first and second grade, despite little formal fraction instruction. Children showed relatively higher performance in nonsymbolic fraction items compared with symbolic items. Gains in understanding the concept of 'half' characterized first grade, with higher gains in understanding "thirds" and "fourths" in second grade. There were large individual differences at all timepoints in informal fraction knowledge. Variation in informal fraction knowledge at the start of first grade predicted later mathematics achievement at the end of second grade, even when including number sense in the model. These findings suggest that early informal fraction knowledge is a foundational number skill, whose growth is intertwined with growth in facility with integers.

Introduction

There is a growing consensus that foundational mathematics includes a set of skills related to early number sense (e.g., Berch, 2005; Dehaene, 2001; Liu, Peng, & Yan, 2025; McIntosh et al., 2005). Early number sense involves interconnected strands of whole number knowledge: number (e.g., cardinality, counting), number relations (e.g., magnitude comparisons, estimation), and number operations (e.g., number composition and decomposition) (Jordan et al., 2022). Children show a wide range of individual differences in number sense, and number sense understanding predicts later mathematics achievement. For example, children's kindergarten number sense predicts their first grade (Jordan et al., 2007), second grade (Locuniak & Jordan, 2008), and third grade (Jordan et al., 2009) mathematics scores. A recent longitudinal meta-analysis of 54 studies showed that the predictive strength of early numeracy increases with longer intervals, suggesting that early numeracy may trigger a positive "snowballing effect" in mathematics (Liu et al., 2025). In particular, whole number line estimation seems important in supporting later mathematics achievement in addition to broader number sense (e.g., Case et al., 1996; Schneider et al., 2018; Siegler et al., 2011).

However, children's early mathematics learning involves more than whole numbers. Young children often use approximate and

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<https://doi.org/10.1016/j.jecp.2026.106461>

Received 13 March 2025; Received in revised form 10 January 2026;

Available online 20 January 2026

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nonsymbolic fractions in their everyday lives without being consciously aware that they are engaging with fractional quantities. For example, they may equally distribute play materials with friends, cut or fold something in half, or play sports separated into quarters. These early competencies can be referred to as informal fraction knowledge, which is a suite of fraction understandings developed through everyday experiences, without explicit instruction, such as equal sharing, understanding equivalence, and iterating equal lengths to create a whole (for review see Viegut et al., 2023). There is a growing body of research that suggests such reasoning about fractional quantities is supported by basic neurocognitive architecture (Jacob et al., 2012; Lewis et al., 2016; Newcombe et al., 2015; Park & Matthews, 2021). Indeed, preschoolers and even infants are able to discriminate between different sized nonsymbolic fractions (e.g., McCrink & Wynn, 2007; Park et al., 2020).

A traditional model of mathematical development conceptualizes mathematics learning as hierarchical and cumulative (e.g., Hiebert, 1998; Siegler & Lortie-Forgues, 2014; Xu et al., 2023), emphasizing informal mathematics knowledge as important precursors to later mathematics learning (for review see Purpura et al., 2013). For fraction learning, recent longitudinal research has identified informal fraction knowledge as the largest predictor to later formal fraction knowledge, even after controlling for other mathematics competencies (Schadl & Ufer, 2023; Viegut et al., 2023). Having foundational fraction knowledge, in turn, is a key predictor of later mathematics achievement (e.g., Bailey et al., 2012; Booth & Newton, 2012; Resnick et al., 2016; Siegler et al., 2012), especially algebra (e.g., Barbieri et al., 2021; Siegler et al., 2012). Mastering fractions requires learners to deepen their understanding of numerical properties (e.g., Siegler & Lortie-Forgues, 2014; Stafylidou & Vosniadou, 2004; Steffe, 2001) and understand relational reasoning (DeWolf et al., 2016). Thus, the potential role of informal fraction knowledge in early foundational mathematics skills should not be overlooked (Misquitta, 2011; Viegut et al., 2023), whilst focusing *only* on whole number sense and whole number line estimation as core early number competencies (e.g., Powell & Fuchs, 2012).

Informal fraction knowledge may support a range of key early mathematics competencies in different ways. For example, equal sharing can help children grasp that a whole can be divided into equal parts that represent a fraction (Clements & Sarama, 2014). Children as young as three years old can successfully solve division problems when they are presented with manipulatives in an equal sharing context (Hamamouche et al., 2020). Understanding that equal parts sum to make up a whole is required for understanding how whole numbers are constructed and deconstructed (e.g., breaking 10 into 5 and 5). Part-whole composition is seen in 5- and 6-year-olds, the average age of a kindergartener in the US (Sophian & McCorgray, 1994). This understanding can lay the foundation for operations of addition, subtraction, multiplication, and division (Empson & Levi, 2011).

Informal fraction knowledge may also promote flexible thinking as children develop an understanding of equivalence (e.g., two fourths is the same as one half), which is essential for understanding that numbers can be represented in different ways (e.g., $2 + 3$ is the same as $4 + 1$; Behr, 1983). Recognizing equivalent fractions across different nonsymbolic formats (e.g., one half can be represented with a rectangle or a circle) may also facilitate representational flexibility, which supports mathematics problem solving (see Acevedo Nistal 2009 for a review). Another component of informal fraction knowledge, iterating equal lengths to produce a whole, may support broad understanding of measurement, including in 4- to 8-year-old children (MacDonald & Lowrie, 2011; Solomon et al., 2015).

Viegut et al. (2023) examined the role of informal fraction knowledge as a foundational mathematics skill. In this study, 109 children were tracked across first grade. It was found that informal fraction knowledge at the start of first grade provided a small, but independent, contribution in predicting overall mathematics achievement at the end of first grade, even after controlling for whole number sense, whole number line estimation, executive function, and spatial skills. However, it is not clear whether the relations identified in Viegut et al.'s (2023) study would persist into future school years. Recall that mathematics instruction during first and second grade emphasizes whole number learning. Longer longitudinal research is required to characterize the role of informal fraction knowledge as a foundational mathematics skill across these early years.

Current study

The aim of the current study was to examine the potential role of informal fraction knowledge as a foundational number skill. In Viegut et al. (2023), children were assessed on informal fraction knowledge, mathematics, and cognitive skills at the beginning of first grade, and then informal fraction knowledge and mathematics again at the end of first grade. In the current study, we had the opportunity to follow the same cohort of children on informal fraction knowledge and mathematics for a third time at the end of second grade.

Examining the *durability* and *robustness* of earlier effects across time is important. Much of the literature in education and psychology may identify significant relations in the short term, but there is comparatively little follow-up work that examines whether these effects persist over longer developmental periods. Indeed, a well-documented concern in the field is that early effects often fade, attenuate, or change in character when examined longitudinally (e.g., Bailey et al., 2016; De Smedt et al., 2009; Lavelle-Hill et al., 2024). The current study will provide evidence that the mechanisms proposed below are not limited to a narrow developmental window, but instead reflect stable processes with broader relevance.

Research question 1: continued growth in informal fraction knowledge

The aim of research Question 1 is to explore the nature and magnitude of change in informal fraction knowledge over grades one (i.e., ~6–7 years old in the US) and two (i.e., ~7–8 years old in the US). For example, the magnitude of change may vary across fraction representation types (discrete versus continuous) and fraction size (half, third, fourth). Informal fraction knowledge can involve fractions that are represented as discrete sets of objects (e.g., pieces of chocolate or demarcated area models) versus continuous

relations (e.g., whole pizzas or a number line). It can also involve fractions of different sizes. Young children tend to be more accurate when reasoning about continuous representations versus discrete representations (Boyer et al., 2008) and half versus other fractions (Brizuela, 2006). Viegut et al. (2023) found that although first grade children improved their overall understanding of discrete and continuous items, and items of different sizes, the relative performance remained: children were better on continuous versus discrete and half versus other sizes.

Given these fraction concepts are not systematically taught in second grade, we hypothesized that this gap in performance will remain; however, it is unclear if there would be the same continued growth. It is possible that some development in informal fraction understandings will occur as children increasingly interact with their peers and their environment that are already rich in mathematical stimuli (e.g., equal sharing in homes and classrooms, older children and adults' use of mathematical language in relation to fractions, toys that focus on partitioning such as pretend food cutting practice, etc.). This finding may help to explain why children show large individual differences in fraction understanding at the start of formal fraction instruction in fourth grade (Resnick et al., 2016). Notably, the gap between children with high versus low fraction understanding continues to widen and leads to cascading mathematics difficulties throughout the rest of their school years (Siegler et al., 2012; Siegler & Pyke, 2013).

Research question 2: predicting second grade informal fraction knowledge

The aim of Research Question 2 is to examine mathematics and cognitive predictors of growth in early fraction knowledge. There are different multi-component models describing how mathematics is supported both by domain-specific mathematics knowledge and domain-general cognitive processes; such models often highlight the roles of number sense, whole number line estimation, executive function (inhibition and cognitive flexibility), visuospatial working memory, spatial skills, and vocabulary (e.g., Geary, 2011; Hawes et al., 2019; LeFevre et al., 2010). For example, visuospatial working memory, inhibition, and cognitive flexibility are all consistently found to support early years mathematics and fraction knowledge (e.g., Allen et al., 2019; Blair & Razza, 2007; Hawes et al., 2019; Leib et al., 2023; Park et al., 2021; Reuhkala, 2001; Verdine et al., 2014). There is a long history connecting spatial skills and mathematics (e.g., for review see Resnick et al., 2020), even in the early years (Resnik and Lowrie, 2023). Two spatial skills that are especially relevant to the current study are *spatial scaling* (mentally mapping distances between different sized spaces) and *proportional reasoning* (recognizing two ratios are equivalent), because both support fraction understanding (Möhring et al., 2016). Vocabulary is also an important part of both early whole number learning (Donlan et al., 2007) and fractions (Brizuela, 2006), in part because language helps label, encode, and explain mathematics concepts (e.g., quantity, relational properties, place value structure).

Viegut et al. (2023) found that, although the above mathematics and cognitive skills were separately correlated with informal fraction knowledge at the end of first grade, only number sense and informal fraction knowledge explained unique variance in a simultaneous regression model. Given both first and second grade focus on whole number instruction (whereas the bulk of fraction instruction begins in third grade), we hypothesize that the relations observed by Viegut et al. (2023) will extend through to the end of second grade. However, it is possible that the relation between cognitive skills and fractions understanding is driven by maturation or content (and not instruction), in which case we may find that they are predictive in second grade.

Research question 3: predicting second grade math achievement

The aim of Research Question 3, our main research question, is to examine whether and to what extent informal fraction knowledge at the start of first grade (Fall) predicts math achievement at the end of second grade (Spring the next school year), when controlling for whole number and cognitive skills. Based on the research above outlining the central role of fraction understanding in mathematics, we hypothesize that informal fraction knowledge will predict later mathematics achievement, along with other whole number competencies and cognitive skills. This finding would highlight a high level of continuity between informal fraction knowledge predicting general mathematics achievement across first and second grade. The durability provides stronger evidence that informal fraction understanding should be considered a foundational mathematics skill along with whole number sense (in contrast to most research focusing solely on whole number sense).

Method

Transparency and openness

We report all data exclusions and all measures in the study, and we follow Journal Article Reporting Standards (JARS; Kazak, 2018). All deidentified data, analysis code, and nonproprietary research materials are available on Open Science Framework (OSF) at this link (will be made available upon acceptance). We used R, Version 4.1.3 for data analysis, including the *tidyverse* (Wickham et al., 2019), *ggpubr* (Kassambara, 2023), *lm.beta* (Behrendt, 2022), *nanian* (Tierney & Cook, 2023), and *psych* (Revelle, 2023) packages. This study's design and its analysis were not preregistered.

Participants

At the start of first grade (Fall), 107 children participated in Viegut et al.'s (2023) study. At the end of second grade (Spring), we invited all children who were still enrolled in the same school ($n = 99$) to participate in this follow-up study. Of this sample, parents/guardians of $n = 80$ children consented to participate. Little's Missing Completely at Random (MCAR) test showed that patterns of

missingness were random and thus likely unrelated to observed and unobserved values on study variables, $\chi^2(96) = 90.1, p = .650$. Thus, missing data are considered ignorable (Glas, 2010), and listwise deletion does not bias estimates for this mechanism. We therefore report all analyses with the 80 children (Mean age at time 1 = 6.68 years, $SD = 0.34$ year) for whom we have both first and second-grade data. Among the 80 children in the analytic sample, four children missed one of the four sessions: one in the middle of first grade (winter) and three at the end of first grade (spring). As these data were also MCAR ($\chi^2(71) = 79.0, p = .241$), we used listwise deletion to exclude these 4 participants from regression analyses. Additionally, one child had an unrealistically low KeyMath score in 2nd grade (more than 3 standard deviations below the group mean, and 13 points lower than the same child's score the previous year), possibly due to being distracted during testing. We treated this outlier as missing data and excluded this child from the analysis predicting 2nd grade KeyMath. To note, we also report consistent results using multiple imputation with the *mice* package (van Buuren & Groothuis-Oudshoorn, 2011) for all missing data in Supplement B.

According to parent/caregiver report, the study sample included 37 girls and 43 boys. Participants were 61.3% White, 16.3% Asian, 3.8% Black or African American, 1.3% Hispanic, 7.5% Multiracial, 2.5% Other, and 7.5% not reported. Children came from three independent or parochial schools in the mid-Atlantic region of the United States serving predominantly middle-income families. The curricula used by the schools included *McGraw-Hill Reveal*, *McGraw Hill Math Connects*, and *Sadlier Progress in Math*. All focus on whole number instruction, with one section at the end of each 1st grade textbook describing fractional amounts (e.g., partitioning circles and rectangles) and one section in each 2nd grade textbook introducing unit fractions. During the first-grade and second grade testing periods, fraction-related topics were not presented in the curricula. At one of the schools, children were separated for second grade math into grade-level math and advanced math. Therefore, 14 of the 80 children in our sample were receiving third grade math instruction during the second grade, which likely included more fraction concepts. We control for advanced math status in our analyses.

Measures

Informal fraction knowledge

To assess informal fraction knowledge, children were given an informal fraction knowledge measure developed by Viegut et al. (2023). The measure includes a range of nonsymbolic (no fraction words or symbols) and symbolic (fraction words and/or symbols) items designed to efficiently capture variance of fraction understanding at this age (see Fig. 1 for examples). There were broad categories of items, such as equal sharing, equivalence, iteration, symbolic fraction word mapping, and symbolic fraction notation mapping. Items varied by fraction representation (i.e., there were 20 nonsymbolic vs. 23 symbolic items) and size (i.e., there were nine items involving halves, six involving thirds, and eight involving fourths).

The experimenter presented each item one at a time, using a colorful picture booklet and reading each question aloud. There are 43 items, and the score is the percent of items answered correctly. The informal fraction knowledge measure takes about 15 min to complete. Internal consistency for the current sample was $\alpha = .81$, 95% CI [.75, .87] at Time 1; $\alpha = .80$, 95% CI [.74, .86] at Time 2; and $\alpha = .89$, 95% CI [.86, .92] at Time 3.

To reduce the possibility of ceiling effects at the end of second grade, we added seven additional items involving symbolic fraction notation to the end of the *Informal Fraction Knowledge* measure at the final time point at the end of second grade. These items included mapping symbolic fractions to a nonsymbolic representation, comparing symbolic fractions, and choosing which of three symbolic fractions was equivalent to one-half or one whole. For all analyses in which we report growth in fraction knowledge over time, we considered children's performance on only the original 43 items to ensure the scale was the same (research question 1). However, we

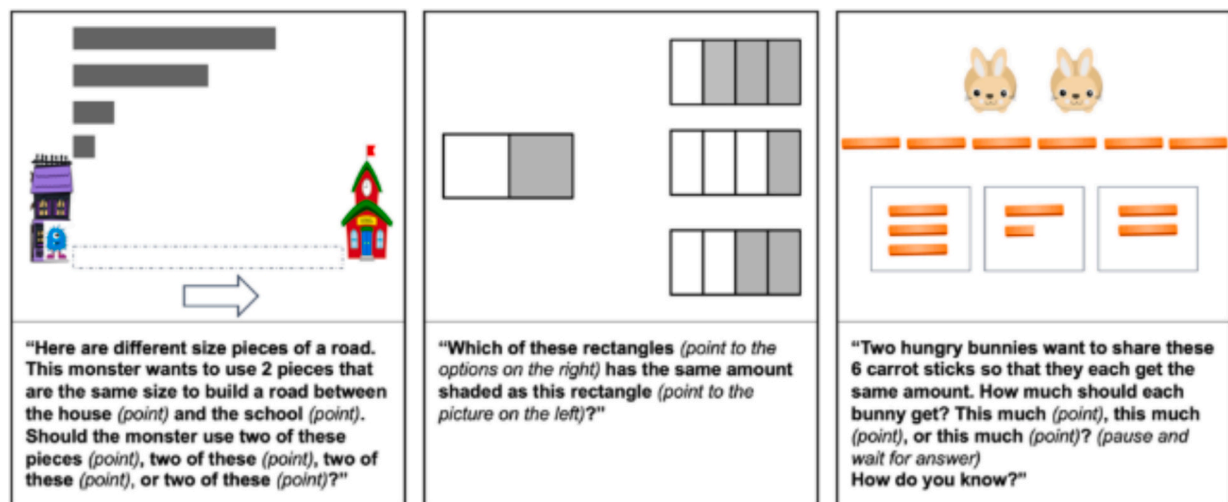


Fig. 1. Examples of items from the Informal Fraction Knowledge Measure.

included the seven challenge items in the regression analyses predicting end-of-year second grade fraction knowledge (research question 2).

Whole number sense

We used the first-grade number and number relations sections of the *Screener for Early Number Sense* (SENS; Jordan et al., 2024) to assess children's whole number sense at the start of first grade. The SENS is a standardized measure of symbolic number knowledge, with the number and number relations sections focusing on counting, number recognition, and number comparison (17 total items). The score was the percent of items answered correctly. The SENS takes approximately ten minutes in total to complete. Internal consistency for the current sample was $\alpha = .84$, 95% CI [.80,.89].

Whole number line estimation

To assess whole number line estimation skills, we asked children to identify where individual numbers would be located on a 0–100 number line (Booth & Siegler, 2006). The number line (25 cm/9.84in) was presented in the center of an iPad screen, with the following numbers appearing above the number line one at a time in a random order: 5, 11, 18, 25, 33, 40, 46, 52, 58, 65, 71, 78, 85, 93 and 99. Following common practice with children this age, we began with a practice item with feedback, to ensure the children understood what to do (Booth & Siegler, 2006). Scores were calculated using mean percent absolute error (PAE), which is the average of the overall variance between the children's response and the correct location, divided by the length of the number line (in this case, 100). Higher PAE scores represent lower accuracy (i.e., higher error rates). We used a log transformation of PAE to correct a skewed distribution (Bailey et al., 2012). The whole number line estimation task took approximately five minutes to complete. Internal consistency for the current sample at fall of first grade was $\alpha = .89$, 95% CI [.86,.92].

Executive functions

We used a touch screen version of the *Hearts and Flowers* task to evaluate cognitive flexibility and inhibitory control (Davidson et al., 2006). The *Hearts and Flowers* task comprises three blocked trials: Hearts (the child presses a button on the same side to where the stimulus appeared), Flowers (the child presses a button opposite to where the stimulus appeared), and Mixed (the child follows the appropriate rule from a mix of Flower and Heart stimuli). Prior to each set of trials, six practice trials with feedback were provided. There were 20 items for each set of trials, where the stimuli appeared for 5000 ms (five seconds) for Hearts and Flowers blocks, and 6000 ms (six seconds) for the Mixed block (or until the student responded). As the children were highly accurate on all tasks, especially the Mixed task which is more challenging ($M = 95.3\%$), we used reaction time (RT) to measure the students' executive function (Camerota et al., 2019). As did Camerota et al. (2019), we calculated a change in RT to measure cognitive flexibility ($\Delta RT_{\text{Mixed}} = RT_{\text{Mixed}} - RT_{\text{Hearts}}$) and inhibitory control ($\Delta RT_{\text{Flowers}} = RT_{\text{Flowers}} - RT_{\text{Heart}}$). The standardized ΔRT_{Mixed} and the standardized $\Delta RT_{\text{Flowers}}$ results were correlated ($r = .53$, $p < .001$). Both scores were added together, to create a composite score for executive function. The *Hearts and Flowers* task took approximately six minutes to complete. Internal consistency for the current sample was $\alpha = .92$, 95% CI [.89,.94].

Visual-spatial working memory

We used *PathSpan* (LeFevre et al., 2010), which is a touch screen adaptation of the Corsi Block test (Corsi, 1972), to assess visual spatial working memory (vsWM). *PathSpan* is a non-verbal task that tests both backward and forward vsWM span. We told the children that the activity was a 'copying game.' For forward vsWM span, the children saw nine green flashing dots that appeared in a sequence, and we asked them to copy the sequence in the same order. We used the same procedure for the backward vsWM span, but with the instruction to touch the dots in the reverse order. The experimenter demonstrated a practice sequence prior to commencing both vsWM span tasks. After two correct trials, the number of dots presented increased; and after two incorrect trials, the task ended. The correct number of sequences was measured to calculate performance. The activity took approximately nine minutes to complete. Test-retest reliability for this test was previously reported at $r = .56$ (LeFevre et al., 2010).

Spatial scaling

To assess spatial scaling, we administered the *Spatial Scaling Task* (Frick & Newcombe, 2012), modified for use on a large iPad. We told children that chickens hid their eggs in a field and the farmer needed their help to find them. There were 24 total trials, where children were presented with the "field", which was a green referent space, along with a map that illustrated where the egg was hidden. Half of the trials used a rectangular shaped field and map (16 trials) and half of the trials used a circular shaped field and map (8 trials). The maps were the same size (1:1) or required scaling (e.g., the size correspondence was 1:2, 1:4). Children tapped on the screen using a stylus to indicate where they thought the egg was hidden. The trial was repeated once if the student did not answer within 10 s. Each participant scored from 0 (least accurate) to 3 (most accurate) for each answer, depending on how close their estimate was to the correct position in the space. The highest score possible was 72 points, with final scores being calculated as a percentage. The *Spatial Scaling Task* took approximately five minutes to complete. The internal consistency for the current sample was $\alpha = .60$, 95% CI [.47,.72].

Spatial proportional reasoning

We assessed spatial proportional reasoning with a task modified from Möhring et al. (2016) for use with iPads. Children learned about a bear named Harry, who liked his drinks to taste a certain amount of cherry. Children were presented with a rectangle that was 2 cm (0.79 in) wide but had differing lengths. The rectangle was split into two colors: red for cherry juice and blue for water. Below the rectangle, there was a 12 cm (4.72 in) line with a slider that the students could move. There was one cherry on the left of the line and many cherries on the right. The children were asked to use the slider to show how much the juice mixture shown above would taste like cherries (i.e., mapping the stimulus proportion onto a continuing rating scale). Children were given three practice trials with feedback to ensure they understood the task following the same procedure as Viegut et al. (2023). There were 16 tasks in total, with different amounts of juice (3, 4, 5, 6 units) and total amount (6, 12, 18, 24 units) combined to create a factorial design. The overall final score was the average PAE. This task took approximately five minutes to complete. Internal consistency for the current sample was $\alpha = .73$, 95% CI [.65,.82].

Mathematics achievement

We used the numeration section of the *KeyMath-3* Diagnostic Test (Form A) to assess general mathematics achievement (Connolly, 2007). *KeyMath-3* is a standardized test for K-12 students. The numeration section includes items on basic math concepts, such as place value, number quantity and number order. Items cover increasingly difficult topics such as rounding and fractions. Children began with the first question and ended after getting four questions wrong in a row. Therefore, completion time varied. Performance was calculated as total correct. The numeration test has a reported split-half reliability of $r = .84$ for second graders.

Vocabulary

Children's oral vocabulary comprehension was assessed with the Picture Vocabulary Test of the NIH Toolbox for Assessment of Neurological and Behavioral Function (Slotkin et al., 2012). This standardized assessment measures comprehension of single words. Words varied by difficulty and were measured with an auditory word-picture matching paradigm. Four pictures were displayed on the iPad screen while a word was recited via the iPad. Children were instructed to touch the picture that matched the word. Once a child selected the picture, the next set of images appeared. The task took approximately four minutes to complete. Test-retest reliability based on the test's norms is $r = .84$.

Procedure

Parents/guardians originally provided informed consent to have their children participate in three one-on-one sessions with an experimenter throughout first grade. Parents/guardians also reported children's background characteristics (age, race/ethnicity, gender) at this time. Parents/guardians of all participating children who were still at the same school in second grade were separately invited to consent to allow their children to participate in the second-grade follow-up study.

Children participated in 30-minute sessions one-on-one with an experimenter in a quiet area of their school at four time points: Start of First Grade (fall), Middle of First Grade (winter), and End of First Grade (spring), and End of Second Grade (spring). At each

Table 1
Descriptives of all variables.

Measure	Start of First Grade (fall)		Middle of First Grade (winter)		End of First Grade (spring)		End of Second Grade (spring)	
	<i>M</i> (<i>SD</i>)	Range	<i>M</i> (<i>SD</i>)	Range	<i>M</i> (<i>SD</i>)	Range	<i>M</i> (<i>SD</i>)	Range
Informal fraction knowledge (% correct)	59.5 (15.1)	25.6- 90.7	—	—	68.5 (13.8)	27.9- 100	79.0 (13.5)	34.9- 100
Symbolic fraction challenge problems (% correct)	—	—	—	—	—	—	43.8 (32.3)	0–100
Number line estimation (PAE)	12.5 (7.6)	4.0- 40.7	—	—	8.0 (5.6)	1.9- 38.6	—	—
Screener for Early Number Sense (SENS; % correct)	76.3 (21.8)	11.8- 100	—	—	—	—	—	—
Hearts and Flowers (standardized composite score)	—	—	−0.1 (1.7)	−4.6- 6.7	—	—	—	—
Visual-spatial memory span (forward & backward mean)	—	—	4.1 (2.3)	0–8	—	—	—	—
Spatial scaling (% of total possible points)	—	—	59.0 (11.4)	29.2- 81.9	—	—	—	—
Spatial proportional reasoning (PAE)	—	—	23.9 (5.9)	12.5- 45.2	—	—	—	—
Picture vocabulary (standard score)	—	—	67.4 (27.2)	1–99	—	—	—	—
KeyMath-3 Numeration (scaled score; mean = 10)	—	—	—	—	12.5 (3.0)	4–18	11.2 (3.3)	1–18

Note. PAE = Percent Absolute Error. Lower PAE indicates more accurate estimates.

session, children completed math and cognitive activities by responding out loud or touching an iPad. The order and timing of each measure is presented in Table 1.

Results

Data analytic plan

To address the first research question, which examined the nature and magnitude of change in informal fraction knowledge over grades one and two, we conducted two mixed ANOVAs. We first examined whether the magnitude of change varied by representation type (symbolic vs. nonsymbolic) of informal fraction knowledge by conducting a two (Representation type) by three (Time point: start of first, end of first, start of second) mixed ANOVA. We then examined whether the magnitude of change varied by fraction size (halves, thirds, fourths) by conducting a three (Fraction size) by three (Time point: start of first, end of first, start of second) mixed ANOVA.

Although research question one makes a clear distinction between different aspects of fraction knowledge (i.e., representation type and size), we use the total score on the informal fraction knowledge measure in the subsequent analyses. Examining the possibility of different pathways for these variations is beyond the scope of the current study, which focuses on how early informal fraction knowledge may be a foundational mathematics skill that warrants greater attention. We acknowledge there is literature to suggest that the distinction between symbolic and nonsymbolic representations may be meaningful (Lewis et al., 2016), and have subsequently included four regression analyses examining the role of representation within Supplement C. With regards to fraction size, however, there is no previous literature that would suggest similar distinctions, and have subsequently not examined this point further.

To determine which mathematics and cognitive skills predicted growth in early fraction knowledge (research question 2), we conducted a multiple regression predicting second grade informal fraction knowledge by the mathematics and cognitive measures (described above). To assess whether informal fraction knowledge at the start of the first grade continued to predict math achievement at the end of second grade (research question 3), we conducted a multiple regression predicting second grade math achievement by first grade informal fraction knowledge, controlling for a range of mathematics and cognitive measures.

We include gender as a covariate in the regression analyses because it has been found as a predictor of early number competence in previous studies (e.g., Jordan et al., 2006). Statistical assumptions for regression (i.e., linearity, homoscedasticity, independence of errors, and normality of residuals) were met in all models. There was no evidence of multicollinearity among the predictors, as all *VIF* values were well below 5 (O'Brien, 2007). We further explain each of these analyses and sets of results below.

Continued growth in informal fraction knowledge (Research Question 1)

Means and standard deviations for all variables are shown in Table 1. The two (Representation type: nonsymbolic vs. symbolic) $\times$ three (Time: start of first grade, end of first grade, and end of second grade) mixed ANOVA showed a significant main effect of time ($F(2,152) = 120.9, p < .001$), a significant main effect of representation type ($F(1,76) = 161.5, p < .001$), but no Time $\times$ Representation type interaction ($F(2,152) = 0.81, p = .45$). Thus, children improved significantly over time and these improvements were similar on nonsymbolic and symbolic items. Expanding our previous work (Viegut et al., 2023), we continue to see large individual differences in children's informal fraction knowledge and pattern of growth, as shown in Fig. 2. Post hoc Bonferroni-corrected comparisons of growth between time points showed that children significantly improved both from the start of first grade ($M = 59.48\%$, $SD = 15.08$) to end of first grade ($M = 68.50\%$, $SD = 13.77$), $t(76) = 6.8, p < .001$, Hedge's $g = 0.57$, as well as from end of first grade to end of second grade ($M = 79.01\%$; $SD = 15.27$), $t(76) = 9.43, p < .001$, Hedge's $g = 0.78$. Children had higher scores on nonsymbolic versus symbolic items (collapsed across time points), but they showed growth on all subtests of the informal fraction knowledge assessment between first and second grades, as shown in Fig. 3.

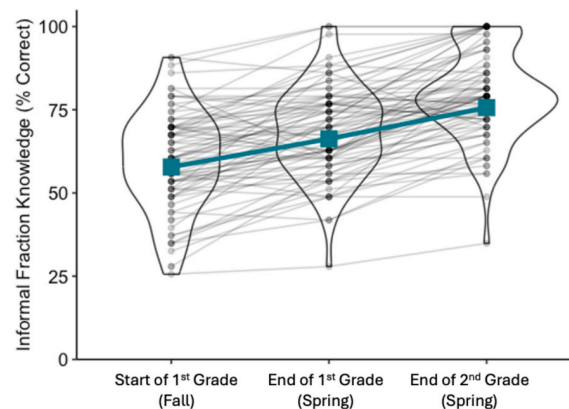


Fig. 2. Growth in Informal Fraction Knowledge Across First and Second Grade. Note. Squares represent the group's average accuracy at each time point.

The 3 (Fraction size: halves, thirds, fourths) $\times$ 3 (Time) mixed ANOVA revealed a main effect of time point ($F(2, 152) = 57.36, p < .001$) showing growth over time collapsed across fraction size. There was also a main effect of fraction size ($F(1.63, 123.7) = 159.96, p < .001$) with children more accurate with halves than other fractions collapsing across time points. There was a significant interaction between fraction size and time point ($F(4, 304) = 3.19, p = .014$). Follow-up Bonferroni comparisons showed that children improved significantly with halves over the first grade year ($g = 0.57, p < .001$) but not with thirds ($g = 0.24, p = .344$) or fourths ($g = 0.18, p = .751$), and children improved significantly with thirds ($g = 0.60, p < .001$) and fourths ($g = 0.60, p < .001$) over the second grade year but not with halves ($g = 0.30, p = .053$).

As expected, the subset of 14 children who were in the advanced math class in second grade showed descriptively stronger fraction knowledge at each time point ($M_{Gr1Fall} = 67.4\%$, $M_{Gr1Spring} = 78.6\%$, $M_{Gr2Spring} = 95.0\%$; $g_{Gr1Fall-Gr1Spring} = .86$; $g_{Gr1Spring-Gr2Spring} = 1.49$) than children who were not in advanced math ($M_{Gr1Fall} = 58.7\%$, $M_{Gr1Spring} = 66.3\%$, $M_{Gr2Spring} = 75.8\%$; $g_{Gr1Fall-Gr1Spring} = .53$; $g_{Gr1Spring-Gr2Spring} = .74$). We did not have sufficient power to conduct a statistical analysis on this subset, so we present descriptive findings. A comparison of effect sizes (see Fig. 4) reveals more similar magnitudes of growth from start of first grade (fall) to end of first grade (spring) in the two groups but descriptively larger growth from the end of first grade (spring) to the end of second grade (spring) for the small group of students in advanced mathematics. To account for these potential differences, we conduct exploratory analyses controlling for advanced math status in our main regression analyses.

When only including children who were not taking advanced math class, the two (Representation type) $\times$ three (Time) mixed ANOVA still showed a significant main effect of time ($F(2,124) = 95.4, p < .001$) and representation type ($F(1,62) = 141.0, p < .001$), and still showed no Time $\times$ Representation type interaction ($F(2,124) = 3.05, p = .51$).

When only including children who were not taking advanced math class, the three (Fraction size) $\times$ three (Time) mixed ANOVA still showed a significant main effects of time ($F(2,124) = 40.5, p < .001$) and fraction size ($F(2, 124) = 132.9, p < .001$), but no longer showed a significant Time $\times$ Fraction size interaction ($F(4,248) = 2.19, p = .07$). That is, children taking regular math class (not advanced) improved similarly on problems with halves, thirds, and fourths across the three time points.

Although fraction instruction typically begins later, we also wanted to rule out the possibility that students' improvements in informal fraction instruction were due to class instruction. Subsequently, we investigated the intraclass correlation (ICC) to test how much variance in fraction knowledge was attributable to classrooms (our sample was too small to model classroom-level effects using multilevel modeling; Maas & Hox, 2005). The ICCs for first grade fraction knowledge were very small (ICC = .004 at fall and ICC = .003 at spring), suggesting that any variability in instruction between classrooms was not driving the growth that we observed. The ICC for fraction knowledge at the end of second grade was much higher (ICC = .36), but this was driven almost entirely by the 14 children who were in the "advanced math" classroom in 2nd grade, whom we know received some fraction-related instruction. When we excluded those children, the ICC for fraction knowledge at the end of second grade dropped to $< .001$, suggesting that almost no variability in fraction knowledge was attributable to 2nd grade classroom differences.

Predicting second grade informal fraction knowledge (Research Question 2)

Next, we investigated which first grade skills predicted informal fraction knowledge at the end of second grade (spring). A complete correlation matrix for all variables is available in Supplement A. We used multiple regression to predict informal fraction knowledge at the end of second grade (spring) by whole number sense at the start of first grade (fall) and an array of general cognitive skills at the middle of first grade (winter). Because we wanted to focus on predictors of informal fraction learning during the first two years of elementary school rather than capturing correlates of general academic skills at the start of first grade, we controlled for informal fraction knowledge at the start of first grade (fall). Results are shown in Table 2.

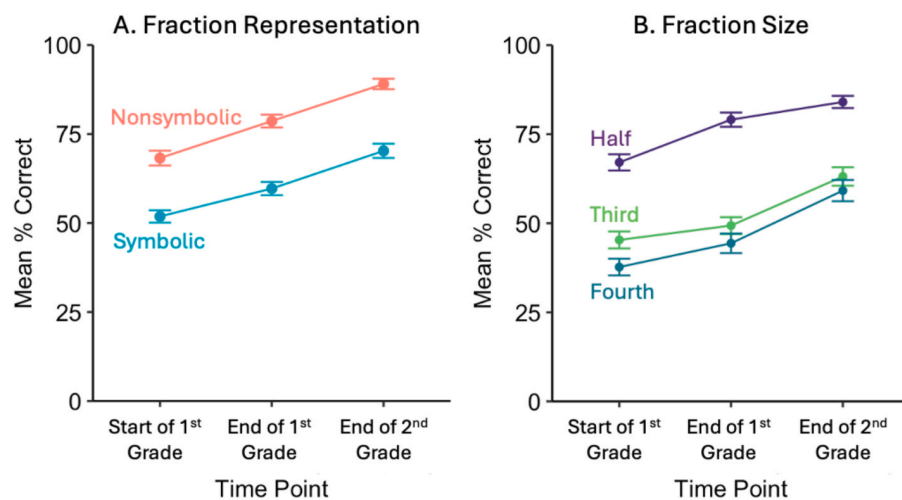


Fig. 3. Children's Knowledge Growth by Fraction Representation and Size. Note. Error bars represent the standard error of the mean.

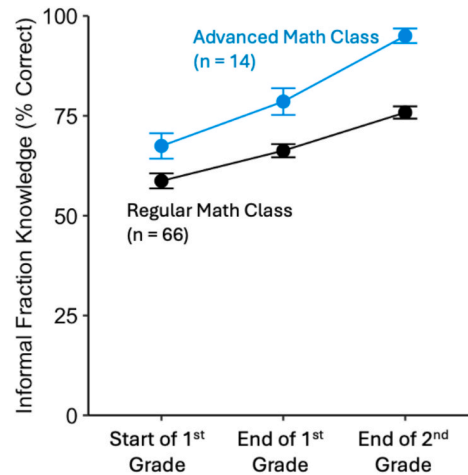


Fig. 4. Comparison of Children's Growth by Second Grade Advanced Math Status. Note. Error bars represent the standard error of the mean.

As expected, informal fraction knowledge at the start of first grade was predictive of informal fraction knowledge at the end of second grade, $\beta = 0.32$, $SE = 0.12$, $p = .010$. Whole number sense at the start of first grade was also a significant predictor of informal fraction knowledge at the end of second grade, with a larger effect size than informal fraction knowledge in first grade, $\beta = 0.49$, $SE = .09$, $p < .001$. Compared to a child with an average SENS score at the start of first grade, a child scoring one standard deviation above the mean on SENS was estimated to score about half of a standard deviation better on the informal fraction assessment at the end of second grade, when controlling for all other variables. Neither number line estimation nor any of the cognitive skills were significant predictors of end of second grade informal fraction knowledge, when considered simultaneously. No gender differences were found in informal fraction knowledge in second grade. All variables together explained about 57% of the variance in end-of-second-grade informal fraction knowledge, $F(9,69) = 10.2$, $p < .001$.

To make sure our results were not driven by the 14 students taking advanced math in second grade, we re-ran this regression model controlling for advanced math status, $F(10, 68) = 14.0$, $p < .001$, $R^2 = .67$. As shown in Table 2, the direction and statistical significance of results were consistent. The effect size of whole number sense (SENS) on later informal fraction knowledge was weaker, although still large, when controlling for advanced math status, $\beta = 0.35$, $SE = .08$, $p = .006$. Advanced math status also predicted informal fraction knowledge at the end of second grade when controlling for all other variables, $\beta = 0.36$, $SE = 3.1$, $p < .001$.

Predicting second grade math achievement (Research Question 3)

Lastly, we conducted regression analyses to test whether differences in children's informal fraction knowledge at the start of first grade (fall) predicted their KeyMath numeration scores in the end of second grade (spring), when controlling for gender (dummy code: Girl = 1; Boy = 0), first grade whole number sense (SENS) and number line estimation, and general cognitive skills (i.e., executive function, working memory, spatial scaling, spatial proportional reasoning, and receptive vocabulary), $F(9, 68) = 8.63$, $p < .001$. Results are shown in Table 3.

As we hypothesized, both start of first grade (fall) number sense ($\beta = 0.50$, $SE = 0.02$, $p < .001$) and informal fraction knowledge ($\beta = 0.31$, $SE = 0.03$, $p = .019$) explained significant variance in math achievement scores at the end of second grade (spring), controlling for gender. Effect sizes (i.e., standardized beta weights) were medium to large: children who started first grade performing one standard deviation above average on the SENS were estimated to score half of a standard deviation better on KeyMath at the end of

Table 2
Regression predicting informal fraction knowledge at the end of second grade (spring).

Predictor	Model 1 β (SE)	Model 2 β (SE)
Girl	−0.10 (2.64)	−0.07 (2.33)
Executive Function (Hearts and Flowers)	−0.12 (0.76)	−0.10 (0.67)
Working Memory	−0.08 (0.61)	−0.05 (0.54)
Spatial Scaling	−0.07 (0.11)	−0.08 (0.10)
Spatial Proportional Reasoning	−0.16 (0.23)	−0.08 (0.20)
Picture Vocabulary	−0.01 (0.05)	0.01 (0.05)
Fall First Grade SENS	0.49*** (0.09)	0.35** (0.08)
Fall First Grade Whole Number Line (log-transformed PAE)	0.01 (3.17)	0.01 (2.80)
Fall First Grade Informal Fractions	0.32* (0.12)	0.33** (0.11)
Advanced Math Class (1 = yes, 0 = no)	–	0.36*** (3.10)
R^2	.57	.67

Table 3
Multiple regression predicting end of second grade (spring) KeyMath.

Predictor	Model 1 β (SE)	Model 2 β (SE)
Girl	−0.20* (.58)	−0.18* (.55)
Executive Function (Hearts and Flowers)	−0.09 (.16)	−0.07 (.15)
Working Memory	−0.05 (.13)	−0.03 (.12)
Spatial Scaling	0.00 (.02)	−0.01 (.02)
Spatial Proportional Reasoning	−0.09 (.05)	−0.03 (.05)
Picture Vocabulary	−0.03 (.01)	−0.01 (.01)
Fall First Grade SENS	0.50*** (.02)	0.40** (.02)
Fall Whole Number Line (log-transformed PAE)	0.08 (.70)	0.07 (.66)
Fall First Grade Informal Fractions	0.31** (.03)	0.31* (.03)
Advanced Math Class (1 = yes, 0 = no)	—	0.27** (.71)
R^2	.53	.59

Note. One participant was dropped from these analyses because their KeyMath score was more than 3 standard deviations below the sample mean and this observation showed unacceptably large leverage on the regression estimates, as indicated by a Cook's distance above 0.2. PAE = Percent Absolute Error.

second grade, holding all else constant (boys are the referent). A one standard deviation increase on the Informal Fraction Measure at start of first grade corresponded to an increase of about one-third of a standard deviation on KeyMath at the end of second grade, holding all else constant. These differences are meaningful, corresponding to at least 30% of the annual gains that occur in first grade on U.S.-based nationally normed tests of mathematics (see Bloom et al., 2008). There were also significant differences between boys and girls on KeyMath scores ($\beta = -0.20$, $SE = 0.58$, $p = .031$), with girls estimated to score one-fifth of a standard deviation below boys, holding all else constant. None of the cognitive predictors explained significant variance in second grade spring KeyMath scores, when controlling for all other measures. All findings were consistent when controlling for advanced math status, as shown in Table 3. The model including all predictors combined explained about half of the variance in KeyMath scores, $F(10, 67) = 9.7$, $p < .001$, $R^2 = .59$.

Discussion

In the current study, we extended Viegut et al. (2023), who followed children's fraction understanding from the start of first grade (fall) to the end of first grade (spring), by continuing to follow these children through to the end of second grade (spring). We found that children's informal fraction knowledge continues to improve across first and second grade, despite direct fraction instruction not being a part of the curriculum (see Participants section for details on focus on whole number versus fraction instruction). Initial informal fraction knowledge and whole number sense at the start of first grade (fall), each uniquely predicted later informal fraction knowledge and overall mathematics achievement at the end of second grade (spring). These findings demonstrate the robustness and durability of our theoretical account that informal fraction knowledge should be considered an early foundational mathematics skill. Below, we discuss the implications for theories of fraction development and its potential role as a foundational numerical skill in early childhood education.

Growth in informal fraction knowledge

At the start of first grade, children already show large individual differences in informal fraction knowledge (Viegut et al., 2023). The current study sheds light on how these individual differences grow over time. Although there is continued growth in understanding both nonsymbolic and symbolic concepts over first and second grade, the children in our sample had higher performance on the nonsymbolic fraction items. This suggests that nonsymbolic fraction concepts may emerge earlier or be easier. This interpretation is consistent with the broader literature, which has shown that children often demonstrate nonsymbolic understandings prior to symbolic understandings (e.g., Levine et al., 1992; Jordan et al., 2013). However, given the measure used in this study did not match nonsymbolic and symbolic items for task difficulty, our comparisons should be interpreted cautiously and primarily as converging with existing developmental findings, rather than as a definitive test of relative proficiency.

With regards to different kinds of fractions (half, thirds, fourths), children improved more on understanding “half” over first grade, whereas they improved more on understanding “thirds” and “fourths” over second grade. These findings show that, from an early age, children are continually refining their understanding of fractions, even when instruction is focused on whole numbers.

This finding contrasts most theories of fraction development, which suggest that fraction learning comes later, after whole number understanding – whether it be as part of a segmented (e.g., Bailey et al., 2015; DeWolf et al., 2016; Ni & Zhou, 2005) or a continuous (Siegler & Lortie-Forgues, 2014) process. Indeed, cross-nationally, curricula tend to focus on whole number learning from preschool through third grade, with the bulk of fraction instruction occurring between fourth and sixth grades (e.g., ACARA, 2020; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Instead, our finding aligns with cognitive models of nonsymbolic ratio, which suggest that individuals are able to intuitively reason about ratio from a very young age (Lewis et al., 2016). Such reasoning may be supported by a single generalized magnitude system which then differentiates to support fractions and whole numbers under separate systems (Newcombe et al., 2015) or there may always be separate neurocognitive architectures (Lewis et al., 2016). More research is required to examine the factors that influence early fraction learning. If children are

not receiving formal fraction instruction, they may be learning about fractions informally through everyday experiences, such as baking with caregivers, equal sharing amongst friends, or playing sport. They may also learn fraction concepts indirectly, through instruction on whole number (e.g., the number line) or other mathematics concepts (e.g., learning about time, partitioning shapes).

Predicting later informal fraction knowledge

Number sense at the start of first grade (fall) was the largest predictor of informal fraction knowledge at the end of first grade (spring; Viegut et al., 2023) and at the end of second grade (spring; current study). The consistent and strong relation between number sense and informal fraction knowledge highlights the importance of whole number sense in supporting fraction learning. Number sense and informal fraction knowledge may both involve reasoning about magnitudes (e.g., Siegler et al., 2011), and the use of nonsymbolic, visual or concrete representations (e.g., number lines, finger counting, partitioning shapes, sharing materials). They also both involve the composition and decomposition of numbers (Empson & Levi, 2011; Sophian & McGorray, 1994). For example, understanding that six is the same as two plus four, or six cookies can be shared equally among three people by giving two cookies to each person.

Our results suggest that number sense and informal fraction understanding are developing and being learned at the same time across first and second grade. Subsequently, future research should examine the impact of weaving fraction and whole number instruction across early years learning (Resnick & Stieff, 2024). For example, what would be the impact of including unit fractions in a counting sequence or as part of a whole number line? We do not know if and how early informal fraction understanding supports later formal fraction learning. However, by the time children enter fourth grade (the start of formal fraction instruction), there are already large individual differences in formal fraction skills, which continue to widen (Resnick et al., 2016; Gesuelli & Jordan, 2024). Early fraction intervention may help create a pathway for explicitly building on informal understandings of fraction concepts towards later formal fraction understandings.

Examination of early fraction intervention will also inform theories of numerical development and the role of fractions (see above). Examining the malleability of fraction understanding in the early years can clarify whether fraction knowledge builds directly on whole number understanding, concurrently with whole number understanding, or requires a conceptual shift in numerical thinking (which may also be linked with development). Early fraction interventions can help identify which cognitive mechanisms are most critical for grasping fraction concepts, how fraction knowledge and whole number knowledge interact as foundational mathematics skills.

Although executive functions, visuospatial working memory, spatial scaling, proportional reasoning, and vocabulary all predict later formal fractions knowledge (for review see Viegut et al., 2023), here we see significant, but relatively low, correlations between these predictor measures and informal fraction knowledge. One possible reason could be that these cognitive skills contribute to both informal and formal fraction knowledge, in a fashion that changes over development. In the case of spatial skills, informal fraction tasks may draw primarily on perceptual spatial processes (e.g., visually quantifying parts, such as a half or a third), whereas formal symbolic fraction tasks may require more conceptual spatial skills, including spatial scaling, proportional reasoning, and mental transformation, all of which might support mapping between symbolic magnitudes and spatial representations. Future studies could examine if simple perceptual skills (e.g., symmetry detection) predict informal fraction knowledge, if these early spatial skills predict the rate of change in fraction understanding, and if training perceptual versus conceptual skills improves informal versus formal fraction understanding. Future research might also consider how these cognitive skills support whole number versus fraction understanding.

Predicting later mathematics achievement

Informal fraction knowledge and number sense at the start of first grade (fall) each provided unique variance in predicting later mathematics achievement at the end of second grade (spring), with medium to large effect sizes. Although none of the other cognitive variables were predictive in the model (despite being correlated with mathematics), this may be due to the whole number sense being a powerful predictor and likely subsuming the variance (Viegut et al., 2023), which further highlights the potential role of informal fraction knowledge as a separate foundational skill. Notably, the predictive utility of informal fraction knowledge increases between the beginning of first grade and the end of first grade (Viegut et al., 2023) to the end of second grade (current study). This highlights the relation between informal fractions knowledge and mathematics achievement is not only durable and robust, but may continue to strengthen two years later.

Current educational policies emphasize screening and instruction around number sense because it predicts later mathematics achievement (e.g., National Association for the Education of Young Children (NAEYC) & National Council of Teachers of Mathematics (NCTM), 2002; NCTM, 2000; Norris, 2024). Such policies, and subsequent educational practice, do not explicitly mention rational number knowledge. However, the current study demonstrates that informal fraction knowledge predicts mathematics learning, which suggests it could be beneficial to incorporate this topic into the early mathematics curriculum.

Limitations and future directions

We used the same methods as Viegut et al. (2023), and so the current study shares the same limitations (see Viegut et al., 2023 for review around measuring fraction instruction, design of the informal fraction knowledge measure, and generalizability of findings across different populations). For example, we did not measure if, when, and how fraction concepts may have been introduced in informal or formal learning environments (e.g., the home) or possibly in the classroom (even though typically formal fraction

instruction begins later). It is also possible that students learned something about fractions, or launched motivation to spontaneously focus on fractions in everyday life, simply by participating in the study. To better characterize the malleability of early informal fraction understanding, more research is needed to capture the kinds of experiences children are having with fraction concepts. Our future work will investigate first and second grade curricula to better understand what fraction-related information young students are exposed to. Future work might also map the trajectory from informal to formal fraction knowledge more fully, by examining how early informal fraction knowledge supports later formal fraction learning. Such work will be important to empirically assess our theoretical model that informal fraction knowledge constitutes a key developmental precursor to later mathematical achievement.

The informal fraction measure consisted of the same items at each time point. Although reusing the same assessment raises the possibility of retest and familiarity effects, the substantial length of time between testing (approximately one year) and that the curriculum did not target the specific fraction concepts assessed during the intervening period, mitigates this concern. We elected to use identical items to maximize measurement invariance and maintain comparability of the construct across time. Future research should develop psychometrically validated parallel forms with link items.

Another limitation is the attrition in samples from first to second grade. Attrition was due to children moving to another school or parents not consenting to participate the follow up year. As reported above, the data were missing at random, and so likely do not impact the observed findings. Nevertheless, to address any potential concerns, we also completed multiple imputation for all missing data (see [Supplement B](#)) and found consistent results with the exception of a gender difference.

Conclusions

Our findings support the hypothesis that informal fraction knowledge is a foundational number skill. Mathematics learning is hierarchical, where foundational number skills provide essential building blocks for later, increasingly advanced mathematics knowledge (e.g., Hiebert, 1988; Siegler & Lortie-Forgues, 2014; Xu et al., 2023). These early foundational number skills are linked to academic achievement throughout elementary school (Duncan et al., 2007; Nguyen et al., 2016), middle school (Bailey et al., 2014; Claessens & Engel, 2013), and high school (Watts et al., 2014). The current study demonstrates that young children are growing in their understanding of informal fractions over the first two years of elementary school, and that this understanding uniquely predicts mathematics achievement over time. Given that fractions are an important aspect to our daily lives into adulthood as well as for learning more advanced mathematics (National Mathematics Advisory Panel, 2008), more research is needed to explore how an early instructional focus on fractions may improve children's mathematics learning trajectory.

CRediT authorship contribution statement

Ilyse Resnick: Writing – review & editing, Writing – original draft, Supervision, Methodology, Investigation, Funding acquisition, Conceptualization. **Alexandria A. Viegut:** Writing – review & editing, Writing – original draft, Visualization, Project administration, Methodology, Investigation, Formal analysis, Conceptualization. **Christina Areizaga Barbieri:** Writing – review & editing, Writing – original draft, Visualization, Supervision, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Conceptualization. **Nora S. Newcombe:** Writing – review & editing, Writing – original draft, Supervision, Methodology, Funding acquisition, Conceptualization. **Nancy C. Jordan:** Writing – review & editing, Writing – original draft, Supervision, Software, Resources, Methodology, Funding acquisition, Conceptualization.

Funding

This research was supported by the National Science Foundation Awards 2,000,495 and 2000424.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary data

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.jecp.2026.106461>.

Data availability

We have shared a link to the data/code within the Transparency and Openness section of the Methods section.

References

- Acevedo Nistal, A., Dooren, W., Clarebout, G., Elen, J., & Verschaffel, L. (2009). Conceptualising, investigating and stimulating representational flexibility in mathematical problem solving and learning: A critical review. *ZDM*, 41(5), 627–636. <https://doi.org/10.1007/s11858-009-0189-1>
- Allen, K., Higgins, S., & Adams, J. (2019). The relationship between visuospatial working memory and mathematical performance in school-aged children: A systematic review. *Educational Psychology Review*, 31, 509–531. <https://doi.org/10.1007/s10648-019-09470-8>
- Curriculum, A. (2020). *Assessment and Reporting Authority (ACARA)*. Mathematics: Sequence of content.
- Bailey, D. H., Hoard, M. K., Nugent, L., & Geary, D. C. (2012). Competence with fractions predicts gains in mathematics achievement. *Journal of Experimental Child Psychology*, 113(3), 447–455. <https://doi.org/10.1016/j.jecp.2012.06.004>
- Bailey, D. H., Nguyen, T., Jenkins, J. M., Domina, T., Clements, D. H., & Sarama, J. S. (2016). Fadeout in an early mathematics intervention: Constraining content or preexisting differences? *Developmental Psychology*, 52(9), 1457–1469. <https://doi.org/10.1037/dev0000188>
- Bailey, D. H., Watts, T. W., Littlefield, A. K., & Geary, D. C. (2014). State and trait effects on individual differences in children's mathematical development. *Psychological Science*, 25(11), 2017–2026. <https://doi.org/10.1177/0956797614547539>
- Bailey, D. H., Zhou, X., Zhang, Y., Cui, J., Fuchs, L. S., Jordan, N. C., Gersten, R., & Siegler, R. S. (2015). Development of fraction concepts and procedures in U.S. and Chinese children. *Journal of Experimental Child Psychology*, 129, 68–83. <https://doi.org/10.1016/j.jecp.2014.08.006>
- Barbieri, C. A., Young, L. K., Newton, K. J., & Booth, J. L. (2021). Predicting middle school profiles of algebra performance using fraction knowledge. *Child Development*, 92(5), 1984–2005. <https://doi.org/10.1111/cdev.13568>
- Behr, M. (1983). Rational number concepts. In R. Lesh, & M. Landau (Eds.), *Acquisition of mathematics concepts and processes* (pp. 91–126). Academic Press.
- Behrendt, S. (2022). *lm.beta: Add Standardized Regression Coefficients to Linear-Model-Objects (R package version 1.7-2)* [Computer software].
- Berch, D. B. (2005). Making sense of number sense. *Journal of Learning Disabilities*, 38(4), 333–339. <https://doi.org/10.1177/00222194050380040901>
- Blair, C., & Razza, R. P. (2007). Relating effortful control, executive function, and false belief understanding to emerging math and literacy ability in kindergarten. *Child Development*, 78(2), 647–663. <https://doi.org/10.1111/j.1467-8624.2007.01019.x>
- Bloom, H. S., Hill, C. J., Black, A. R., & Lipsey, M. W. (2008). Performance trajectories and performance gaps as achievement effect-size benchmarks for educational interventions. *Journal of Research on Educational Effectiveness*, 1(4), 289–328. <https://doi.org/10.1080/19345740802400072>
- Booth, J. L., & Newton, K. J. (2012). Fractions: Could they really be the gatekeeper's doorman? *Contemporary Educational Psychology*, 37(4), 247–253. <https://doi.org/10.1016/j.cedpsych.2012.07.001>
- Booth, J. L., & Siegler, R. S. (2006). Developmental and individual differences in pure numerical estimation. *Developmental Psychology*, 42(1), 189–201. <https://doi.org/10.1037/0012-1649.41.6.189>
- Boyer, T. W., Levine, S. C., & Huttenlocher, J. (2008). Development of proportional reasoning: Where young children go wrong. *Developmental Psychology*, 44(5), 1478–1490. <https://doi.org/10.1037/a0013110>
- Brizuela, B. M. (2006). Young children's notations for fractions. *Educational Studies in Mathematics*, 62(3), 281–305. <https://doi.org/10.1007/s10649-005-9003-3>
- Camerota, M., Willoughby, M. T., & Blair, C. B. (2019). Speed and accuracy on the Hearts and Flowers task interact to predict child outcomes. *Psychological Assessment*, 31(8), 995–1005. <https://doi.org/10.1037/pas0000725>
- Case, R., Okamoto, Y., Griffin, S., McKeough, A., Bleiker, C., Henderson, B., Stephenson, K. M., Siegler, R. S., & Keating, D. P. (1996). The role of central conceptual structures in the development of children's thought. *Monographs of the Society for Research in Child Development*, 61(1/2), i–295. <https://doi.org/10.2307/1166077>
- Claessens, A., & Engel, M. (2013). How important is where you start? Early mathematics knowledge and later school success. *Teachers College Record*, 115(6), 1–29. <https://doi.org/10.1177/016146811311500603>
- Clements, D. H., & Sarama, J. (2014). Learning trajectories: Foundations for effective, research-based education. In A. P. Maloney, J. Confrey, & K. H. Nguyen (Eds.), *Learning over time: Learning trajectories in mathematics education* (pp. 1–30). IAP Information Age Publishing.
- Connolly, A. J. (2007). *KeyMath 3: Diagnostic Assessment*. San Antonio, TX: Pearson.
- Corsi, P. M. (1972). *Human Memory and the Medial Temporal Region of the Brain*. McGill University [PhD thesis].
- Davidson, M. C., Amso, D., Anderson, L. C., & Diamond, A. (2006). Development of cognitive control and executive functions from 4 to 13 years: Evidence from manipulations of memory, inhibition, and task switching. *Neuropsychologia*, 44(11), 2037–2078. <https://doi.org/10.1016/j.neuropsychologia.2006.02.006>
- De Smedt, B., Janssen, R., Bouwens, K., Verschaffel, L., Boets, B., & Ghesquière, P. (2009). Working memory and individual differences in mathematics achievement: A longitudinal study from first grade to second grade. *Journal of Experimental Child Psychology*, 103(2), 186–201. <https://doi.org/10.1016/j.jecp.2009.01.004>
- Dehaene, S. (2001). Precise of the number sense. *Mind and Language*, 16(1), 16–36. <https://doi.org/10.1111/1468-0017.00154>
- DeWolf, M., Bassok, M., & Holyoak, K. J. (2016). A set for relational reasoning: Facilitation of algebraic modeling by a fraction task. *Journal of Experimental Child Psychology*, 152, 351–366. <https://doi.org/10.1016/j.jecp.2016.06.016>
- Donlan, C., Cowan, R., Newton, E. J., & Lloyd, D. (2007). The role of language in mathematical development: Evidence from children with specific language impairments. *Cognition*, 103(1), 23–33. <https://doi.org/10.1016/j.cognition.2006.02.007>
- Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., & Japel, C. (2007). School readiness and later achievement. *Developmental Psychology*, 43(6), 1428–1446. <https://doi.org/10.1037/0012-1649.43.6.1428>
- Empson, S., & Levi, L. (2011). *Extending Children's Mathematics: Fractions and Decimals: Innovations in Cognitively Guided Instruction* (pp. 178–216). Portsmouth, NH: Heinemann.
- Frick, A., & Newcombe, N. S. (2012). Getting the big picture: Development of spatial scaling abilities. *Cognitive Development*, 27(3), 270–282. <https://doi.org/10.1016/j.cogdev.2012.05.004>
- Glas, C. A. W. (2010). Missing Data. In P. L. Peterson, & E. Baker (Eds.), *International Encyclopedia of Education Third Edition* (pp. 283–288). (1–8; No. 7). Elsevier. <https://doi.org/10.1016/B978-0-08-044894-7.01346-4>
- Geary, D. C. (2011). Cognitive predictors of achievement growth in mathematics: A 5-year longitudinal study. *Developmental Psychology*, 47(6), 1539. <https://doi.org/10.1037/a0025510>
- Gesuelli, K. A., & Jordan, N. C. (2024). Fraction arithmetic development: An examination of students' patterns of growth and errors across the intermediate grades. *Journal of Educational Psychology*, 116(3), 377. <https://doi.org/10.1037/edu0000828>
- Kassambara, A. (2023). *ggpubr: ggplot2-Based Publication Ready Plots* (Version 0.6. 0) [R package].
- Hamamouche, K., Chernyak, N., & Cordes, S. (2020). Sharing scenarios facilitate division performance in preschoolers. *Cognitive Development*, 56, Article 100954. <https://doi.org/10.1016/j.cogdev.2020.100954>
- Hawes, Z., Moss, J., Caswell, B., Seo, J., & Ansari, D. (2019). Relations between numerical, spatial, and executive function skills and mathematics achievement: A latent-variable approach. *Cognitive Psychology*, 109, 68–90. <https://doi.org/10.1016/j.cogpsych.2018.12.002>
- Hiebert, J. (1998). A theory of developing competence with written mathematical symbols. *Educational Studies in Mathematics*, 19(3), 333–355. <https://doi.org/10.1007/BF00312451>
- Jacob, S. N., Vallentin, D., & Nieder, A. (2012). Relating magnitudes: The brain's code for proportions. *Trends in Cognitive Sciences*, 16(3), 157–166. <https://doi.org/10.1016/j.tics.2012.02.002>
- Jordan, N. C., Devlin, B. L., & Botello, M. (2022). Core foundations of early mathematics: Refining the number sense framework. *Current Opinion in Behavioral Sciences*, 46, Article 101181. <https://doi.org/10.1016/j.cobeha.2022.101181>
- Jordan, N. C., Kaplan, D., Locuniak, M. N., & Ramineni, C. (2007). Predicting first-grade math achievement from developmental number sense trajectories. *Learning Disabilities Research & Practice*, 22(1), 36–46. <https://doi.org/10.1111/j.1540-5826.2007.00229.x>
- Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology*, 45(3), 850–867. <https://doi.org/10.1037/a0014939>
- Jordan, N. C., Klein, A., & Huang, C. (2024). *SENS: Developing a screener for early number sense*. Austin TX: Hammill Institute.

- Kazak, A. E. (2018). Editorial: Journal article reporting standards. *American Psychologist*, 73(1), 1–2. <https://doi.org/10.1037/amp0000263>
- Lavelle-Hill, R., Frenzel, A. C., Goetz, T., Lichtenfeld, S., Marsh, H. W., Pekrun, R., Sakaki, M., Smith, G., & Murayama, K. (2024). How the predictors of math achievement change over time: A longitudinal machine learning approach. *Journal of Educational Psychology*, 116(8), 1383–1403. <https://doi.org/10.1037/edu0000863>
- LeFevre, J.-A., Fast, L., Skwarchuk, S.-L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: Longitudinal predictors of performance. *Child Development*, 81(6), 1753–1767. <https://doi.org/10.1111/j.1467-8624.2010.01508.x>
- Leib, E. R., Starr, A., Younger, J. W., Bunge, S. A., Uncapher, M. R., Rosenberg-Lee, M., & Project iLead Consortium. (2023). Testing the whole number interference hypothesis: Contributions of inhibitory control and whole number knowledge to fraction understanding. *Developmental Psychology*, 59(8), 1407–1425. [10.1037/dev0001557](https://doi.org/10.1037/dev0001557).
- Lewis, M. A., Matthews, P. G., & Hubbard, E. M. (2016). Neurocognitive architectures and the nonsymbolic foundations of fractions understanding. *Elsevier EBooks*, 141–164. <https://doi.org/10.1016/b978-0-12-801871-2.00006-x>
- Liu, Y., Peng, P., & Yan, X. (2025). Early numeracy and mathematics development: A longitudinal metaanalysis on the predictive nature of early numeracy. *Journal of Educational Psychology*, 117(6), 863–883. <https://doi.org/10.1037/edu0000925>
- Locuniak, M. N., & Jordan, N. C. (2008). Using kindergarten number sense to predict calculation fluency in second grade. *Journal of Learning Disabilities*, 41(5), 451–459. <https://doi.org/10.1177/002221940832112>
- Maas, C. J., & Hox, J. J. (2005). Sufficient sample sizes for multilevel modeling. *Methodology*, 1(3), 86–92. <https://doi.org/10.1027/1614-2241.1.3.86>
- MacDonald, A., & Lowrie, T. (2011). Developing measurement concepts within context: Children's representations of length. *Mathematics Education Research Journal*, 23(1), 27–42. <https://doi.org/10.1007/s13394-011-0002-7>
- McCrink, K., & Wynn, K. (2007). Ratio abstraction by 6-month-old infants. *Psychological Science*, 18(8), 740–745. <https://doi.org/10.1111/j.1467-9280.2007.01969.x>
- McIntosh, A., Reys, B. J., & Reys, R. E. (2005). A proposed framework for examining basic number sense. In *Subject Learning in the Primary Curriculum* (pp. 209–221). Routledge.
- Misquitta, R. (2011). A review of the literature: Fraction instruction for struggling learners in mathematics. *Learning Disabilities Research & Practice*, 26(2), 109–119. <https://doi.org/10.1111/j.1540-5826.2011.00330.x>
- Möhring, W., Newcombe, N. S., & Frick, A. (2016). Using mental transformation strategies for spatial scaling: Evidence from a discrimination task. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 42(9), 1473–1479. <https://doi.org/10.1037/xlm0000240>
- National Association for the Education of Young Children (NAEYC) & National Council of Teachers of Mathematics (NCTM). (2002). *Early childhood mathematics: Promoting good beginnings*. Joint position statement.
- National Governors Association Center for Best Practices & Council of Chief State School Officers (2010). *Common Core State Standards for Mathematics*. Washington, DC: Author.
- National Mathematics Advisory Panel. (2008). *Foundations for success. The final report of the National Mathematics Advisory Panel*. Washington, DC: Department of Education.
- National Council of Teachers of Mathematics. (NCTM). (2000). *Principles and standards for school mathematics: Executive summary*. National Council of Teachers of Mathematics.
- Newcombe, N. S., Levine, S. C., & Mix, K. S. (2015). Thinking about quantity: The intertwined development of spatial and numerical cognition. *Wiley Interdisciplinary Reviews: Cognitive Science*, 6(6), 491–505. <https://doi.org/10.1002/wcs.1369>
- Nguyen, T., Watts, T. W., Duncan, G. J., Clements, D. H., Sarama, J. S., Wolfe, C., & Spitler, M. E. (2016). Which preschool mathematics competencies are most predictive of fifth grade achievement? *Early Childhood Research Quarterly*, 36, 550–560. <https://doi.org/10.1016/j.ecresq.2016.02.003>
- Ni, Y., & Zhou, Y.-D. (2005). Teaching and learning fraction and rational numbers: The origins and implications of whole number bias. *Educational Psychologist*, 40(1), 27–52. https://doi.org/10.1207/s15326985sep4001_3
- Norris, K. (2024, June 19). *Screening That Counts: Why Australia Needs Universal Early Numeracy Screening*. The Centre for Independent Studies. <https://www.cis.org.au/publication/screening-that-counts-why-australia-needs-universal-early-numeracy-screening/>.
- O'Brien, R. M. (2007). A caution regarding rules of thumb for variance inflation factors. *Quality & Quantity*, 41(5), 673–690. <https://doi.org/10.1007/s11135-006-9018-6>
- Park, Y., & Matthews, P. G. (2021). Revisiting and refining relations between nonsymbolic ratio processing and symbolic math achievement. *Journal of Numerical Cognition*, 7(3), 328–350. <https://doi.org/10.5964/jnc.6927>
- Park, Y., Viegut, A. A., & Matthews, P. G. (2020). More than the sum of its parts: Exploring the development of ratio magnitude versus simple magnitude perception. *Developmental Science*, 24(3). <https://doi.org/10.1111/desc.13043>
- Powell, S. R., & Fuchs, L. S. (2012). Early numerical competencies and students with mathematics difficulty. *Focus on Exceptional Children*, 44(5), 1. <https://pmc.ncbi.nlm.nih.gov/articles/PMC3756513/>.
- Resnick, I., Harris, D., Logan, T., & Lowrie, T. (2020). The relation between mathematics achievement and spatial reasoning. *Mathematics Education Research Journal*, 32(2), 171–174. <https://doi.org/10.1007/s13394-020-00338-7>
- Resnick, I., Jordan, N. C., Hansen, N., Rajan, V., Rodrigues, J., Siegler, R. S., & Fuchs, L. S. (2016). Developmental growth trajectories in understanding of fraction magnitude from fourth through sixth grade. *Developmental Psychology*, 52(5), 746–757. <https://doi.org/10.1037/dev0000102>
- Resnik, I., & Lowrie, T. (2023). Spatial reasoning supports preschool numeracy: Findings from a large-scale nationally representative randomized control trial. *Journal for Research in Mathematics Education*, 54(5), 295–316. <https://doi.org/10.5951/jresmetheduc-2022-0051>
- Resnick, I., & Stieff, M. (2024). The role of theory in research and practice: Characterizing cognition and context within mathematics education. *Springer International Handbooks of Education*, 271–292. https://doi.org/10.1007/978-3-031-51474-6_11
- Reuhkala, M. (2001). Mathematical skills in ninth-graders: Relationship with visuo-spatial abilities and working memory. *Educational Psychology*, 21(4), 387–399. <https://doi.org/10.1080/01443410120090786>
- Revelle, W. (2023). psych: Procedures for Personality and Psychological Research. Northwestern University, Evanston, <https://CRAN.r-project.org/package=psych>. R package version 2.3.6.
- Schneider, M., Merz, S., Stricker, J., De Smedt, B., Torbeyns, J., Verschaffel, L., & Luwel, K. (2018). Associations of number line estimation with mathematical competence: A meta-analysis. *Child Development*, 89(5), 1467–1484. <https://doi.org/10.1111/cdev.13068>
- Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., Susperreguy, M. I., & Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological Science*, 23(7), 691–697. <https://doi.org/10.1177/0956797612440101>
- Siegler, R. S., & Lortie-Forgues, H. (2014). An integrative theory of numerical development. *Child Development Perspectives*, 8(3), 144–150. <https://doi.org/10.1111/cdep.12077>
- Siegler, R. S., & Pyke, A. A. (2013). Developmental and individual differences in understanding of fractions. *Developmental Psychology*, 49(10), 1994–2004. <https://doi.org/10.1037/a0031200>
- Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology*, 62(4), 273–296. <https://doi.org/10.1016/j.cogpsych.2011.03.001>
- Slotkin, J., Kallen, M., Griffith, J., Magasi, S., Salsman, J., & Nowinski, C. (2012). NIH toolbox. *Technical Manual*. [Google Scholar].
- Solomon, T. L., Vasilyeva, M., Huttenlocher, J., & Levine, S. C. (2015). Minding the gap: Children's difficulty conceptualizing spatial intervals as linear measurement units. *Developmental Psychology*, 51(11), 1564–1573. <https://doi.org/10.1037/a0039707>
- Sophian, C., & McCorgay, P. (1994). Part-whole knowledge and early arithmetic problem solving. *Cognition and Instruction*, 12(1), 3–33. https://doi.org/10.1207/s1532690xci1201_1
- Stafylidou, S., & Vosniadou, S. (2004). The development of students' understanding of the numerical value of fractions. *Learning and Instruction*, 14(5), 503–518. <https://doi.org/10.1016/j.learninstruc.2004.06.015>

- Steffe, L. P. (2001). A new hypothesis concerning children's fractional knowledge. *The Journal of Mathematical Behavior*, 20(3), 267–307. [https://doi.org/10.1016/s0732-3123\(02\)00075-5](https://doi.org/10.1016/s0732-3123(02)00075-5)
- Tierney, N., & Cook, D. (2023). Expanding tidy data principles to facilitate missing data exploration, visualization and assessment of imputations. *Journal of Statistical Software*, 105(7), Article 10.18637/jss.v105.i07.
- Van Buuren, S., & Groothuis-Oudshoorn, K. (2011). mice: Multivariate imputation by chained equations in R. *Journal of statistical software*, 45, 1–67. <https://doi.org/10.18637/jss.v045.i03>
- Verdine, B. N., Irwin, C. M., Golinkoff, R. M., & Hirsh-Pasek, K. (2014). Contributions of executive function and spatial skills to preschool mathematics achievement. *Journal of Experimental Child Psychology*, 126, 37–51. <https://doi.org/10.1016/j.jecp.2014.02.012>
- Viegut, A. A., Resnick, I., Miller-Cotto, D., Newcombe, N. S., & Jordan, N. C. (2023). Tracking informal fraction knowledge and its correlates across first grade. *Developmental Psychology*, 59(10), 1739–1756. <https://doi.org/10.1037/dev0001581>
- Watts, T. W., Duncan, G. J., Siegler, R. S., & Davis-Kean, P. E. (2014). What's past is prologue: Relations between early mathematics knowledge and high school achievement. *Educational Researcher*, 43(7), 352–360. <https://doi.org/10.3102/0013189x14553660>
- Wickham, H., Averick, M., Bryan, J., Chang, W., McGowan, L. D. A., François, R., & Yutani, H. (2019). Welcome to the Tidyverse. *Journal of Open Source Software*, 4(43), 1686. <https://doi.org/10.21105/joss.01686>
- Xu, C., Di Leonardo Burr, S., & LeFevre, J. A. (2023). The hierarchical relations among mathematical competencies: From fundamental numeracy to complex mathematical skills. *Canadian Journal of Experimental Psychology*, 77(4), 284–295. <https://doi.org/10.1037/cep0000311>