

Comments on Danielle Macbeth's *Realizing Reason*

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Realizing Reason is a long and dense book. The prospect of working through it may appear daunting. But its broad historical synthesis, intellectual ingenuity, and philosophical depth reward the effort. The study is full of provocative perspectives, illuminating connections, and ambitious theses. As I was guided along its various pathways, I felt my intellectual purview repeatedly being enhanced and extended. In short I learned a great deal from this book, and it has inspired me to pursue a number of its ideas further.

Realizing Reason is principally a work of epistemology. However, even this may be a somewhat misleading characterization. The book also has metaphysical and ontological interests. It is concerned with the world on which cognition in its various modes of intentionality bears. More fundamentally – in a Hegelian spirit that is variously engaged with and sometimes critical of members of the Pittsburgh school – the book is concerned with understanding cognition as something that occurs within a form of life. Above all in the case of those living beings that possess the special capacity for culture, it is concerned with cognition and its intentional objects or contents within forms of life that are located in traditions of inquiry.

Realizing Reason is especially interested in the Western tradition of mathematical practice and the epistemology of this practice. Mathematical practice and epistemology are of concern here insofar as mathematics aspires to be a form of inquiry that is purely rational. Macbeth argues that this aspiration has in fact been realized, and that it was first fully realized in the nineteenth century through the development and reconceptualization of the field as *Denken in Begriffen* (thinking in concepts) by Bolzano, Riemann, and their heirs. In a phrase then *Realizing Reason* is, *pace* Kant, a defense of pure reason.

Most of the book is devoted to a 'narrative of truth and knowing' that endeavors to illustrate and explain this unfolding and realization of pure reason. The narrative mainly focuses on 'essential moments' (3) in Western mathematical and mathematical-philosophical history, beginning with Euclid's geometry,

moving through Descartes' unification of algebra and geometry, Kant's critical account of mathematical judgment as a priori but based in intuition and synthetic, and culminating in Frege's *Begriffsschrift*, a work whose non-natural concept language and notation finally enabled us, as Macbeth writes, 'to realize the standpoint of pure reason' (422).

On page 1 Macbeth states that we are seeking here 'an adequate conception of mathematical practice as a mode of intellectual inquiry'. Adequacy is taken to require satisfaction of at least the following two conditions: the conception must be 'naturalistic', that is, 'fully compatible with our best scientific understanding'; and it must 'enable us to understand ourselves as rational beings answerable in our judgments to the norm of objective truth' (56). I underscore the phrase 'objective truth'. Macbeth boldly claims that what the fully purified mathematical practice of the nineteenth century achieved was a power of knowing objective reality. Compare her following statement: 'Our aim is to understand our capacity for knowledge of things as they are in themselves' (295). The claim that this cognitive achievement has occurred is one of the book's most ambitious theses. If cogent, it would bring closure to or at least a framework for closure to the central preoccupation of modern philosophy.

In the following comments I want to engage two issues. One is a critical point that relates to the inception of the narrative. The other relates to the endpoint of the narrative. Minimally this second point is an attempt to clarify and to seek confirmation that I have clarified certain commitments made there. If I do have the account properly in view, then perhaps this line of thought may also serve to put pressure on these commitments. I'm not sure about this.

Regarding the first issue, I am taking the opportunity to exercise my background in ancient philosophy. *Realizing Reason* as a whole and the narrative of mathematical practice that is central to it is organized in three parts, called 'Perception', 'Understanding', and 'Reason' respectively. These rubrics indicate defining epistemological characteristics of the developmental stages of the Western mathematical tradition that extends from antiquity through early modernity to the nineteenth century. Macbeth claims that for the ancient Greeks perception is the paradigm of knowledge and thus also for their understanding of mathematical knowledge. For example to achieve knowledge of the equilateral triangle is to come to see what the nature of this geometric figure is. And one achieves this by coming to see how the figure is constructed. The so-called proofs (*apodeixeis*) that accompany Euclidean diagrams should rather be understood as scripts or instructions that enable one to reason 'within' the diagrams and thereby to see the structural relations of part and whole that obtain among the objects the diagram illustrates.

In support of the claim that the Greeks conceive of mathematical knowledge on the model of perception Macbeth appeals, among other things, to the inquiry into the nature of *epistêmê* in Plato's *Theaetetus*. She suggests that although this dialogue ends aporetically, Plato provides many indications that he views knowledge on the model of perception. For example she writes:

The most direct evidence that Plato thinks that knowledge is a kind of perception, and in particular a matter of seeing with the mind's eye, is a remark he has Socrates make at the end of the discussion of Theaetetus' claim that knowledge is (sensory) perception. Socrates says: 'we are not going to grant that knowledge is perception, *not at any rate on the line of inquiry which supposes that all things are in motion*' (183c), suggesting thereby that knowledge is a kind of perception of what is unchanging, a mental grasp of, for instance, mathematical and the Forms. (189, n.36, Macbeth's italics)¹

I think this is wrong, for Plato and for many other ancient philosophers. For example in the final refutation of the identification of knowledge and perception at *Theaetetus* 184–6 Plato has Socrates argue that whereas *epistêmê* is of what is, *aesthêsis* cannot grasp being (185cff). More generally, as I understand it, the inquiry in *Theaetetus* is structured as a dialectical ascent – albeit one that does end inconclusively – from *aesthêsis* to *doxa* to true *doxa* with a *logos*. Compare the account of cognitive ascent with which Aristotle's *Metaphysics* opens. Again this begins with *aesthêsis* and ends with *epistêmê* and *sophia* (980a–981b).

Now one may admit this, but still maintain that the way the ancients conceive of *epistêmê* or *sophia* essentially conforms to the way they conceive of perception as a cognitive grasp and perhaps more precisely as a seeing of objects. I assume that this will be Macbeth's response. Greek epistemology is essentially characterized by a certain conception of intentionality. According to this conception, our mode of intentionality consists of an immediate apprehension of the objective world; and again perception is paradigmatic. Accordingly *epistêmê* or rather the exercise of the power of *epistêmê* is taken to consist in an immediate apprehension of objects, albeit perhaps objects of a special kind, for example, mathematical or Forms, however these may be conceived.

I suggest that this construal misses the central essential feature of ancient epistemology. For many of the principal ancient philosophers *epistêmê* and *sophia* are discursive and in particular require an explanation (an *aitia*) of the fact known.² For example recall how Aristotle distinguishes those practiced builders who have experience (*empeiria*) from the master craftsman who possesses *epistêmê* on the grounds that the former grasp that something is the case, but not the explanation (*aitia*) for its being so (981a28–29). As Aristotle writes at the beginning of *Posterior Analytics*: 'We think we know (*epistasthai*) each thing in an unqualified way when we think we cognize, of the *aitia* of the thing [known], that it is the *aitia* of that thing' (71b9–12). This conception of *epistêmê* as dependent on an aetiological or explanatory account is first introduced in Plato's *Meno*, in the process of mathematical inquiry into the problem of doubling the square that Socrates conducts with the slave-boy (82b ff). The reasoning through which the slave-boy is guided is not merely supposed to cause him to see the solution, it is supposed to constitute an understanding of the solution and thus *epistêmê* of it.

In short, for the ancients *epistêmê* is explanatory knowledge. One does not have *epistêmê* of something until one has attained an explanation of it. In *Meno*

Plato has Socrates explicitly and without retraction characterize *epistêmê* as true *doxa* with a reasoning of the *aitia* (98a; cp. *Phaedo* 96a ff.). Since sense perception per se does not constitutively consist of a grasp of the *aitia* of what is perceived, sense-perception – even if veridical or, as for example in the case of Epicurus, necessarily veridical – cannot constitute *epistêmê*.

Now in personal communication with Macbeth about this issue, she emphasized to me that the key distinction to which she wants to draw attention is that between objects and propositions. For the ancients, she suggests, knowledge is of objects, not propositions. I admit that in Plato there is textual evidence that encourages such a view, in particular in *Republic*. However if one considers Plato's treatments of judgment (*doxa*) in the later dialogues, I believe one finds compelling reason to believe that he distinguishes the attitude of judgment from the content of judgment and that he distinguishes the form of the content as propositional. For example in *Philebus* Socrates characterizes judgment (*doxa*) as an operation in which a metaphorical scribe inscribes sentences (*logoi*) into the soul, and he explicitly distinguishes this operation from that of imagination, according to which a metaphorical painter depicts images in the soul (38e ff). And in the account of false judgment in *Sophist* the Eleatic Stranger distinguishes the form of the doxastic content in terms of the distinction between a noun or subject (*onoma*) and a verb or predicate (*rhêma*) (261eff).³

Leaving Plato aside, much more decisive evidence for a conception of the content of *epistêmê* and *doxa* as propositional comes from the Stoics. For the Stoics the content of a judgment (*doxa*, *krisis*) and of epistemic judgment is a so-called assertible (*axiôma*). An assertible, as they describe it, is a 'self-complete sayable (*lekton*) that can be stated as far as itself is concerned' (Sextus Empiricus *PH* 2.104); it is 'that by saying which we make a statement' (Diogenes Laertius 7.66); it is 'that by saying which we either speak truly or speak falsely' (Sextus Empiricus *M* 8.73). I underscore that an assertible is characterized as self-complete (*autotelê*) as opposed to deficient (*ellipê*). Deficient are those sayables (*lekta*) 'that have an unfinished expression, for example, "writes" (*graphêi*) – for we go on to ask: Who? Self-complete are those that have a finished expression, for example, "Socrates writes" (*Sokratês graphêi*)' (DL 7.63)⁴ It is worth underscoring here that Stoicism was the dominant philosophical outlook of antiquity for over five hundred years.

In sum, however much the objects of Euclidean geometry are ultimately bound to sense perception, the ancient philosophers' conception of *epistêmê* is, I think, not.

Now, assuming this criticism holds, I suggest the following simple remedy: divorce the interpretation of Euclidean geometry from mainstream ancient philosophical epistemology.⁵

Let me now turn now to the second issue I want to engage. This is the idea that in the nineteenth century a purely rational power of knowing objective reality was achieved. Compare this claim, as Macbeth invites us to do, with the

claim that animals have evolved a non-rational power of perceiving objective reality.

Regarding this latter claim, Macbeth appeals to the following familiar line of thought:

The process of evolution by natural selection does not only realize living things. Nature, the world in which animals are to be found, acquires thereby a new sort of significance as well. What are otherwise merely physical things, for instance, various rock formations, bodies of water ... come to have the significance of ... affordances for animals, where an affordance is 'what [the environment] provides or furnishes, either for good or ill' and 'implies the complementarity of the animal and the environment. Animals and environments co-evolve; they emerge together and neither is intelligible without the other.' (31, citing Gibson, *The Ecological Approach to Visual Perception*, 127)

This view of animal-environment complementarity is epistemologically significant since, as it is here written, neither the animal nor the environment is intelligible independently of the other. It is also epistemologically significant in that it supports a certain conception of the animal's cognitive relation to the environment. Again, Macbeth writes: 'Instead of conceiving the animal as representing things around it in response to inputs or stimuli to its sense organs, an animal, on Gibson's account, is directly perceptually aware of affordances, biologically significant aspects of its environment' (31). Observe two points that are being conveyed here. One concerns the nature of the objects of perception, namely, that these are affordances, biologically significant aspects of the animal's environment. The other concerns the immediacy of the relation between perceiver and perceptual object: again, the animal is 'directly perceptually aware' of these objects. I take it that the point about the immediacy or directness of the perceptual relation to the perceptual object is not that this is a necessary occurrence, but rather a common one. After all animals can and do misperceive. So there is a difference between the power of perception being constitutively or essentially for the (direct) apprehension of certain sorts of objects, namely affordances, and perception typically being immediately of such objects.

Now, the Gibsonian conception of perception provides a way of thinking of and understanding certain cognitive powers as powers of knowing objective reality. Indeed Macbeth introduces this conception in chapter 1 to serve as a convenient analogue to the account of the development of the purely rational mathematical power of knowing objective reality that the narrative from Euclid to Frege subsequently charts. Compare her following claim: 'The emergence of rational beings ... is at the same time the emergence of the world as such, the objective reality on which thought aims to bear' (27). More precisely, if the analogy is to hold, it is the emergence of those aspects of objective reality that complement those cognitive powers that are rational. Accordingly I further infer: the emergence of beings with the capacity for pure rationality is at the same time the emergence of constituents of objective reality on which pure reason aims to bear.

What then are the constituents of objective reality on which pure reason aims to bear? The answer is that they are concepts – concepts, that is, in the Fregean (non-psychologistic) sense. More precisely the concepts on which pure reason aims to bear are of two kinds: logical and mathematical. This claim is significant in two respects. First the division of concepts into logical and mathematical owes to the fact that, contrary to the aim of logicism, mathematical concepts are not reducible to logical concepts. Macbeth maintains, intelligible unities; hence their decomposition into logical constituents, insofar as that is possible, destroys their intelligibility. Second, logic itself is not empty formalism. Like mathematics, logic is contentful. So the concepts on which pure reason aims to bear furnish the content or subject matter of the exact sciences of logic and mathematics. And in virtue of that provision, which includes the properties that concepts have and the relations in which they stand to one another, they facilitate fruitful inquiry within these sciences.

Consider now Macbeth's following statement:

A concept, which is the *Bedeutung* of some word, is something objective; it is not up to us to decide what concepts there are in mathematics and logic. Because it is not, one's proposed definition [of a given concept] can fail to designate any concept. It is, then, a fully objective matter what ... bonds⁶ actually obtain among concepts. (398)

Regarding the nature of such conceptual entities Macbeth further writes:

In mathematics the reality [that one can come to know] is not of course empirical reality, how things actually are, but instead how they could be ... The concepts of mathematics, insofar as they are concepts of pure reason, are concepts of how empirical reality can be. The concepts of mathematics (and logic) are not, then, in the world in the way that the entities fundamental physics studies, including even the space-time of general relativity, are in the world (though if they were, the conception we are after here would be correctly described as a form of Platonism). Nor is it correct to describe them as mental entities, though we can, through our grasp of the relevant senses [that is, Fregean senses], come to be in a cognitive relation of knowing them. They are fully objective insofar as they are the possible ways things could most objectively be, the ways any rational being might in time come to recognize as the possibilities of things. (416)⁷

I find it odd that Macbeth writes that if the concepts of mathematics (and logic) were in the world in the way that the entities fundamental physics studies are in the world, the conception here would be correctly described as a form of Platonism. Surely Plato does not think that mathematical and Forms are in the world in the way that the elements of physics are in the world. For Plato mathematical and Forms are, so to speak, super-natural and extra-cosmic. They are outside of space and time – though 'outside' here is of course metaphorical. Properly speaking these entities are not located anywhere. Yet they exist.

So I am driven to conclude that Macbeth is committed to a form of Platonism about the subject matter of mathematics and logic. I wonder what she will say

about this. Perhaps she will not flinch. But let me underscore and attempt to clarify a couple further points relating to this commitment.

I suggested that the Gibsonian account of animal-affordance complementarity and the epistemological, precisely perceptual, import of this complementarity is introduced to serve as an analogue to the power of pure reason and its content, namely concepts, concepts that are objective. But observe several distinctions between the Gibsonian account of perception and Macbeth's account of pure reason. Animal-affordance complementarity comes into being through evolution, a historical process. Pure reason-concept complementarity does not. Indeed it cannot. While creatures such as ourselves may develop capacities of pure rationality – this of course is precisely what the narrative of truth and knowing aims to chart – concepts themselves do not come into being. They are, so far as I can tell, eternal. Consequently the analogue to affordances cannot actually be concepts. We were told that animals are, typically, directly perceptually aware of affordances. But the pure reasoner's cognitive access to concepts is mediated by (Fregean) sense. Accordingly the account should actually be that there is a complementarity of pure reason and senses (of concepts). Garden-variety affordances, for example food stuffs, are then ontologically dependent on constituents of the physical world, whereas senses of concepts are, if I am not descending into a confusion of nonsense here, ontologically dependent on concepts, which is to say ontologically dependent on entities that are not constituents of the physical world.

Finally a point of clarification regarding the notion that (mathematical) concepts are ways that things in the material world (for example, the fundamental elements of physics) can be. As such concepts are mathematical (and logical) properties, where a property is understood in some sort of robust metaphysical sense – in particular as an entity that is not exhausted by its instantiations (if any), but is something over and above them. In one respect such entities are *possibilia*, again ways that things can be. But since, as Macbeth emphasizes, concepts are objective, in another sense they are actual – although not actual in the sense of being members of the physical world.

This is not the dualism of Descartes. And I am not suggesting that Macbeth's account lapses into the sideways-on view that early modernity bequeathed to the nineteenth century. But the account is metaphysically dualist in a robust way all the same. Recall that one of the conditions of adequacy on the conception of mathematical practice as a mode of intellectual inquiry that the book sets out to satisfy is that this conception be 'naturalistic', that is, 'fully compatible with our best scientific understanding'. One might wonder whether or in what sense the commitment to logical and mathematical concepts conforms to the naturalism condition after all.⁸

Notes

1. Cp. 'Perception, with the eyes of the mind of what is unchanging and precisely what it is, nothing more or less, perception that is achieved through just the sort of Socratic examination that is displayed in *Theaetetus* seems to Plato (at least as I read him) to be knowledge properly so called. Knowledge on this account is a cognitive grasp of an object in its nature, as what it is' (189).
2. If 'fact known' is question-begging, then substitute 'what is known'.
3. For an incisive and critical discussion of whether Plato conceives of judgment propositionally, see Crivelli 2012.
4. There are ways, however, in which assertibles differ from propositions, on which cp. S. Bobzien, 'The Stoics', in 'Logic', 1999.
5. It might be worth noting that while perceptual language, notably that of seeing, sometimes figures in the epistemological treatments of the ancients, aspects of early modern and nineteenth-century epistemological language similarly involve sense perceptual and specifically visual terms. For example consider the concepts of clarity and distinctness in Descartes' epistemology and Frege's notion that logical and mathematical concepts have 'clear boundaries'. For example in her discussion Macbeth cites the following passage from Frege's 1885 paper 'On the Law of Inertia': 'In the search for a boundary line, the contradictions, as they emerged, brought to the attention of the searchers that the assumed boundary was still uncertain or blurred ... The real driving force is the perception of a blurred boundary' (377).
6. Macbeth speaks here specifically of 'logical' bonds. I have elided this term because I think may be misleading insofar as it suggests that the bonds are merely logical. Of course deductive reasoning operates through entailments and entailment is a logical relation. But what justifies an entailment is a relation between concepts or kinds. For example, one may infer 'Felix is a mammal' from 'Felix is a cat' because 'cats are mammals' is true. But the relation between cats and mammals is, I take it, not a logical relation.
7. In a footnote Macbeth quotes the following from Frege's *Foundations of Arithmetic* (sec. 87): 'in the external world, in the whole of space and all that therein is, there are no concepts, no properties of concepts, no numbers. The laws of number, therefore, are not really applicable to external things; they are not laws of nature. They are, however, applicable to judgment holding good of things in the external world: they are laws of the laws of nature' (416, n.36).
8. These comments, here very lightly revised, were originally presented at the Greater Philadelphia Philosophy Consortium (GPPC) Author Meets Critics event, held at Haverford College on 19 September 2015. Thanks to Danielle for the invitation to participate then and the suggestion to contribute the comments in this volume of *IJPS*. Thanks to Jim O'Shea for agreeing to include them.

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