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## Poisson Regulation in Mother-Infant Gaze Systems

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**Abstract**—We propose stochastic models for the interactive regulation of gaze on/off each partner's face in mother/infant gaze as well as “turn-taking”. We infer that a Poisson timing mechanism indeed underlies the negative exponential distributions of gaze, providing a simplifying organizational principle for mother-infant communication, enabling both partners to predict the other's behavior. The Poisson rate constants quantify how likely infant or mother is to gaze on or off in comparison to each other. Mothers are far more likely to initiate gaze than infants, and infants are far more likely to terminate gaze than mothers. Initiation of a gaze “turn” (the individual unilaterally initiates gaze) follows a simple Poisson rule for infants, but mothers initiate a gaze turn with the second occurrence of infants' gaze termination. These turn findings suggest that mothers are using more history than infants in gaze turn regulation. For the infant, all three processes (gaze on, gaze off, gaze turn) are regulated by a Poisson process. © 2004 Elsevier Ltd. All rights reserved.

**Keywords**—Poisson models, Gaze, Turn-taking.

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## INTRODUCTION

During mother-infant face-to-face interaction, the timing of gaze on and off the other's face, as well as the timing of taking a turn, is a means of self-regulation for each, as well as communication between the two. The question arises as to whether there are identifiable stochastic structures for the emergence of gaze on, gaze off, and gaze turn-taking for each member of the dyad. According to our turn rule, the first partner to gaze unilaterally instantly becomes the turnholder, and remains so until the other gazes unilaterally, at which point the turn switches. Time series is used to encode both the "on-off" gaze states and turn-taking, for each member of the dyad.

Work since at least 1949 [1,2], in both speech and more recently gaze [3,4] has demonstrated negative exponential distributions of durations of on and off. Such distributions are a necessary condition for a process to be Poisson and these prior findings exhibit this telltale sign.

We use mother-infant gaze patterns as our example, since this modality has seen considerable prior work (see, for example, [3-8]). A description of the temporal model underlying each partner's on-off gaze stream, as well as an analysis of the temporal model underlying gaze turn-taking, constitutes our approach. A stochastic analysis of the time series representations quantifies the approach to modeling regulation.

## SUBJECTS AND DATA

One hundred twenty-eight mother-infant dyads were recruited from a large city hospital based on the criteria of the first full-term birth; uncomplicated pregnancy, delivery, and perinatal assessment; discharged with mother. The demographic range was 49% White, 15% Black, 33% Hispanic, 2% Asian, 1% American Indian. The sample is well educated with 27% having some college, 32% being a college graduate, and 34% having some postcollege education. The sample is comparable to other community samples in the literature.

### Data Source: Gaze On-Off in Face-to-Face Play at Four Months

The infant was seated in an infant seat, with its mother seated opposite. Two videotape cameras generated a split-screen view of the interaction. Mothers were instructed to play with their infants as they would at home, for approximately ten minutes. Gaze on/off each partner's face was coded for the first 150 seconds of uninterrupted interaction in which both partners' faces were visible and the infant was not crying. Slow motion was used to clarify exact timing of events. With three sets of reliability dyads (mother-infant pairs), two coders achieved mean kappas of .78 ( $N = 12$ ), .80 ( $N = 18$ ), and .83 ( $N = 14$ ).

### Data Structure

There are two types of time-series in the present data. The first consists of the raw on-off gaze data and may itself be regarded as two time series of length 150 seconds, one series each for mother and infant, respectively. At each of the 150 seconds, each series is 1 if the given member is gazing at the other's face and is 0 otherwise. A second time series for each partner consists of turn initiations. Each such series is 1 when the subject initiates the turn and is 0 otherwise. For both data structures, the record begins at the first gaze on. The 150 sec. time series were concatenated for the 128 mother-infant dyads. The structure of the gaze on/off series implies that an individual who gains the turn keeps it in spite of subsequent joint speech or joint silence, until the other partner gazes unilaterally, at which point the gaze turn switches.

Table 1. Four time series, two each for infant and mother, defining gaze on-off and turn data. Asterisks indicate turn initiations.

Turn Initiation							*					*								*			
Infant On/Off	0	0	1	1	0	0	0	1	1	1	0	1	1	0	1	0	0	0	0	1	1	0	0
Mother On/Off	0	1	1	1	1	1	0	0	1	1	1	0	0	0	0	1	1	1	0	0	1	1	0
Turn Initiation		*									*					*						*	
Time in Seconds	1	2	3	4	5	6	7	8	9	10	...	...	...										

**On-Off Data**

In the above time series for infant and mother, the waiting times for infant gaze off (that is, duration of “on”), in the first 22 seconds are 2, 3, 2, 1, and 2, whereas, the waiting times for mother gaze off (duration of “on”) are 5, 3, 3, 2. The waiting times for infant gaze on (that is duration of “off”) in the first 22 seconds are 2, 3, 1, 1, 4, and 2, whereas mother gaze on waiting times (that is duration “off”) are 1, 2, 4, 2, and 1.

**Turn Data**

The turn rule states that the first partner to gaze unilaterally instantly becomes the turnholder, and remains so until the other gazes unilaterally, at which point the turn switches. We distinguish between the initiation of the turn vs. its maintenance. In the above time series, asterisks above seconds #8, 12, and 20 indicate that the infant initiates a gaze turn; asterisks below seconds #2, 11, 16, and 22 indicate that mother initiates a gaze turn. Note that the turn initiation waiting time series for *one member* depends on the gaze series *for both*. It is therefore a series that contains, for each member of the dyad, information on the interaction of both. This makes the turn an important systemic unit.

**METHODS**

1. GAZE ON AND OFF. Our approach studies the histograms for the waiting times between recurrences of gaze on and gaze off. This corresponds to studying four histograms: infant gaze-on waiting times, infant gaze-off waiting times, and likewise for the mother, each inferred from the corresponding binary gaze on-off time series.

The motive for this approach is in the possibility that the shape of the histograms will suggest a known stochastic structure. The criterion in the present data is that the histograms are of a negative exponential type. Such histograms are a telltale sign of a Poisson process (see the Appendix), a well-known process characterizing many natural and man-made systems [9–14]. The Poisson model proposed by Mosteller [1] and Verzeano [2] conceives of the speech timing of *monologue* as a sequence of on-off intervals constructed by alternate random draws from frequency distributions of “ons” and “offs”. Note that these early reports did not deal with *dyadic* interaction, as the present study does.

The waiting time density,  $h(t)$ , for a Poisson process has the form  $h(t) = \lambda e^{-\lambda t}$ , where  $\lambda$  is referred to as rate constant. Poisson processes have the property that the mean waiting time between the next occurrences of the root event is  $1/\lambda$ . More generally, the rate constant allows exact predictions of the mean waiting time for a given number of occurrences of the root event to take place. A standard method of testing for a Poisson process is to find the  $\lambda$  that gives a best negative exponential fit to the sample histogram of waiting times. This is done here via nonlinear regression using the BMDP statistical package.

The fact that the waiting time density is of negative exponential type follows from its definition in the Appendix. The probability,  $p$ , of a Poisson event occurring in a small time interval,  $h$ , has the form

$$p = \lambda h + o(h), \quad \text{for some } \lambda > 0,$$

where  $o(h)$  is a term that approaches 0 faster than  $h$ . Hence,  $p$  is essentially  $\lambda h$  for small time intervals  $h$  and the rate constant  $\lambda$  quantifies how likely the given event is to occur in  $h$  units of time.

2. GAZE TURN INITIATIONS. The turn rule is used to generate binary series for the occurrence of each partner's turn initiation from the two partners' binary series. Let  $(i, m)$  denote the (infant, mother) gaze series. Infant turn initiation occurs at time  $t$  if  $(i, m) = (1, 0)$  and if at time  $t - 1$   $(i, m)$  is one of  $(0, 0)$  or  $(0, 1)$  or  $(1, 1)$ . The condition at  $t - 1$  may be put as  $(i, m) \neq (1, 0)$ , a state that implies the infant is a turnholder at  $t$  but not initiating the turn. In compact form, infant turn initiation occurs at time  $t$  if  $(i, m) = (1, 0)$  and if at time  $t - 1$   $(i, m)$  assumes a different value. The definition for mother turn initiation is analogous.

The above definition is used to construct two waiting time turn initiation histograms, one each for infant and mother. These are studied for signs of a stochastic structure, just as with the gaze series. The criteria are the sign of a simple Poisson process, a histogram of negative exponential type, and that of a recurring Poisson process, a histogram of gamma type [9-13]. The waiting time density,  $h(t)$ , for the  $r^{\text{th}}$  occurrence ( $r \geq 2$ , the  $(r - 1)^{\text{st}}$  recurrence) of a Poisson process with rate constant  $\lambda$  has the form

$$h(t) = \frac{\lambda}{(r - 1)!} (\lambda t)^{r-1} e^{-\lambda t}, \quad \text{a gamma type density.}$$

A standard method of testing for a recurring Poisson process is to find the  $\lambda$  and  $r$  that give a best fit of the above form to the sample histogram. The parameters  $\lambda$  and  $r$  are estimated here with nonlinear regression using the BMDP statistical package. It is essential that the *estimate* of  $r$  be near a positive integer for the recurring Poisson model to be considered.

## RESULTS

1. GAZE ON AND OFF. The histograms for the GAZE ON and GAZE OFF waiting times for both mother and infant (see Figures 1-4) suggest the exponential shape that indicates an underlying Poisson process. Table 2 summarizes the GAZE ON and OFF findings. The goodness of fit measure (RMS) reveals that all four histograms are well fit by negative exponentials. Note that the expected error for each parameter ( $\lambda$ ) is at most about 8% of the estimated parameter. This is evidence of an underlying Poisson mechanism for each of the four time series. The overplot in Figures 5-8 of predicted (P) and observed (O) further reveals how well the histograms are fitted by negative exponentials.

The mother's rate constant for going from GAZE ON to GAZE OFF is .219 and the infant's is .407. This reveals that mothers are less likely to pass from GAZE ON to GAZE OFF and tend hold their GAZE ON much longer than infants. Infants are twice as inclined as mothers to suddenly break into GAZE OFF from GAZE ON and tend to remain in the GAZE ON state for less time. This follows from the fact that the mean waiting time between successive Poisson events is  $1/\lambda$ .

The mother's rate constant for going from GAZE OFF to GAZE ON is 1.368 and the infant's is 0.237. The ratio of these rate constants is 5.77, indicating that mothers are about six times more likely to go from gaze off to on than their infants. This also reveals that mothers stay in GAZE OFF for much shorter periods of time than infants.

The infant shows opposite tendencies. The infant and mother rate constants for going from GAZE ON to GAZE OFF are 0.407 and 0.219, respectively, with a ratio of 1.86, indicating that the infants are about two times more likely to go from gaze on to off than their mothers. This suggests a compensatory mechanism between mothers and infants, meaning that as the penchant of one member goes up, that of the other goes down.

INFANT ON HISTOGRAM EACH SYMBOL REPRESENTS 10 OBSERVATIONS

NAME	INTERVAL	FREQUENCY				PERCENTAGE							
		50	100	150	200	250	300	350	400	INT.	CUM.	INT.	CUM.
*1	+	+-----+-----+-----+-----+-----+								360	360	28.3	28.3
*2	+	+-----+-----+-----+-----+-----+								214	574	16.8	45.2
*3	+	+-----+-----+-----+-----+-----+								156	730	12.3	57.4
*4	+	+-----+-----+-----+-----+-----+								85	815	6.7	64.1
*5	+	+-----+-----+-----+-----+-----+								70	885	5.5	69.6
*6	+	+-----+-----+-----+-----+-----+								49	934	3.9	73.5
*7	+	+-----+-----+-----+-----+-----+								40	974	3.1	76.6
*8	+	+-----+-----+-----+-----+-----+								27	1001	2.1	78.8
*9	+	+-----+-----+-----+-----+-----+								29	1030	2.3	81.0
*10	+	+-----+-----+-----+-----+-----+								27	1057	2.1	83.2
*11	+	+-----+-----+-----+-----+-----+								21	1078	1.7	84.8
*12	+	+-----+-----+-----+-----+-----+								26	1104	2.0	86.9
*13	+	+-----+-----+-----+-----+-----+								20	1124	1.6	88.4
*14	+	+-----+-----+-----+-----+-----+								13	1137	1.0	89.5
*15	+	+-----+-----+-----+-----+-----+								5	1142	0.4	89.9
*16	+	+-----+-----+-----+-----+-----+								10	1152	0.8	90.6
*17	+	+-----+-----+-----+-----+-----+								5	1157	0.4	91.0
*18	+	+-----+-----+-----+-----+-----+								11	1168	0.9	91.9
*19	+	+-----+-----+-----+-----+-----+								5	1173	0.4	92.3
*20	+	+-----+-----+-----+-----+-----+								8	1181	0.6	92.9
*21	+	+-----+-----+-----+-----+-----+								8	1189	0.6	93.5
*22	+	+-----+-----+-----+-----+-----+								6	1195	0.5	94.0
*23	+	+-----+-----+-----+-----+-----+								12	1207	0.9	95.0
*24	+	+-----+-----+-----+-----+-----+								4	1211	0.3	95.3
*25	+	+-----+-----+-----+-----+-----+								3	1214	0.2	95.5
*26	+	+-----+-----+-----+-----+-----+								5	1219	0.4	95.9
*27	+	+-----+-----+-----+-----+-----+								1	1220	0.1	96.0
*28	+	+-----+-----+-----+-----+-----+								0	1220	0.0	96.0
*29	+	+-----+-----+-----+-----+-----+								3	1223	0.2	96.2
*30	+	+-----+-----+-----+-----+-----+								3	1226	0.2	96.5
*31	+	+-----+-----+-----+-----+-----+								2	1228	0.2	96.6
*32	+	+-----+-----+-----+-----+-----+								1	1229	0.1	96.7
*33	+	+-----+-----+-----+-----+-----+								3	1232	0.2	96.9
*34	+	+-----+-----+-----+-----+-----+								3	1235	0.2	97.2
*35	+	+-----+-----+-----+-----+-----+								2	1237	0.2	97.3
*36	+	+-----+-----+-----+-----+-----+								1	1238	0.1	97.4
*37	+	+-----+-----+-----+-----+-----+								0	1238	0.0	97.4
*38	+	+-----+-----+-----+-----+-----+								4	1242	0.3	97.7
*39	+	+-----+-----+-----+-----+-----+								4	1246	0.3	98.0
*40	+	+-----+-----+-----+-----+-----+								0	1246	0.0	98.0
*41	+	+-----+-----+-----+-----+-----+								0	1246	0.0	98.0
*42	+	+-----+-----+-----+-----+-----+								1	1247	0.1	98.1
*43	+	+-----+-----+-----+-----+-----+								1	1248	0.1	98.2
*44	+	+-----+-----+-----+-----+-----+								1	1249	0.1	98.3
*45	+	+-----+-----+-----+-----+-----+								1	1250	0.1	98.3

Figure 1. Waiting time histogram for infant gaze on.

INFANT OFF HISTOGRAM EACH SYMBOL REPRESENTS 10 OBSERVATIONS

NAME	INTERVAL	FREQUENCY				PERCENTAGE							
		50	100	150	200	250	300	350	400	INT.	CUM.	INT.	CUM.
*1	+	+-----+-----+-----+-----+-----+								240	240	18.8	18.8
*2	+	+-----+-----+-----+-----+-----+								193	433	15.1	33.9
*3	+	+-----+-----+-----+-----+-----+								155	588	12.1	46.0
*4	+	+-----+-----+-----+-----+-----+								103	691	8.1	54.1
*5	+	+-----+-----+-----+-----+-----+								86	777	6.7	60.8
*6	+	+-----+-----+-----+-----+-----+								69	846	5.4	66.2

Figure 2. Waiting time histogram for infant gaze on.





MOTHER OFF HISTOGRAM EACH SYMBOL REPRESENTS 25 OBSERVATIONS

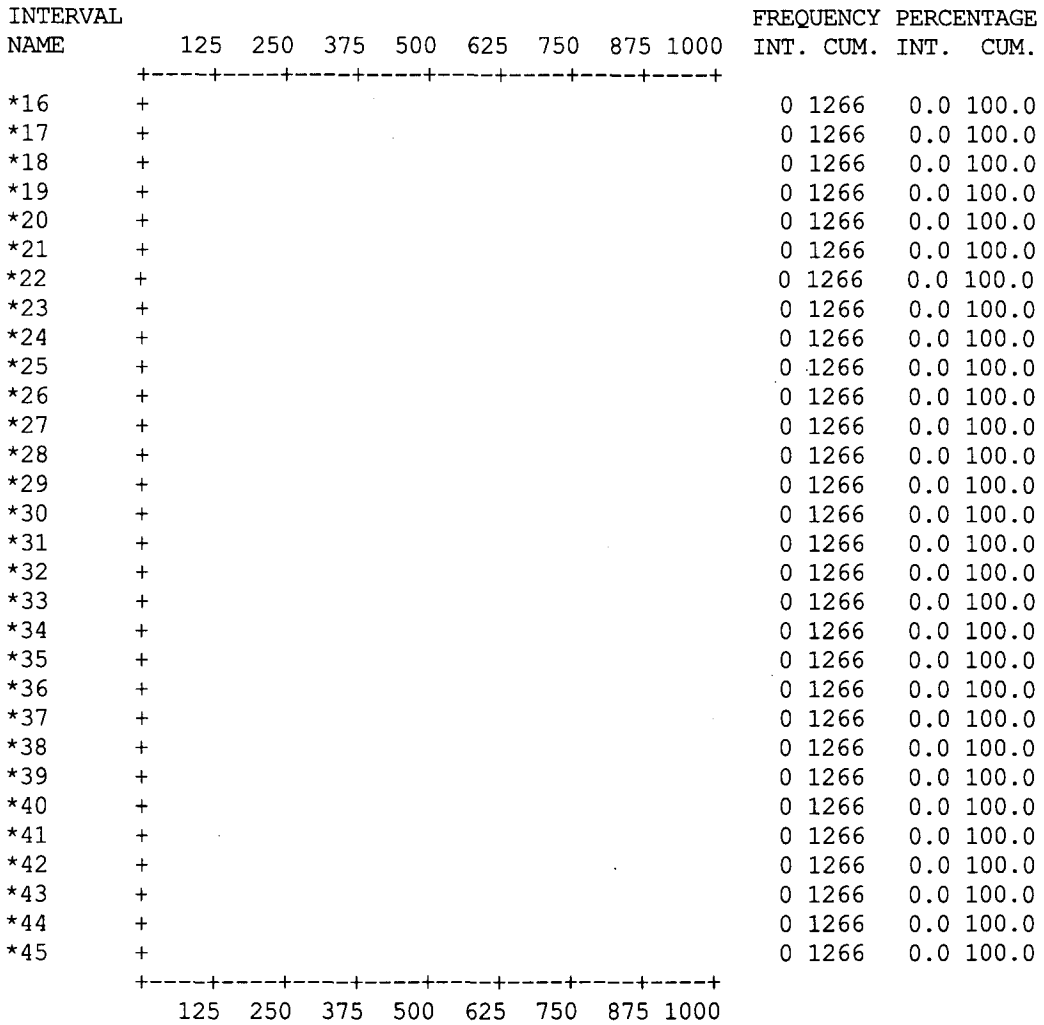


Figure 4. (cont.)

Table 2. Model results for all data pooled.

Group	Model	Parameter(s)	N <sup>1</sup>	Goodness of Fit (RMS)	DF
MOTHER GAZE ON TO OFF	Poisson	0.219 + / - 0.016	1363	0.192072E-03	42
MOTHER GAZE OFF TO ON	"	1.368 + / - 0.108	1266	0.283940E-03	12
INFANT GAZE ON TO OFF	"	0.407 + / - 0.014	1271	0.449780E-04	46
INFANT GAZE OFF TO ON	"	0.237 + / - 0.005	1278	0.140045E-04	51
MOTHER TURN	Gamma <sup>2</sup>	0.365 + / - 0.029 1.039 + / - 0.087	1776	0.102038E-03	48
INFANT TURN	Poisson	0.114 + / - 0.009	282	0.125612E-03	43

<sup>1</sup>N is the number of waiting time intervals used, not the number of subjects.

<sup>2</sup>The parameters of the gamma fit are the estimates of λ and r in the recurring (r<sup>th</sup> occurrence or (r - 1)<sup>st</sup> recurrence) Poisson density h(t) given by

$$h(t) = \frac{\lambda}{(r-1)!} (\lambda t)^{r-1} e^{-\lambda t}.$$



INFANT ON TO OFF POISSON FIT.

RESIDUAL MEAN SQUARE 0.449780E-04

DEGREES OF FREEDOM 46

PARAMETER	ESTIMATE	ASYMPTOTIC STANDARD DEVIATION	TOLERANCE
P1	0.406903	0.013521	1.000000

SERIAL CORRELATION 0.5902

DURBIN-WATSON STATISTIC 0.7910 BASED ON 47 RESIDUALS

PLOT OF PREDICTED (P) VERSUS OBSERVED (O)

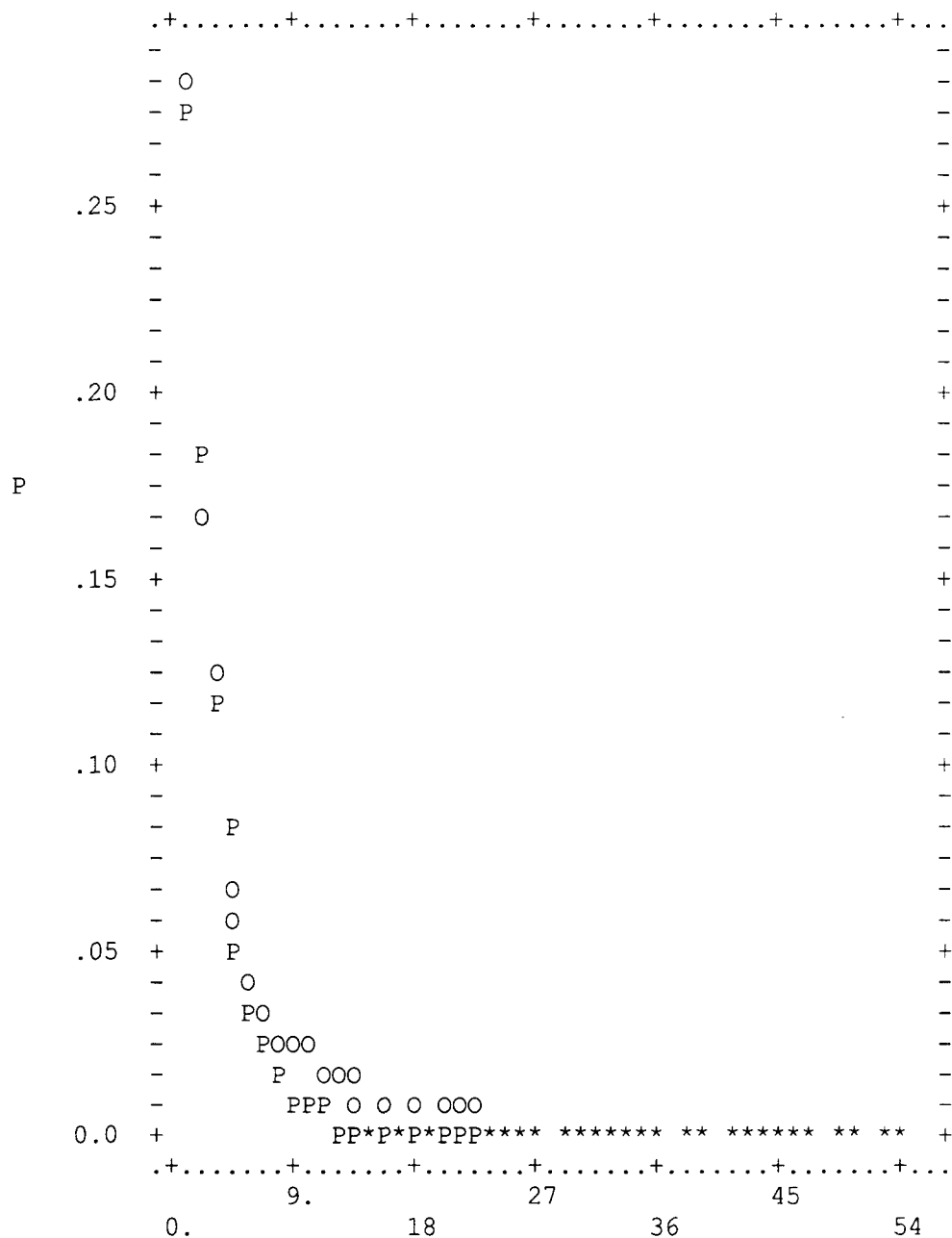


Figure 5. Infant ON Poisson fit. P and O represent predicted by the exponential fit and observed, respectively.

INFANT OFF TO ON POISSON FIT.

RESIDUAL MEAN SQUARE		0.140045E-04	
DEGREES OF FREEDOM		51	
PARAMETER	ESTIMATE	ASYMPTOTIC STANDARD DEVIATION	TOLERANCE
P1	0.237431	0.004710	1.000000

SERIAL CORRELATION 0.5540  
 DURBIN-WATSON STATISTIC 0.8910      BASED ON 52 RESIDUALS

PLOT OF PREDICTED (P) VERSUS OBSERVED (O)

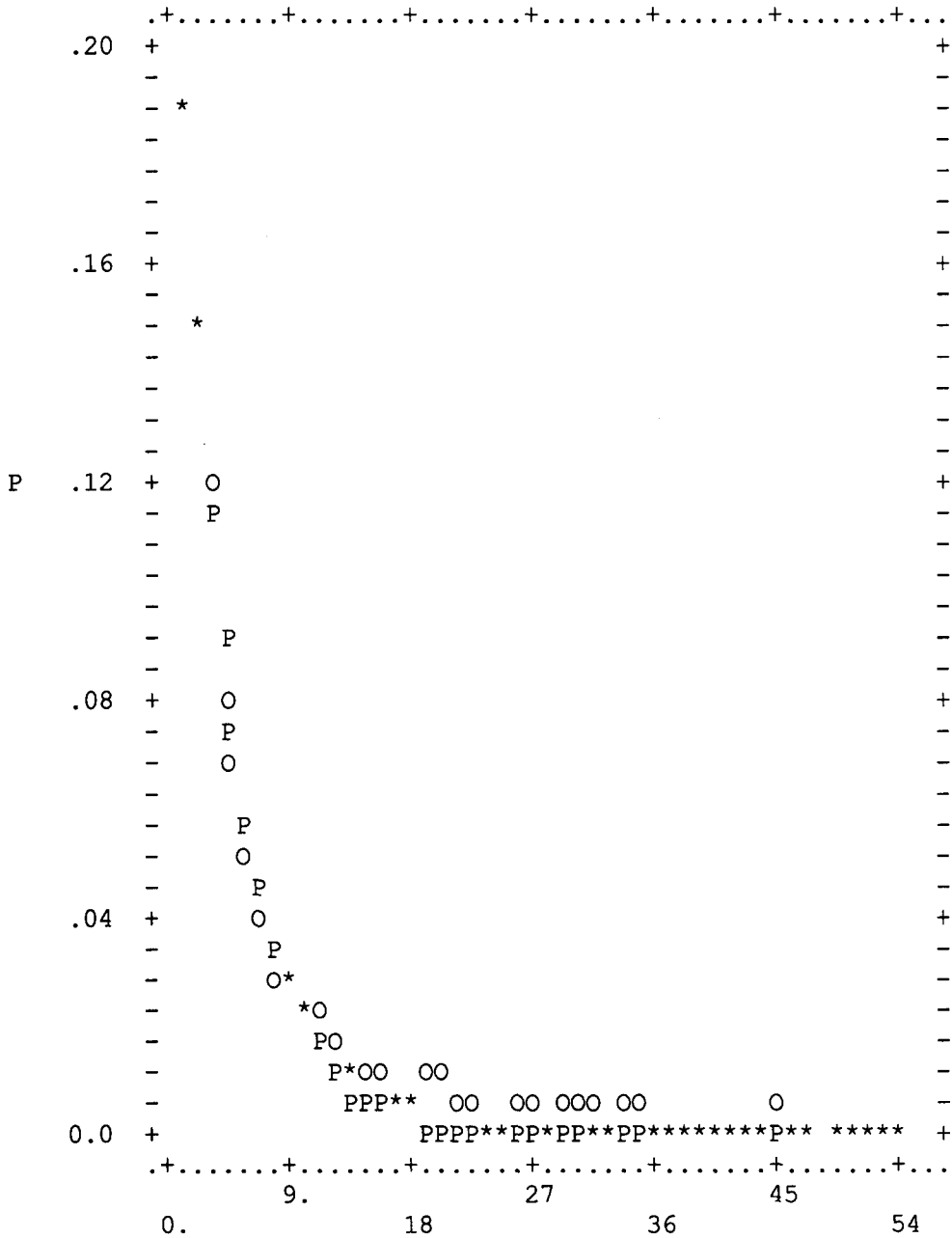


Figure 6. Infant OFF Poisson fit. P and O represent predicted by the exponential fit and observed, respectively.

MOTHER ON TO OFF POISSON FIT.

RESIDUAL MEAN SQUARE 0.192072E-03

DEGREES OF FREEDOM 42

PARAMETER	ESTIMATE	ASYMPTOTIC STANDARD DEVIATION	TOLERANCE
P1	0.218677	0.016384	1.000000

SERIAL CORRELATION 0.6337

DURBIN-WATSON STATISTIC 0.6847 BASED ON 43 RESIDUALS

PLOT OF PREDICTED (P) VERSUS OBSERVED (O)

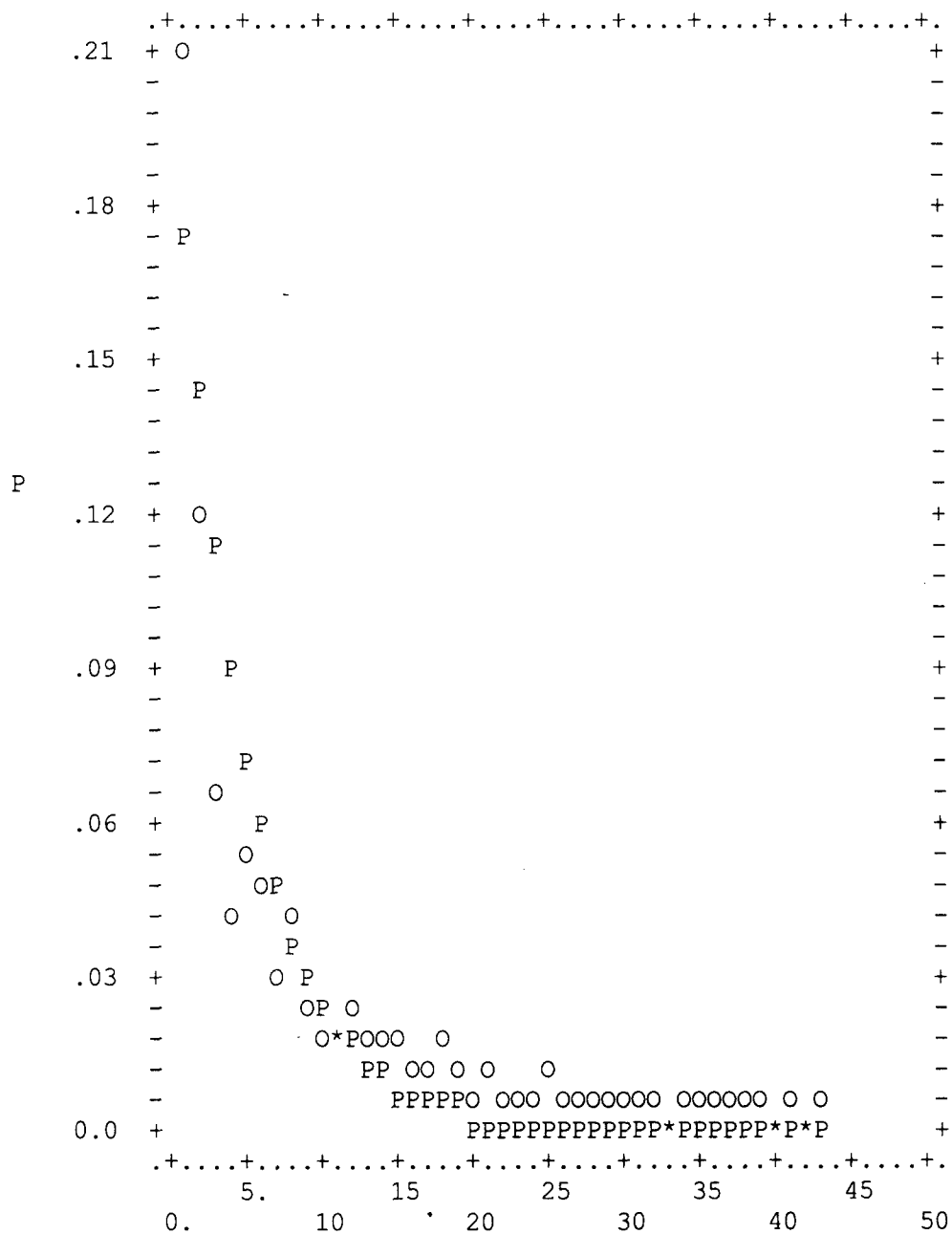


Figure 7. Mother ON Poisson fit. P and O represent predicted by the exponential fit and observed, respectively.

MOTHER OFF TO ON POISSON FIT.

RESIDUAL MEAN SQUARE		0.283940E-03	
DEGREES OF FREEDOM		12	
PARAMETER	ESTIMATE	ASYMPTOTIC	TOLERANCE
		STANDARD DEVIATION	
P1	2.736296	0.107728	1.000000

SERIAL CORRELATION 0.2918  
 DURBIN-WATSON STATISTIC 1.097      BASED ON 13 RESIDUALS

PLOT OF PREDICTED (P) VERSUS OBSERVED (O)

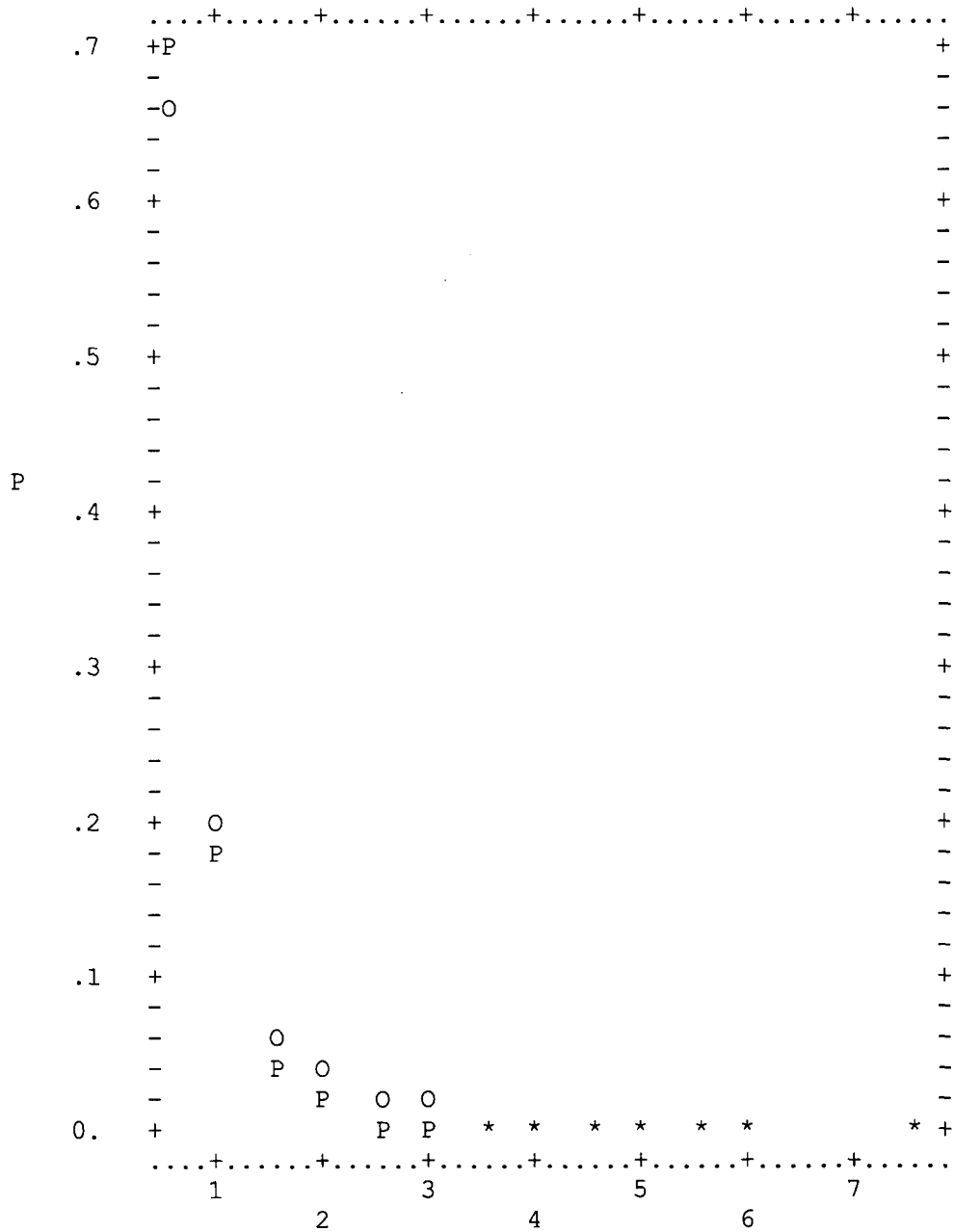


Figure 8. Mother OFF Poisson fit. P and O represent predicted by the exponential fit and observed, respectively.

2. TURN. The histogram for the infant's TURN (Figure 9) again suggests the exponential shape that is telltale of a Poisson process. The parameters in Table 2 reveal that this histogram is in fact well fit by an exponential and the overplot of predicted by regression versus observed (Figure 10) confirms this. The infant turn rate constant is given by  $0.114 + / - 0.009$ .

INFANT ROLE TURN HISTOGRAM												
EACH SYMBOL REPRESENTS 1 OBSERVATIONS												
INTERVAL									FREQUENCY	PERCENTAGE		
NAME	5	10	15	20	25	30	35	40	INT.	CUM.	INT.	CUM.
+-----+-----+-----+-----+-----+-----+-----+-----+												
*1	+								0	0	0.0	0.0
*2	+	#####							34	34	12.1	12.1
*3	+	#####							32	66	11.3	23.4
*4	+	#####							21	87	7.4	30.9
*5	+	#####							14	101	5.0	35.8
*6	+	#####							6	107	2.1	37.9
*7	+	#####							13	120	4.6	42.6
*8	+	#####							11	131	3.9	46.5
*9	+	#####							11	142	3.9	50.4
*10	+	#####							9	151	3.2	53.5
*11	+	#####							8	159	2.8	56.4
*12	+								0	159	0.0	56.4
*13	+	#####							7	166	2.5	58.9
*14	+	###							3	169	1.1	59.9
*15	+	#####							8	177	2.8	62.8
*16	+	#####							5	182	1.8	64.5
*17	+	####							4	186	1.4	66.0
*18	+	##							1	187	0.4	66.3
*19	+	###							3	190	1.1	67.4
*20	+	####							4	194	1.4	68.8
*21	+	#####							5	199	1.8	70.6
*22	+	#####							7	206	2.5	73.0
*23	+	#####							5	211	1.8	74.8
*24	+								0	211	0.0	74.8
*25	+	##							2	213	0.7	75.5
*26	+	###							3	216	1.1	76.6
*27	+	#####							6	222	2.1	78.7
*28	+	##							1	223	0.4	79.1
*29	+	###							3	226	1.1	80.1
*30	+	##							2	228	0.7	80.9
*31	+	##							1	229	0.4	81.2
*32	+	###							3	232	1.1	82.3
*33	+	##							1	233	0.4	82.6
*34	+	##							1	234	0.4	83.0
*35	+	##							1	235	0.4	83.3
*36	+	##							2	237	0.7	84.0
*37	+	####							4	241	1.4	85.5
*38	+	##							2	243	0.7	86.2
*39	+	##							2	245	0.7	86.9
*40	+	##							2	247	0.7	87.6
*41	+	##							1	248	0.4	87.9
*42	+								0	248	0.0	87.9
*43	+								0	248	0.0	87.9
*44	+								0	248	0.0	87.9
*45	+	##							1	249	0.4	88.3
+-----+-----+-----+-----+-----+-----+-----+-----+												
	5	10	15	20	25	30	35	40				

Figure 9. Waiting time histogram for infant turn.

INFANT ROLE TURN POISSON FIT.

RESIDUAL MEAN SQUARE		0.125612E-03	
DEGREES OF FREEDOM		43	
PARAMETER	ESTIMATE	ASYMPTOTIC	TOLERANCE
		STANDARD DEVIATION	
P1	.0113627	0.008561	1.000000

SERIAL CORRELATION 0.5787  
 DURBIN-WATSON STATISTIC 0.8124 BASED ON 44 RESIDUALS

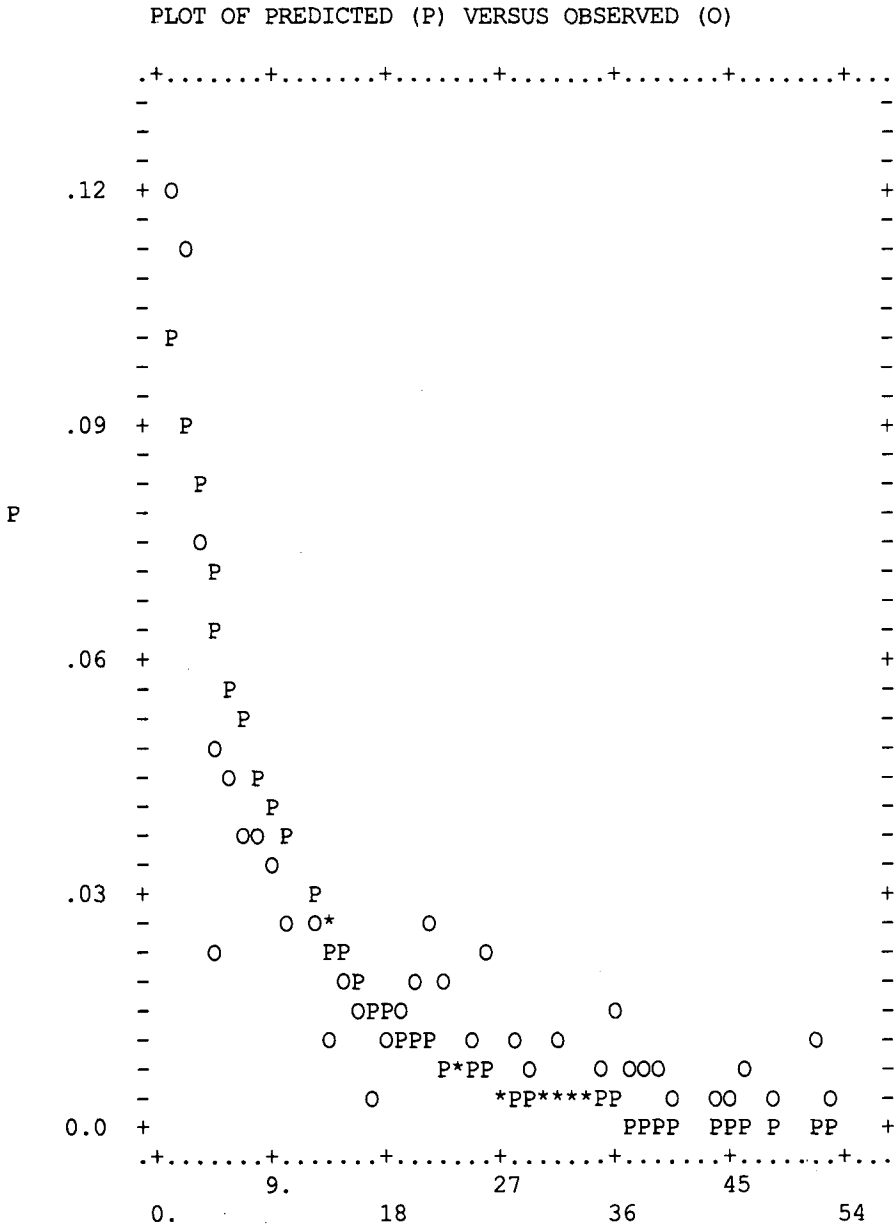


Figure 10. Infant ROLE TURN Poisson fit. P and O represent predicted by the exponential fit and observed, respectively.

The histogram for the mother's TURN (Figure 11) has the shape that is expected from a gamma type function. A numerical fit of the form given earlier finds that the histogram is well fit by an integer value of  $r$ , a necessary condition for the TURN to be a higher-order occurrence of an underlying Poisson process. The parameters  $\lambda$  and  $r-1$  of the fit are estimated at  $0.365 \pm 0.029$  and  $1.039 \pm 0.087$  (Table 2). The goodness of fit of the nonlinear regression is confirmed in

MOTHER ROLE TURN HISTOGRAM  
EACH SYMBOL REPRESENTS 10 OBSERVATIONS

INTERVAL NAME											FREQUENCY		PERCENTAGE	
	50	100	150	200	250	300	350	400	INT.	CUM.	INT.	CUM.		
*1	+										0	0	0.0	0.0
*2	+#####										207	207	11.7	11.7
*3	+#####										235	442	13.2	24.9
*4	+#####										183	625	10.3	35.2
*5	+#####										144	769	8.1	43.3
*6	+#####										156	925	8.8	52.1
*7	+#####										139	1064	7.8	59.9
*8	+#####										102	1166	5.7	65.7
*9	+#####										82	1248	4.6	70.3
*10	+#####										67	1315	3.8	74.0
*11	+#####										54	1369	3.0	77.1
*12	+####										44	1413	2.5	79.6
*13	+####										44	1457	2.5	82.0
*14	+####										41	1498	2.3	84.3
*15	+####										35	1533	2.0	86.3
*16	+###										29	1562	1.6	88.0
*17	+###										29	1591	1.6	89.6
*18	+##										16	1607	0.9	90.5
*19	+H										14	1621	0.8	91.3
*20	+##										19	1640	1.1	92.3
*21	+H										10	1650	0.6	92.9
*22	+H										14	1664	0.8	93.7
*23	+H										10	1674	0.6	94.3
*24	+H										10	1684	0.6	94.8
*25	+H										11	1695	0.6	95.4
*26	+H										9	1704	0.5	95.9
*27	+H										6	1710	0.3	96.3
*28	+H										6	1716	0.3	96.6
*29	+H										6	1722	0.3	97.0
*30	+										1	1723	0.1	97.0
*31	+H										5	1728	0.3	97.3
*32	+H										7	1735	0.4	97.7
*33	+										1	1736	0.1	97.7
*34	+										4	1740	0.2	98.0
*35	+										3	1743	0.2	98.1
*36	+										1	1744	0.1	98.2
*37	+										4	1748	0.2	98.4
*38	+										1	1749	0.1	98.5
*39	+										3	1752	0.2	98.6
*40	+										1	1753	0.1	98.7
*41	+										2	1755	0.1	98.8
*42	+										2	1757	0.1	98.9
*43	+										1	1758	0.1	99.0
*44	+										1	1759	0.1	99.0
*45	+										1	1760	0.1	99.1

Figure 11. Waiting time histogram for mother turn.

the predicted vs. observed overplot in Figure 12. This supports the inference that the mother TURN corresponds to the second occurrence of an underlying Poisson process whose rate is  $0.365 + / - 0.029$ .

We now ask whether any of the infant's behaviors, as measured by the parameters of the Poisson models, fall in the range of the mother's. The latter's rate constant is estimated at  $0.365 + / - 0.029$ , with an estimation interval of (.336, .394). The infant's gaze termination rate

MOTHER ROLE TURN GAMMA FIT.

RESIDUAL MEAN SQUARE		0.102038E-03	
DEGREES OF FREEDOM		48	
PARAMETER	ESTIMATE	ASYMPTOTIC STANDARD DEVIATION	TOLERANCE
P1	0.365246	0.029139	0.183127
P2	1.038612	0.087333	0.183127

SERIAL CORRELATION 0.6967  
 DURBIN-WATSON STATISTIC 0.5538 BASED ON 50 RESIDUALS

PLOT OF PREDICTED (P) VERSUS OBSERVED (O)

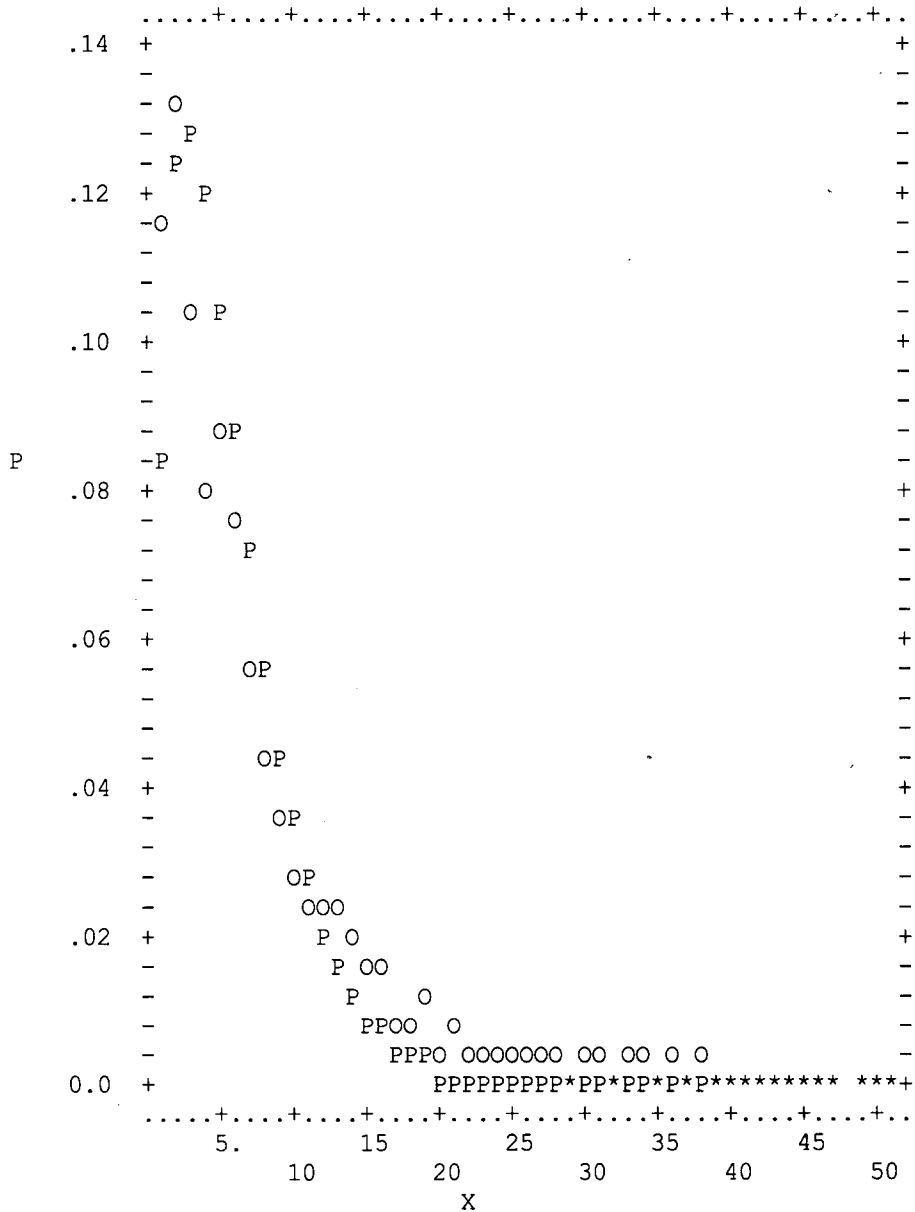


Figure 12. Mother ROLE TURN recurring Poisson fit. P and O represent predicted by the exponential fit and observed, respectively.

constant is estimated at  $0.407 \pm 0.014$ , with an interval of (.393, .421). These two intervals share a small overlap and are in a similar numerical range. In addition, no other such intervals (from among MOTHER GAZE ON TO OFF, MOTHER GAZE OFF TO ON, INFANT GAZE



OFF TO ON, and INFANT TURN) have a comparable property; indeed these others are quite disjoint from (.336, .394). These numerical results suggest that the underlying Poisson process whose second occurrence drives MOTHER TURN is INFANT GAZE TERMINATION. That is, the mother taking her unilateral gaze turn is triggered, on average by the second occurrence of an infant gaze alternation. In other words, mother is likely to initiate a unilateral gaze at her infant after the infant has looked away a second time.

## DISCUSSION—GAZE ON AND OFF

Negative exponential distributions are a well-known feature of sound and silence in monologue and dialogue in adults [1,2,15,16], as well as a feature of mother-infant gaze on and off [3,4]. This paper documents that the necessary condition of negative exponential waiting density for a Poisson timing mechanism is indeed met by the mother-infant gaze data. These Poisson findings *also* confirm a large amount of previous descriptive research which has shown that the mother is more likely to continue gazing than the infant, and the infant “makes and breaks” gaze, with a higher probability of gazing away [4,8]. Since speech and gaze both follow negative exponential distributions, we infer that the functional significance of this timing structure applies to both infants and adults, and across modalities, providing a simplifying organizational principle underlying communication (see [3]).

Although the Poisson model is stochastic, it has associated statistical properties that suggest why nature might select a Poisson process for mother and infant. Compared to normal distributions, negative exponential distributions have one rather than two parameters, such that the mean and standard deviation predict each other, thus, providing a simple mechanism through which expectancies can be created. Much literature suggests that infants can represent these distributions, and that the probabilities can be used by infants to compute expectancies [17]. Such a timing mechanism immediately permits detection of discrepancies from the expected. We presume that the existence of this simple timing mechanism for both gaze on and gaze off provides an advantage for mother and infant since both are highly predictable. That is, expectancy with a normally distributed waiting time depends on estimating both  $\mu$  and  $\sigma$ , but in the exponential case expectancy depends only on estimation of the mean because here  $\mu$  and  $\sigma$  are the same. The estimation of one parameter rather than two is a simpler principle, presumably facilitating the creation of expectancies.

It is surprising that both gaze on and off, as complementary processes, each follow a Poisson rule, both simultaneously organizing the behavior of both partners. There is no *a priori* reason to expect this result. It could easily happen that either process for either partner could have a normal, uniform or other distribution, the first showing a central tendency and the second reflecting much randomness in the choice of gaze state. We propose that a rhythmic process, *not yet identified mathematically in the frequency domain but manifested in time domain* via the Poisson models identified here, underlies gaze on and off within each partner, and that a rhythmic process links the two partners [16,18].

In many, if not all, brain regions, the firing of neurons also follows a Poisson rule. This tempts the speculation that the gaze on/off processes share something of the same regulatory mechanism that fires neurons. It may be that entire brain regions fire as Poisson events, making the principle of recapitulation, so familiar in biology, also true of behavioral systems.

## DISCUSSION—GAZE TURN

The range of values of the INFANT TURN rate constant is not near any other rate constant. The infant’s turn may be a Poisson event unto itself. From extensive work on vocal turn-taking, the turn is known to be at a different level of complexity from any of its component on or off distributions.

The characteristic turn-taking structure of polite adult verbal conversation is an alternation of speaker and listener, and this coordinated exchange of active and passive roles usually emerges from joint silence and only rarely from coaction. In contrast, the characteristic structure for gaze in such conversations is coactive (both gazing), rather than alternating. Since the first second of gaze turn initiation (one partner gazes unilaterally while the other's gaze is off) could be construed as a moment of dyadic "disjoin", it is remarkable that the moment of dyadic disjoin follows the same timing mechanism as the on and off within each partner. At least for the infant, all three processes of on, off, and turn are organized by the same timing mechanism.

As noted in the Results section, the mother taking her unilateral gaze turn is triggered, on average, by the second occurrence of an infant gaze termination. In other words, mother is likely to initiate a unilateral gaze at her infant (a "gaze turn") after the infant has looked away a second time. This agrees with the mother's familiar penchant to secure her infant's gaze.

The contrast of the infant turn following a simple Poisson model and the mother turn following a recurring Poisson model ( $\gamma$ ) suggests that the infant regulates the gaze turn somewhat independently of the mother and that the mother regulates more contingently upon the infant. In fact, the infant's Poisson model is stationary, and hence, has no internal sense of prior states or history. However, a recurring Poisson model, which fits the mother's behavior, has history in it. The sum of these observations suggest the idea that the mother is more contingent upon the infant than vice versa, when the data are approached in this way. They also suggest that the mother's unilateral gaze turn *is organized* in a form of waiting for the infant. These suggestions of differences in contingency structures require formal interactive contingency analyses to be confirmed.

## SUMMARY

This paper presents evidence that mother and infant gaze on and off the partner's face follow known stochastic models. In particular, the emergence of gaze on and off for both infant and mother appear to be Poisson processes. Infant gaze turn appears to regulate as a Poisson process also. However, mother gaze turn behaves as the second occurrence of a Poisson process whose rate is that of the infant's gaze going from on to off.

The prevalence of Poisson models at this behavioral level is not surprising given that many brain events are known to be Poisson. That such models manifest at the higher level of a behavior like gazing agrees with the established biological concept of recapitulation within living systems. Because the Poisson timing structure applies across the lifespan [3], and across modalities (vocalization and gaze), it provides a simplifying organizational principle underlying communication.

## APPENDIX

### TECHNICAL APPENDIX ON POISSON PROCESSES AND THEIR REPETITIONS

#### POISSON PROCESSES

A Poisson model characterizes the instantaneous probability of an event, here a gaze going on or off. The model states that the probability  $p(h)$  of an event occurring within a small time interval  $h$  is given by  $\lambda h + o(h)$ , for some  $\lambda > 0$ , where  $o(h)/h$  approaches 0 as  $h$  approaches 0—that is,  $o(h)$  approaches 0 faster than  $h$  does.

The quantity  $\lambda$  is called a rate constant. It has the specific meaning given above. In the present data, the rate constant for GAZE OFF TO ON times a small time interval is the probability of a change to GAZE ON, from OFF, in the time interval  $h$  and likewise for GAZE ON TO OFF. The rate constants for the present Poisson processes give information on behavior.

Poisson models arise in studies of radioactive emissions, people waiting for service at checkouts, birth and death of bacteria, immigration and emigration of peoples, cars queuing at toll booths, component failure in systems, and elsewhere [9].

The chief sign of a Poisson process is that the waiting times between successive events are distributed as a negative exponential. That is, if  $f(t)$  is the density for the amount of time between successive events, then

$$f(t) = \lambda e^{-\lambda t}, \quad \text{for some } \lambda > 0.$$

This implies that the probability  $p$  of an event occurring with time  $T$ , given an event at a starting time  $t = 0$  is given by

$$p = \lambda \int_0^T e^{-\lambda t} dt.$$

The parameter  $P1$  given in Figures 5–8 and 10 for the Poisson models corresponds to estimators of  $\lambda$ .

### REPEATED POISSON PROCESSES

The waiting time density for a Gamma model can be written:

$$f(t) = \frac{\lambda_1}{\lambda_3} (\lambda_1 t)^{\lambda_2} e^{-\lambda_1 t}.$$

If the process is the  $r^{\text{th}}$  occurrence of a Poisson process then  $\lambda_1$  will be its associated (Poisson) rate constant and  $\lambda_3$  will have the form  $(r - 1)!$  where  $r$  is the number of occurrences of the process and  $\lambda_2 = r - 1$ . Note that for a gamma function to correspond to a waiting time density for repetitions of a Poisson processes the quantity  $\lambda_2$  *must* be an integer.

If  $r - 1 = 1$ , then  $(r - 1)! = 1$  and in this simple case  $\lambda_2 = \lambda_3$ , and hence,  $f$  depends only upon two parameters rather than three. This is the model found with the present data and corresponds to  $r = 2$ .

The parameters  $Pj$  given in Figure 12 for the gamma model correspond to estimators of  $\lambda_j$ ,  $j = 1, 2$ .

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