An Introduction to
the Syntax and Semantics of Modal Auxiliary Constructions

David Conan Wolfsdorf

Purpose and Modality

The count noun "purpose" is at least two ways ambiguous. In one sense, "purpose" denotes a goal-oriented mental state akin to aim or intention. For instance:

> For the past five years, Adam has been working long hours. His purpose is to save enough money to buy a summer house in Connecticut.

We can substitute "purpose" here with "intention," "objective," "goal," or "aim" and maintain the sense of the original:

> For the past five years, Adam has been working long hours. His intention/ objective/goal/aim is to save enough money to buy a summer house in Connecticut.[1]

A second sense of "purpose" is akin to "function," "use," or "role." For example:

> Since they were first widely distributed in the eighties, the purpose of cellphones has changed.

We can substitute "purpose" here with "function," "use," or "role" and maintain the sense of the original:

> Since they were first widely distributed in the eighties, the function/use/ role of cellphones has changed.

We may refer to the two senses of "purpose" as "mental" and "functional." My interest here is principally with functional purposes. More precisely, I am interested in a certain modal entailment of functional purposes. I will clarify this point shortly.

Presently, granted the focus on functional purposes, there is a noteworthy relation between mental purposes and at least some functional purposes. One way that entities have functional purposes is through their relation to mental purposes: mental purposes can endow

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[1] Here and throughout I use Arial font for linguistic examples set off from the main text.

entities with functional purposes. For example, rocks in nature typically have no functional purpose. However, a person can employ a rock as a doorstop, as a weapon, or to crack open a nut. In such cases, the person's mental purpose endows the rock with a functional purpose.

Mental purposes are then a kind of functional purpose provider. There are other kinds of functional purpose providers. This includes other kinds of mental states, but – crucially – it may also include entities that are not mental states. Presently, whatever the range of providers of functional purposes, the basic point is that there is a distinction between a functional purpose and a functional purpose provider, be it a mental purpose or some other entity. Hereafter, I will be concerned with functional purposes, and so I will drop the modifier "functional" unless needed for disambiguation.

Consider now the following idea:

If x has a purpose, there is something that x ought to or must do or some way that x ought to or must be.

Since "ought" and "must" are modal auxiliaries, if the preceding conditional is true, there is an entailment from purpose to modality. I will assume that the conditional is true. Given this, I want to explicate the consequent in the conditional by discussing the semantics and syntax of modal expressions, particularly modal auxiliary constructions.

More precisely, my discussion of modal auxiliary constructions principally aims to clarify two things. First, modal auxiliaries are standardly distinguished in terms of their so-called strength or force. For example, compare "must" and "should" in the following minimal pair:

You must see the new Eakins exhibition.
You should see the new Eakins exhibition.

The modal auxiliary "must" is said to be stronger or more forceful than the modal auxiliary "should." Accordingly, one objective is to clarify the modal strength associated with purposes. It might even be questioned whether all purposes are associated with the same modal strength.

Second, readings of modal auxiliary constructions are standardly distinguished in terms of their so-called flavor. For example, compare the following two "must" constructions:

He must be caught in traffic.
He must be caught and punished.

The first sentence is naturally read in terms of probability. The second sentence is naturally read in terms of legal or deontic obligation. These are said to be distinct "flavors" of modality. Accordingly, a second issue is determining the modal flavor associated with purposes. Once again, it might be questioned whether all purposes are associated with a single modal flavor.

In sum, there is an entailment from purpose to modality. But it is unclear what strength of modality is entailed. In the conditional I presented above, the consequent consists of a disjunction. It is also unclear what flavor of modality is entailed. The conditional is silent on this point.

In order to resolve these questions, we need to understand the syntax and semantics of modal auxiliary constructions. This will be the focus of the remainder of the present chapter.

Modal Auxiliaries

Compare the opening words (1) of the Roman poet Catullus' *Carmen* 13 with the following gloss (1g) and English translation (1e):

1. Cenabis bene …

1g. Dine.Fut.2sg well …[2]

1e. "You will dine well …"

In Latin, the person and number of the verb and the future tense are affixed to the verbal stem *cena-*. The morpheme *-b-* is a future tense infix; the morpheme *-is* is a second person singular suffix.

   Since person and number are affixed to the verbal stem, Latin does not syntactically require an independent personal pronoun – which in this case would be *tu*. The number, person, and tense affixes of the verb are called "inflectional" affixes.[3] So, in (1) *cena-* is inflected in number, person, and tense.

   Relative to Latin, the verbal inflectional system of English is impoverished. Consequently, much of the work that verbal inflection performs in Latin must in English be outsourced to independent elements in the clause. For instance, in (1e) the second person singular pronoun "you" is a required element; and "will" is employed to express future tense.

   In traditional grammar, (1e) "dine" is said to be the "main" verb in the clause, and "will" is said to be an "auxiliary" verb. The idea is that since the main verb lacks the capacity for a future tense inflection, the verb "will" helps "dine" to express future tense.

   Some contemporary syntacticians treat auxiliaries and verbs as exclusive categories.[4] In that case, the phrase "auxiliary verb" is incoherent. I accept the contemporary syntactic distinction and will use "auxiliary" and "verb" to denote exclusive syntactic categories.[5]

   In addition to number, person, and tense, verbal expressions in various languages may be inflected in mood. Mood is the grammatical category whose function is to express the semantic property of modality.[6] Precisely what modality is is debatable. It will be convenient to begin with

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[2] (1g) constitutes a gloss of the morphemes of (1), following standard Leipzig glossing rules.

[3] Inflectional affixes are contrasted with derivational affixes. The former alter a lexical item in a systematic way. The latter create new lexical items; for example: "entertainment" from the verb "entertain" and the deverbal nominalizing suffix "-ment."

[4] E.g. cp. A. Radford, *Syntactic Theory and the Structure of English*, Cambridge University Press, 1997, 49-54.

[5] The terminology here is less important than the theoretical grounds for the distinction. In particular, modal auxiliaries and (lexical) verbs perform distinct syntactic and semantic functions and, crucially, occupy different positions in the clausal structure. For a recent defense of the view that English modal auxiliaries are not verbs, cp. Barbiers and van Dooren (2017). For an older view that English modals are verbs, cp. I. Roberts, "Agreement Parameters and the Development of English Modal Auxiliaries," in *Natural Language and Linguistic Theory* 3 (1985) 21-58.

[6] Cp. R. Huddleston and G. K. Pullum, *The Cambridge Grammar of the English Language*, Cambridge University Press, 2002, 172: "mood is a category of grammar, modality is a category of meaning. Mood is the grammaticalization of modality within the verbal system … As far as English is concerned, historical

the following common view: "Modality is a linguistic category of meaning having to do with the expression of possibility and necessity."[7]

As in the case of number, person, and tense, English employs auxiliaries to express modality in verbal phrases. For example, consider:

2. Janet may be at home.

In fact, "be" can be inflected to express modality. However, its morphological capacities in this respect are limited. In (2), the auxiliary "may" is needed to convey the intended modal content. "May," then, is an example of a modal auxiliary.

(2) describes a possible state of affairs. Accordingly, (2) might be glossed in either of the following ways:

2m. Possibly, Janet is at home.
2m'. It is possible that Janet is at home.

English – the natural language on which we will be focusing – contains modal expressions of various syntactic categories. For example, there are modal nouns such as "likelihood" and "inevitability" as well as "possibility" and "necessity" themselves. There are modal adjectives such as "permissible" and "required" as well as "possible" and "necessary." And there are modal adverbs such as "perhaps" and "certainly" as well as "possibly" and "necessarily."

Here, I focus on the most commonly discussed kinds of modal expressions: modal auxiliaries. Unless necessary for disambiguation, I will refer to modal auxiliaries simply as "modals."

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change has more or less eliminated mood from the inflectional system, with irrealis mood confined to the 1st/3rd person singular *were*, which is moreover usually replaceable by the ordinary preterite form *was*. The main mood system, therefore, is analytic rather than inflectional, marked by the presence or absence of special words, the modal auxiliaries."

[7] K. von Fintel, "Modality and Language," in D. Borchert, ed., *Encyclopedia of Philosophy*, Macmillan, 2006, 20-27. Here, von Fintel follows Kratzer: "modality has to do with necessity and possibility." (A. Kratzer, "The Notional Category of Modality," in *Modals and Conditionals*, Oxford University Press, 2012, 28) Cp. J. Van der Auwera and V. A. Plangian, modality comprises "the semantic domains that involve possibility and necessity as paradigmatic variants." ("Modality's Semantic Map," *Linguistic Typology* 2 (1998) 79-124, at 80) Contrast P. Portner's tentative, more expansive, definition: "modality is the linguistic phenomenon whereby grammar allows one to say things about, or on the basis of, situations which need not be real." (*Modality*, Oxford University Press, 2009, 1) Temporal posteriority (futurity) and anteriority (past tense-ness) as well as modality are forms of what C. Hockett calls "displacement." Displacement is the property of natural language that enables talk of what is non-actual. Hockett suggests that displacement is one of thirteen "design features" of natural languages: "Man is apparently almost unique in being able to talk about things that are remote in space or time (or both) from where the talking goes on. This feature – 'displacement' – seems to be definitely lacking in the vocal signaling of man's closest relatives, though it does occur in bee-dancing." ("The origin of speech," *Scientific American* 203 (1960) 88-96, at 90) von Fintel (2006) makes this point, citing from a later work of Hockett.) Given the range of our cognitive capacities, insofar as the function of natural language is to facilitate communication, we need to and are able to communicate not only about the here and now.

Modals and Modal Verbs

Modals have different syntactic properties than verbs. Consider the following three.[8] The object complement of a modal is a verb, whereas the object complement of a verb may be a nominal or non-finite verbal expression:

Janet may *want chocolate*.
Janet may *leave*.

Janet wants *chocolate*.
Janet wants *to leave*.[9]

In interrogatives, verbs require an auxiliary, for example "do" – so-called *do*-support – whereas modals do not:

Does Janet want chocolate?
* Wants Janet chocolate?

May Janet leave?
* Does may Janet leave?

The prefixed asterisk is used here, as standardly, to indicate syntactic ill-formedness.
And whereas modals are negated without an auxiliary, verbs require one:

Janet may not want to leave.

* Janet wants not to leave.
Janet does not want to leave.

A class of modal verbs, including "have (to)" and "need (to)," performs a similar semantic function to modals proper. Compare the following:

Janet must/should/might/may/could be at home.

Janet has/needs to be at home.

As the examples indicate, "has" and "needs" differ from "must," "should," "may," "might," and "could" in that the non-finite complement of "has" and "needs" requires the infinitival tense particle "to":

* Janet has/needs be at home.

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[8] Cp. Radford (1997) 50-51.
[9] Strictly "to travel" is a tense phrase, since the non-finite verb "travel" is here preceded by the infinitival tense phrase marker "to."

The distinction between the two classes of expressions is sometimes described as between modals proper and "semi-modals." But this terminology is misleading. Semantically, so-called semi-modals are just as modal as modals proper. What distinguishes the two classes is syntactic: so-called semi-modals are verbs. I'll call them "modal verbs."

Modal verbs differ from modals in the following additional four respects. First, modal verbs are inflected in the third person singular:

> I/you/we/they *have*/*need* to be at home.
> Janet *has*/*needs* to be at home.

Contrast:

> I/you/Janet/we/they *must*/*should*/*may*/*might* to be at home.

Second, modal verbs, qua verbs, require auxiliaries to form negations and interrogatives:

> Janet *does* not have/need to be at home.
> *Does* Janet have/need to be at home?

Observe also the following scopal difference between the negation of a modal and a modal verb. The following pair of positive statements have very similar meanings:

> Janet must eat more vegetables.
> Janet has to eat more vegetables.

Not so the following pair of negations:

> Janet must not eat more chocolate.
> Janet does not have to eat more chocolate.

"Must" scopes above the negation; "has to" scopes below it.

Third, modal verbs, qua verbs, admit future and past tense forms:

> Janet *had*/*needed* to be at home.
> Janet *will have*/*will need* to be at home.

Modals do not; for example:

> * Janet *will must* be at home.
> * Janet *musted* be at home.

Fourth, modal verbs themselves admit modals, whereas modals do not; for example:

> Janet might/may/should/must have to serve jury duty.

Janet might/may/must need to refill her transit card.[10]

* Janet might/may/should/must might/may/should/must serve jury duty/refill her transit card.

Finally, the modal "ought" deserves special notice. "Ought" may be characterized more precisely as a semi-verbal modal. At least, the non-finite verbal complement of "ought," like other modal verbs, must be preceded by the infinitival tense particle "to":

Janet ought to be at home.
* Janet ought be at home.

However, "ought" shares with other modals the following four properties. First, "ought" is not inflected in the third person singular:

I/you/Janet/we/they ought to be at home.

Second, in negations and interrogatives "ought" does not require an auxiliary:

I ought not to be at home.
Ought I to be at home?

Third, "ought" does not admit future and past tense forms:[11]

* I will ought to be at home.
* I oughted/have oughted to be at home.

Fourth, "ought" does not admit other modals as modifiers:

* She may/should/must/could ought to be at home.

In sum, I have distinguished modals and modal verbs as distinct syntactic categories. And I have observed that the modal "ought" shares most, but not all of its syntactic properties with other modals.

Introduction to the Logical Form of Modal Constructions

The semantics of an expression depends on its syntax. For example, although the following two sentences have the same constituents, their meanings differ because the relations between the constituents differ:

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[10] It is interesting to observe that "should need to" is infelicitous here, though it is acceptable, e.g., in "If Janet should need to refill her transit card, there is machine at Suburban Station."
[11] On the development of modal "ought," cp. R. Nordlinger and E. C. Traugott, "Scope and Development of Epistemic Modality: Evidence from *Ought To*," *English Language Linguistics* 1 (1997) 295-317.

>Ronan loves Katherine.
>Katherine loves Ronan.

Even words belonging to the same general syntactic category, for example noun, may differ in meaning insofar as they are members of distinct sub-categories. For example, singular count nouns and mass nouns have different kinds of denotations. Consider "shoe," which must refer to a single shoe, in contrast to "footwear," which may refer to a plurality of shoes:

>He removed the shoe from patio.
>He removed the footwear from the patio.

Consequently, insofar as we are interested in the semantics of modals and modal constructions, we should be interested in their syntax.

A distinction is often drawn between the superficial syntax of an expression, say, a sentence, and its deep or true syntactic structure.[12] The term "logical form" is often used to denote the true syntactic structure. A common example employed to illustrate this distinction derives from the phenomenon of quantifier scope. For instance, consider the following sentence:

>3. All the products have some defect.

(3) admits two readings, depending on whether the quantifier "all" takes scope over the quantifier "some" or vice versa. On both readings, each product is defective. But the former reading – formalized as (3a) below – is consistent with each product having a different defect, whereas the latter reading – formalized as (3b) – entails that there is some defect that all of the products share:

>3a. $\forall x\ [Px \rightarrow \exists y\ [Dy \wedge Ryx]]$
>3b. $\exists y\ [Dy \wedge \forall x\ [Px \rightarrow Ryx]]$

Since, according to the superficial syntax of (3), the "all" phrase precedes the "some" phrase, the reading of (3) as (3b) entails that the true syntax differs from the superficial syntax. In short, (3) is syntactically and therefore also semantically ambiguous.

Consideration of the syntax of modal constructions will help to clarify their semantics. Moreover, as we will see below, such constructions are subject to ambiguities. And in fact some interpreters have argued that some of these ambiguities are explicable on syntactic grounds.

In considering the syntax of modal constructions, I will first consider a bipartite interpretation of their structure, specifically in relation to what is referred to as the modal "force" or "strength" of such constructions. Subsequently, I will consider a tripartite interpretation of their structure, specifically in relation to what is referred to as the modal "flavor" of such constructions.

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[12] Ultimately, I will need to recast this description in terms of the more recent commitments of the minimalist program. Cp: "Beyond PF and LF, the MP discards other linguistic levels, specifically D-Structure and S-Structure, and attempts to reanalyze any empirical evidence that might be used to motivate them." (R. Freidin and H. Lasnik, "Some Roots of Minimalism in Generative Grammar," in C. Boeckx, ed., *The Oxford Handbook of Linguistic Minimalism*, Oxford University Press, 2011, 1-26, at 4)

Modal Force and the Bipartite Syntax of Modal Constructions

Recall:

2. Janet may be at home.

And recall the following two glosses of (2):

2m. Possibly, Janet is at home.
2m'. It is possible that Janet is at home.

These glosses suggest that (2) is a modalized version of the following sentence:

2p. Janet is at home.

Assuming so, consider the following interpretation of (2). First, assume that modals such as "may" are propositional operators. Accordingly, "may" is a function from a proposition to a proposition. Let O symbolize the modal operator, and let p symbolize the proposition that is its argument, namely (2p). Then, letting q stand for (2):

$$O(p) = q.$$

Accordingly, the modal operator O takes p as its argument and modalizes it.[13]

In formal semantic theory of modality, the proposition p or the corresponding natural language sentence that is the argument of the modal operator O – for example (2p) – stands for what is called "the prejacent proposition" or simply "the prejacent." The term *prejacens* or *praeiacens* derives from Medieval Latin.[14] The literal meaning of the Latin term is "lying before" or "lying in front of." Since the prejacent proposition p is the proposition that serves as the argument of the modal operator, we may conceive of it as logically prior to the modalized proposition q that is the value of the function.

Compare the present bipartite structural interpretation of a modal construction such as (2) with the formalization of the sentence "Possibly p" or "It is possible that p" in contemporary modal logic:

$$\Diamond p.$$

Here, the diamond symbol stands for a modal operator akin to the English adverb "possibly" or the clause "it is possible." Indeed, the present bipartite structural interpretation of modal verbal constructions derives from contemporary modal logic.

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[13] Cp. A. Kratzer: "Syntactically, modal words are sentence operators at some level of logical form. If you prefix a sentence with a modal, you get another sentence." ("Modality," in A. von Stechow and D. Wunderlich, eds., *Semantics: An International Handbook of Contemporary Research*, de Gruyter, 1991, 639-50, at 641)

[14] Curiously, although this point is commonly made in this context, it does not seem that Medieval logicians tended to use the term *prejacens* in their accounts of modality. Thanks to Sarah Uckelman and Catarina Dutilh Novaes (p.c.) for their input here.

Now, consider replacing "may" in (2) with "must":

4. Janet must be at home.

Whereas "may" introduces (mere) possibility, "must" introduces necessity. Accordingly, (4) might be glossed in either of the following ways:

4m. Necessarily, Janet is at home.
4m'. It is necessary that Janet is at home.

Granted this, consider the standard modal logical formalization of sentences such as (4m) and (4m'):

$\Box p$.

Here, the box symbol stands for a modal operator akin to the English adverb "necessarily" or the clause "it is necessary."

More precisely, both in standard modal logic and in formal semantic theory of modality, the symbols $\Diamond$ and $\Box$ and English expressions such as "possibly"/"it is possible" and "necessarily"/"it is necessary" are interpreted as quantifiers over possible worlds. Focusing on the natural language expressions, "possibly"/"it is possible" as well as the modal "may" are interpreted as existential quantifiers over possible words. For instance, consider (2) again. Let us symbolize the prejacent "Janet is at home" as p. And let us, as standardly, use the symbol w as a possible world variable. Then, (2) may be formalized as:

$\exists w\ [p(w)]$.

This formula may be read as stating that there is some world in which Janet is at home. And this captures the intuitive meanings of "Janet may be at home," "Possibly, Janet is at home," and "It is possible that Janet is at home."

Assuming that a proposition p denotes a set of possible worlds, namely those in which p is true, we could just as well formalize (2) as:

$\exists w\ [w \in p]$.

This formula states that there is some world w such that w is a member of the set of worlds in which p is true.

Turning to (4), the necessity expressed by "must" is interpreted in terms of universal quantification over possible words:

$\forall w\ [w \in p]$.

This formula may be read as stating that every world w a member of the set of worlds in which p is true. Instantiating (4), the formula states that in all possible worlds Janet is at home, which captures the intuitive meanings of "Janet must be at home," "Necessarily, Janet is at home," and "It is necessary that Janet is at home."

The distinction between the meanings of "may" and "must," like those between "possibly" and "necessarily," is referred to as one of modal "force" or "strength." I'll hereafter use the term "force." "Must" is said to be a more forceful modal than "may." As we have said, the truth of a sentence modalized by "must" requires that the prejacent be true in all possible worlds, whereas the truth of a sentence modalized by "may" requires that the prejacent merely be true in some possible world. So, the force of a modal is interpreted in terms of the idea that "all" entails "some," but not vice versa. The quantity of worlds in which the prejacent is required to be true according to the meaning of "may" is a proper subset of the quantity of worlds in which the prejacent is required to be true according to the meaning of "must."

Generalizing, with respect to their force, modals are divisible into at least two classes: possibility and necessity modals:

Possibility modals: may, might, can, could

Necessity modals: must, should, ought.[15]

Observe that the modal verbs "have (to)" and "need (to)" also have necessity modal meanings.

Granted this, in the case of necessity modals and modal verbs, note that "should" and "ought" appear to be weaker than "must" and "have (to)." Support for this claim derives from the following examples:

You should send a thank you note, but you don't have to.

Employees must wash their hands; non-employees ought to.[16]

Consequently, a sub-categorial distinction among necessity modals is commonly drawn between so-called strong and weak ones. Precisely how this distinction should be interpreted is a crucial question, but will not discuss further here.

In sum, the preceding discussion has focused on a bipartite structural interpretation of the syntax or logical form of modal constructions. The ensuing discussion will adhere to the interpretation of modals as propositional operators and as quantifiers over possible worlds. However, it will propose replacing the bipartite structural interpretation of the modal constructions with a tripartite structural interpretation.

The replacement owes to a distinction between artificial language quantification, such as occurs in modal logic, and natural language quantification, which is the object that formal semantics studies. Whereas artificial language quantifiers are (typically) unary, natural language quantifiers are (typically) binary. Since we are concerned with the meaning and use of modal constructions in English, a natural language, the fact that they are binary requires that we add a structural component to the interpretation of their logical form, namely their so-called restrictor. Precisely, then, we will be complicating rather than wholly replacing the preceding bipartite

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[15] More cautiously, "ought" and "should" are standardly treated as necessity modals, but, there are alternative interpretations.

[16] This example is taken from K. von Fintel and S. Iatridou, "How to Say *Ought* in Foreign: The Composition of Weak Necessity Modals," in J. Guéron and J. Lecarme, eds., *Time and Modality*, Springer 2008, 115-41, at ?.

structural interpretation. In the course of doing so, I will introduce a distinction among modal constructions referred to as "modal flavor."

Modal Flavor

Consider the following "may" construction:

5. Janet may attend the board meeting.

(5) admits the following two readings:

5e. There is some likelihood that Janet will attend the board meeting.

5r. Janet is allowed/permitted to attend the board meeting.

(5e) is standardly referred to as an "epistemic" reading of (5). For reasons that will become clear below, I will instead refer to it as an "evidential" reading. The reading of (5) as (5r) has been variously described as "root," "deontic," "circumstantial," and "priority." I will initially refer to it as "root" and clarify this term below.[17]

The distinction between evidential and root readings of (5) is referred to in terms of "flavor" of modality. The fact that (5) admits these two readings might be explained by the suggestion that "may" is ambiguous, having both an evidential and a root sense. If so, then flavor, like force, is lexically encoded in the modal, and modals are at least two-ways polysemous.

But this is not how modal flavor is standardly interpreted.[18] Instead, modal flavor is standardly interpreted as a function of the broader textual or discursive context in which the modal occurs. Some support for this claim derives from the fact that, in English and in numerous languages, various modals systematically admit evidential and root readings. For example, consider the following "must" construction:

6. Janet must be at the board meeting.

(6) may be read in either of the following ways:

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[17] The source of the term "root" is T. Ronald Hoffmann, "Past Tense Replacement and the Modal System," in A. Oettinger, ed., *Mathematical Linguistics and Automatic Translation*, Harvard Computational Laboratory. Report NSF-17, Harvard University, 1966; reprinted in J. Mccawley, ed., *Notes from the Linguistic Underground*, Brill, 1976, 85-100. A. Verhulst, I. Depraetere, L. Heyvaert, "Source and strength of modality: An empirical study of root *should, ought to,* and *be supposed* to in Present-day British English," *Journal of Pragmatics* 55 (2013) 210-25, n.1, suggest that the term "root" reflects "the fact that root modality is more 'basic' than epistemic modality in that epistemic modal meanings have historically developed from nonepistemic ones." Cp. R. Nordlinger and E. Closs-Traugott, "The scope and development of epistemic modality: evidence from *ought to*," *English Language and Linguistics* 1 (1997) 295-317.

[18] The *locus classicus* for rejecting the view that modals are ambiguous because there are various modal flavors is A. Kratzer, "What 'Must' and 'Can' Must and Can Mean," *Linguistics and Philosophy* 1 (1977) 337-55; reprinted with minor revisions in A. Kratzer, *Modals and Conditionals*, Oxford University Press, 2012, 4-20, esp. at 4-7.

6e. The evidence entails that Janet is at the board meeting.

6r. Janet is required to be at the board meeting.

Assuming that "may," "must," and other modals (as well as modal verbs) are univocal, strictly so-called evidential or root readings of sentences such as (5) and (6) must be understood as readings of the contexts in which "may" and "must" occur. Consequently, phrases such as "evidential (or epistemic) modality" and "root modality" must be interpreted to denote occurrences of a modal in an evidential or root context. The contextual feature responsible for determining the context as evidential or root is referred to as the "modal base" of the modal.[19]

To a first approximation, the modal base is constituted by a set of propositions. In a case of evidential modality, the propositions constitute information that the speaker takes to support the modal statement. To illustrate this point, consider (5) again:

5. Janet may attend the board meeting.

Kratzer suggests that we can paraphrase the evidential reading of (5) using an "in view of …" phrase, as follows:

5e'. In view of the available evidence, Janet may attend the board meeting.

The "in view of …" phrase suggests the contents of the modal base. For example, the propositions constitutive of this base may include: {Janet has expressed interest in attending the meeting, Janet is free at the time of the meeting, etc.}. As the term "evidence" in (5e') suggests, in a case of evidential modality the key feature of the propositions constituting the modal base is that these propositions play the role of evidence. Precisely, they constitute evidence on the basis of which the likelihood or probability of the prejacent – here, "Janet attends the board meeting" – is determined according to the meaning, precisely the force, of the modal term.

In an evidential instance of (5), no explicit "in view of" or kindred phrase need occur. But some such information must be implicit in the conversational background in which (7) occurs. Granted this, we can now, more precisely, say that the contextual feature responsible for engendering an evidential modal reading of a modal construction is that the modal base plays the role of evidence in the modal statement.[20]

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[19] As we will see, the modal base is one of two components of the so-called "conversational background" of the modal. The other component of the conversational background is called the "ordering source." We will come to this below.

[20] But cp. S. Yalcin, "Modalities of Normality," in N. Charlow and M. Chrisman, eds., *Deontic Modality*, Oxford University Press, 2016, 230-55, at 236: "Kratzer (1977) suggested that it is possible to explicitly control the restriction of a modal with an *in view of*-phrase … [But] I deny it is generally true that *in view of*-phrases systematically semantically control the interpretation of modals. The connection between the interpretation of modals and *in view of*-phrases is loose and indirect at best, as the following examples illustrate: … In view of what we know, you can't be parked here (deontic reading of the modal clearly available …" In fact, cp. L. Matthewson, "Modality," in M. Aloni and P. Dekker, eds., *The Cambridge Handbook of Formal Semantics*, Cambridge University Press, 2016, 525-59, at n.12, discussing F. Nauze, *Modality in Typological Perspective*, University of Amsterdam, dissertation in linguistics, 2008, 157: "Even in the presence of an overt adverbial phrase the conversational background can be given by the

Below, I will clarify how the modal base is incorporated into the syntax of the modal construction. Presently, I turn to the root of readings of (5) and (6). Consider the root reading of (5), which we may paraphrase using a "given" participial phrase as:

5r'. Given the board's attendance rules, Janet may attend the board meeting.

In this case, the propositions constitutive of the modal base saliently include those stating the relevant attendance rules. Here, the propositions do not serve as evidence to which the speaker appeals to assert the probability of the prejacent. Rather, the propositions specify a set of factual conditions or circumstances with which the prejacent is compatible (in the case of a possibility modal such as "may") or of which it is a consequence (in the case of an necessity modal such as "must").[21] Again, no explicit "given" or "in view of" or kindred phrase need occur. But some such information must be implicit in the conversational background in which the modal construction occurs.

In sum, we may say that the difference between root and evidential modality lies in the kind of relation that holds between the modal base and the prejacent. In the case of root modality, the prejacent is simply consistent with or entailed by the modal base. The case of evidential modality is more complicated. Here, there is a lack of direct evidence for the truth-value of the prejacent. So, the modal base serves as indirect evidence on the basis of which the truth-value of the prejacent is inferred. Crucially, even when a necessity modal is employed, the inference to the truth-value of the prejacent is based on indirect evidence. For example, assume that a speaker looks outside his window and sees that it is raining:

7a. It is raining.
7b. # It must be raining.[22]

The prefixed pound symbol is used here, as standardly in linguistics, to indicate semantic infelicity. (7b) is infelicitous precisely because the speaker has direct evidence for the truth-value of the prejacent – in this case, (7a). Contrast a speaker with no direct view of the weather outside, but upon seeing someone come inside soaking wet and closing an umbrella. In that context, (7b) is felicitous.

Focusing now on root modality, above I noted that one encounters an array of terms to describe such readings: "circumstantial," "priority," and "deontic" as well as "root." This diversity reflects the fact that within the secondary literature there is irresolution over whether

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context, as pointed out by Nauze … The modal [in the following example] has an epistemic interpretation, in spite of the deontic adverbial. *Context: We are discussing the upcoming trial of the suspected criminal Jockl. The trial has not yet beg[u]n and we wonder what the outcome will be.* In view of what the law provides, Jock *may* be executed." And cp. A. Kratzer, *Modals and Conditionals*, Oxford University Press, 2012, 21: "What I overlooked in the earlier work was that there are important differences between different adverbial phrases contributing conversational backgrounds for different types of modals."

[21] Observe that sentences such as (5e') and (5r') provide further support for the thesis that modals are univocal. If "may" lexically encoded an evidential or a root reading, then the "In view of …" phrases would yield redundancies.

[22] I owe this example to K. von Fintel and S. Iatridou, "LSA 220: Morphology, syntax, and semantic of modals syllabus," p.12.

root modality, which is to say, non-evidential modality is a single basic flavor of modality or whether there are several basic non-evidential flavors of modality. For example, consider the following "can" construction:

8. Janet can drive the vehicle.

(8) admits the following two readings:

8p. Janet is allowed/permitted to drive the vehicle.
8d. Janet has the ability/skill to drive the vehicle.

Both (8p) and (8d) are non-evidential modal readings. (8d) is typically referred to as a "dynamic"[23] reading of (8). Granted this, is dynamic modality a distinct flavor of modality from root modality? Or is dynamic modality a sub-type of root modality?

Presently, for convenience, I will maintain that there are two basic flavors of modality: evidential and root. However, later in the discussion will I revisit this idea. For now, broadly, the flavor of modality that "purpose" entails belongs to what I am now calling root rather than evidential modality. Insofar as we maintain this conclusion, we can say that a purpose is a weak necessity root modal kind.

The Tripartite Structure of Restricted Quantifiers

A distinction between quantifiers in natural languages and in certain artificial languages, for example first-order predicate logic and modal logic, is that quantifiers in natural language are (typically) binary, whereas those in certain artificial languages are (typically) unary.[24] For instance, consider the predicate-logical formulae:

9a. $\forall x \, [Fx]$
9b. $\exists x \, [Fx]$.

(9a) is to be read as stating that all individuals x are such that they are F (or members of the set F). (9b) is to be read as stating that there are some individuals x such that they are (members of the set) F. The quantifiers $\forall x$ and $\exists x$ are unary insofar as they only require a single argument, here Fx, to form a proposition.

Furthermore, in both (9a) and (9b), the individuals x over which the quantifiers range are not restricted.[25] That is, all individuals in the domain of discourse are included. To be sure, the truth of (9b) requires that some, not all, of the individuals in the domain of discourse are F; nonetheless, (9b) would be made true by any members from the set of all individuals being F.

In contrast, in natural languages, quantifiers are (typically) binary and their domains are (typically) restricted. For instance, consider the following sentence:

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[23] From the Greek δύναμις meaning "power, ability."
[24] Cp. Z. Szabó, "Bare Quantifiers," *Philosophical Review* 120 (2011) 247-83.
[25] At least, they are not restricted beyond the restriction, built into the model, that x range over individuals (as opposed, to say, propositions, properties, or worlds).

10. Everything is on sale.[26]

To a first approximation, (10) might be formalized according to same sort of bipartite structure as (9a):

10b. $\forall x\ [Sx]$.

(10b) is to be read as stating that all individuals x are such that they are (members of the set) S. However, in an ordinary discursive context, (10) would be employed to convey a narrower content. Precisely, the domain of individuals would be restricted. For example, (10) might be uttered by a sales representative at a department store or by a homeowner at a yard sale. Accordingly, the quantifier phrase, here "everything," would not range over every single thing in the universe – whatever sense might be made of that content – but over a contextually specified subset. In the examples just mentioned, the subset would be restricted to purchasable items in the department store or to items being sold at the yard sale.

Such restricted quantificational expressions are interpreted as tripartite structures consisting of one quantifier and two predicate expressions, viz.:

$[Q : P1]\ P2$.

The first predicate $P1$ is called the "restrictor" on the quantifier Q. The semantic function of the restrictor is to restrict the domain of the quantifier. The second predicate $P2$ is called the "nuclear scope" of the quantifier.[27] Given this, the semantic function of the quantifier serves to specify a relation between two sets, one determined by the predicate in the restrictor and one determined by the predicate in the nuclear scope. To illustrate, consider (12) as it occurs the department store context:

10d. Everything purchasable in the department store is on sale.

Let the set of things purchasable in the department store be symbolized as P. Then, the formal tripartite structural interpretation of (10) would be:

10t. $[\forall x : Px]\ Sx$.

Here, the expression Px following the colon restricts the domain of individuals over which the universal quantifier expression ranges. The restriction is to members of the set P. Accordingly, (10t) is to be read as stating that for each individual x such that x is (a member of the set) P, x is (a member of the set) S. So, the universal quantifier relates the sets P and S according to the subset relation, such that the set of individuals purchasable in the department store is a subset of the set of the individuals on sale:

$P \subseteq S$.

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[26] This example is borrowed from D. Westerståhl, "Quantifiers in Formal and Natural Languages," in D. M. Gabbay and F. Guenthner, eds., *Handbook of Philosophical Logic*, vol. 14, Springer, 2007, 223-338.
[27] This interpretation is ultimately indebted to I. Heim, *The Semantics of Definite and Indefinite Noun Phrases*, University of Massachusetts, dissertation in linguistics, 1982; published by Garland Press, 1989.

In contrast, assume that the existential quantifier is employed:

10e. **Some things purchasable at the department store are on sale.**

(10e) would be formalized as:

$$[\exists x : Px]\ Sx.$$

This formula is to be read as stating that for some individuals x such that x is (a member of the set) P, x is (a member of the set) S. Accordingly, the existential quantifier relates the sets P and S so that the cardinality of their intersection is at least as great as 1:

$$|P \cap S| \geq 1.$$

In other words, at least one individual is both purchasable at the department store and on sale, which is what "Some things purchasable at the department store are on sale" entails.[28]

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The Tripartite Structure of Modal Constructions

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Having clarified the tripartite structural interpretation of restricted quantifier expressions in natural language, I return now to modal constructions. The basic idea here is that due to the role that the modal base plays in generating distinct flavors of modality, modal constructions should be interpreted as tripartite structures. For example, consider (5) – "Janet may attend the board meeting" – read in terms of the following root paraphrase:

5r'. **Given the rules of the board, Janet may attend the board meeting.**

Again, the "given" phrase indicates the set of propositions that constitute the modal base. This set of propositions determines a set of so-called *accessible worlds*, namely those possible worlds in which the propositions that constitute the modal base are true.[29] Note that since the propositions in the modal base are true, they are true in the actual world. So, the actual world is an accessible world. Intuitively, the notion of an accessible world is a possible world just like the actual world with respect to the propositions of the modal base.

Let us symbolize the set of accessible worlds as A. Granted this, given that "may" is an existential quantifier over possible worlds, once again let us symbolize the modal operator as $\exists w$. Let j stand for the prejacent of (5): "Janet attends the board meeting." Then, we can represent the root reading of (5) as:

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[28] I note that, by incorporating connectives, universal and existential quantificational expressions in first-order predicate logic can convey restricted quantificational contents. For example, (10t) can be formalized as $\forall x\ (Px \rightarrow Sx)$; and (10e) can be formalized as $\exists x\ (Px \wedge Sx)$. However, vis-à-vis natural language, this expressive power is ultimately limited as the quantifiers cannot convey such natural contents as "most/many/few Fs are Gs." And again, the quantifiers require only a single predicate to form a sentence.

[29] S. Kaufmann notes that in the relevant literature there is no established name for this set of worlds. He refers to it as the *modal background*. ("The Limit Assumption," *Semantics and Pragmatics* 10 (2017) 1-29, at 2)

5t. $[\exists w : w \in A]\ w \in j.$[30]

(5t) is to be read as stating that there is some world w such that w is a member of the set of accessible worlds A and w is a member of j. In short, in some accessible world j.

Observe now that in terms of its semantic function the modal base serves as the restrictor on the quantifier. Precisely, it restricts the set of possible worlds over which the quantifier ranges to the accessible worlds, that is, to those worlds in which all the propositions constituting the modal base are true.

Finally, it should be clarified that while the modal base is contextually determined and in that respect a pragmatic contribution to the content of the sentence asserted, on the assumption that natural language quantifiers denote binary relations, the modal base is required to render a propositional content. Consequently, the modal base contributes to the truth-conditions of the modal construction; and so (5t) can be taken to represent the semantics of the root reading of (7).

Ordering Source

The modal base is a proper part of the so-called "conversational background" of a modal construction. More precisely, it is one of two parts of the conversational background. The other part is called the "ordering source." As we will see, an ordering source must also be included in the restrictor. To appreciate why the modal base alone does not suffice to capture the intuitive meaning of a modal construction and therefore why an ordering source is also required, consider the following example:

11. Janet must pay a fine.

To be clear, I assume a root reading of (11). Granted this, observe that the prejacent of (11) is:

11p. Janet pays a fine.

Now, let's fill out the example by introducing the following propositions constitutive of the modal base:

b1. There is a legal speed limit.
b2. Violators of the speed limit are fined.
b3. Janet was caught exceeding the speed limit.

Since "must" is a necessity modal, the meaning of (11) is that in all accessible worlds – again, in those possible worlds where the propositions constituting the modal base are all true – the prejacent is true. But, in fact, the prejacent (11p) is *not* true in all accessible worlds

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[30] A more common formalization is: $\exists w' \in \bigcap A(w): w' \in j$. The symbol $\bigcap$ denotes the generalized intersection function. The formula $\bigcap A(w)$ returns the intersection of the set of worlds at which the propositions of A are true. In other words, it returns the set of worlds in which all of the propositions of the modal base are true, which is to say the set of accessible worlds. The existential quantification then asserts that there is some world w' that is a member of that set.

determined by the modal base (b1)-(b3). In other words, (b1)-(b3) does not entail (11p). For example, in some possible worlds where (b1)-(b3) are true, Janet is a scoff-law.

In an effort to remedy this defect, consider adding to the modal base a proposition such as:

a1. All citizens obey the law.

Two basic problems follow from this attempted remedy. One is that, assuming Janet is a citizen, (a1) is inconsistent with (b2) and (b3): in a world where all citizens obey the law, there are no violators. So, there are no possible worlds where (a1) and (b1)-(b3) are true. The inclusion of inconsistent propositions within a modal base would wreak logical havoc. On the one hand, any false proposition can be inferred from a contradiction. Likewise, since any conditional is true if its antecedent is false, the modal base could consist of any number of bizarre contents such as: If someone breaks the speed limit, he learns Gaelic.[31] More basically, assuming with classical logic that there are no worlds in which inconsistent propositions are true, then there are no worlds for a necessity modal to quantify over. Consequently, a sentence such as (11) would be trivially true.[32]

A second basic problem with the attempted remedy of adding (a1) to the modal base is simply that (a1) is false: not all citizens obey the law. Accordingly, we could not remedy the problem by adding to the modal base the following alternative to (a1):

a2. All citizens who break the law pay the appropriate penalty.

In this case, (a2) is consistent with (b1)-(b3); but again, like (a1), (a2) is simply false.

A different problem that Kratzer observes with the present interpretation of modal constructions relates to the fact of so-called graded modality. Consider the following felicitous sentences, which involve evidential modal readings:

There is a high probability that it is raining.
There is a greater probability that it is raining than that it is snowing.[33]

Evidence may be taken to support a proposition to varying degrees. Graded modality is particularly common among evidential readings such as these. But it also occurs among root readings. Yet the interpretation of modal constructions currently on the table – limited as it is to universal and existential quantification – has no way to explain "intermediate" grades between

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[31] Cp. Kratzer (1991).

[32] Cp. Matthewson (2016) 530.

[33] Cp. J. Nuyts: "Epistemic modality consists of the estimation of the likelihood that (some aspect of) a … state of affairs is/has been/will be true (or false) in the context of the possible world[s] under consideration. This estimation of likelihood is situated on a scale [whose poles are] certainty that the state of affairs applies … [and] certainty that it does not apply, with intermediary positions on … the scale." (*Epistemic Modality, Language, and Conceptualization: A Cognitive-Pragmatic Perspective*, John Benjamins, 2001, 21-22). Cp. "The speaker asserts that a proposition is possibly or necessarily true, relative to some information or knowledge. If the proposition is only possibly true, the propositional attitude is that of uncertainty; if it is necessarily true, the proposition is that of a high degree of certainty." (J. van der Auwera and A. Ammann, "Overlap between situational and epistemic modal marking," in M. Haspelmath et al., eds., *The World Atlas of Language Structures Online*, Munich, 2008, chapter 76.)

mere possibility and necessity.[34] (Note that if the modal force of "ought" must in fact be interpreted as intermediate between "may" and "must," then the present framework will be incapable of interpreting "ought.")

The solution that Kratzer offers to these problems is to add a set of propositions to the conversational background, but not to the modal base. So, again, Kratzer proposes that the conversational background has two components: the modal base and, what she (following Lewis[35]) calls, the ordering source. These two components differ in content and function. Whereas the propositions constituting the modal base are true – and hence, as it is said, modal bases are "realistic" – the propositions constituting the ordering source needn't be (for example, (a1)); and they may be inconsistent (for example, the conjunction of (a1) and (a2)).

The function of the ordering source is to order the accessible worlds according to degrees of conformity to a relevantly ideal world.[36] I say a "relevantly" ideal world just because different sorts of ideals are relevant to different modal flavors and sub-flavors. In the case of (11), the ideal world is one in which all citizens obey the law, again (a1). But since (11) entails that Janet broke the law, the accessible worlds closest to the ideal are those in which violators pay the appropriate penalty, (a2). As Matthewson puts it, "the set of [accessible] worlds is then ordered according to how many of the ordering source propositions are true in each world…"[37]

In the formalization of the denotation of a modal, there are basically two ways of including an ordering source. The difference between the two turns on whether one accepts or rejects the so-called Limit Assumption.[38] According to the Limit Assumption, there always is an accessible world or set of worlds that comes closer to the ideal than any other worlds. For the sake of simplicity, I will assume the Limit Assumption. In that case, let O stand for the set of propositions that constitutes the ordering source. Granted this, we can say that, given two worlds w and w', w is at least as close as w' to the ideal determined by O if and only if all propositions in O that are true in w' are also true w. Formally, we use the symbol $\leqslant$ to mean "at least as close as"; and we add to it the subscript O to specify "at least as close as, in terms of the propositions of O":[39]

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[34] Matthewson (2016, 531) discusses a third problem. Consider the sentence: "Isabella must have climbed Mount Washington." If in all accessible worlds, i.e. all worlds in which the evidence from the actual world obtains, Isabella climbed Mount Washington, then she climbed Mount Washington in the actual world. So, "Isabella must have climbed Mount Washington" implies "Isabella climbed Mount Washington." But surely the modalized sentence can be true, while the non-modalized sentence can be false.

[35] D. Lewis, "Ordering Semantics and Premise Semantics for Counterfactuals," *Journal of Philosophical Logic* 10 (1981) 217-34.

[36] In this context, interpreters sometimes also speak of "best" worlds. Since my overarching project is an attempt to explain evaluative language in terms of the teleological language of purposiveness and that teleological language in terms of modal language, explanation of modal terms using evaluative language is clearly circular. However, my use of "ideal" here is merely a convenience. "Ideal" (or "best") worlds are simply those accessible worlds that satisfy the most conditions of the ordering source. And I assume that the proposition(s) defining the ordering source can be stated in non-modal, non-teleological, and non-evaluative language.

[37] Cp. Matthewson (2016) 531.

[38] On which, cp. D. Lewis, *Counterfactuals*, Blackwell, 1973, 19-21; R. Stalnaker, *Inquiry*, MIT, 1984, 140-1.

[39] For the following formalization, cp. Kratzer (2012) 39; Matthewson (2016) 531.

$$\forall w,w'\,[w \leqslant_O w' \leftrightarrow \{p : p \in O \wedge w' \in p\} \subseteq \{p : p \in O \wedge w \in p\}]$$

$\{p : p \in O \wedge w' \in p\}$ is a set making operation. Precisely, it determines a set of propositions that meets the following two conditions: any member of the set is both a member of the ordering source O and a member of w'. In other words, the set consists of all of the ideal propositions that are true in w'. Likewise, $\{p : p \in O \wedge w \in p\}$ constitutes the set of all ideal propositions that are true in w. And the relation between the two sets is that those in w' are identical to or are a proper subset ($\subseteq$) of those in w. In short, w is at least as close to the ideal as w'.[40]

     Now, let us wed the original formalizations of possibility and necessity modals to the ordering source. We can do so simply, if we first stipulate an operation that selects the set of ideal worlds from those available given the modal base. Call the operator that performs this operation I.[41] Consequently, we can define the denotation of a necessity modal as follows:

     For a world w, modal base A, and ordering source O:

$$[\![\text{must } p]\!]^{w,A,O} = \forall w' \in I_O(\bigcap A(w)): w' \in p.$$

Note that $[\![\ ]\!]$ are denotation function brackets. I've used Arial font for the expression within the denotation brackets to indicate that this is the object language. The superscripts on the right bracket are assignment variables. Accordingly, the formula can be read as stating that the denotation of "must p," given a world w, modal base A, and ordering source O assignment, is as follows: for all worlds w' that are members of the set of worlds that I determines, namely the ideal worlds given the modal base A and according to ordering source O, p is true at w'.

     A possibility modal can be formalized using the existential instead of the universal quantifier as:

$$[\![\text{may } p]\!]^{w,A,O} = \exists w' \in I_O(\bigcap A(w)): w' \in p.$$

Conclusion

     In sum, according to the standard Kratzerian theory, English language necessity and possibility modals are interpreted as universal and existential quantifiers over possible worlds respectively. The quantifiers are binary. They are restricted by a conversational background that consists of two components: a realistic modal base and an in principle non-realistic ordering source. Necessity modal constructions, for example, those of the form "must p," state that p is a members of all of the most ideal worlds. Possibility modal constructions, for example, those of the form "may p," state that p is a member of some of the most ideal worlds.

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[40] If we do not admit the Limit Assumption, the formalization of the ordering source is as follows: $\forall w, w'$. $\exists w'' : w'' \leqslant_O w' \wedge (w \leqslant_{O(w)} w' \rightarrow w \in p)$

[41] Formally, given $\forall X \subseteq W$, $I_O(X) = \{w \in X : \neg\exists w' \in X.\ w' \leqslant_O w\}$. That is, the function I_O takes a set of worlds X and returns a subset – the ideal worlds – any member w of which is such that there is not some world w' that is a member of X that outranks, i.e. satisfies more of the conditions of the ordering source O, than w.