Verbal Labels Influence Children's Processing of Decimal Magnitudes

Jing Tian, Kexin Ren, & Elizabeth A. Gunderson

In press, *Journal of Applied Developmental Psychology*

Temple University, Department of Psychology and Neuroscience, Philadelphia, PA USA

Corresponding author, Jing Tian. Temple University, Department of Psychology and Neuroscience, 1701 N. 13th Street, Philadelphia, PA 19122. jing.tian@temple.edu. 215-204-0855. ORCiD, 0000-0003-2585-8866. Twitter @_TIANJing.

Kexin Ren, cathy.ren@temple.edu.

Elizabeth A. Gunderson, liz.gunderson@temple.edu ORCiD, 0000-0001-7108-2283. Twitter @GundersonLab.

Acknowledgments

This research was supported by NSF CAREER DRL-1452000, NSF ECR 1760144, and James S. McDonnell Foundation Grant #220020546 to Elizabeth Gunderson. We thank the children and teachers who gave their time to this research and the research assistants who managed data collection and data cleaning: Victoria Bartek, Ying Lin, Yiqiao Wang, Jorge Carvalho Pereira, Daniel Keefer, and Madyson Kolbes. This study was not preregistered. All testing scripts, data, and analysis code have been made publicly available on the OSF and can be accessed at https://osf.io/qfby8/.

Abstract

Verbal labels for math concepts influence multiple aspects of math learning. In this study, we examined the influence of point labels (e.g., .42 as “point four two”), decomposed labels (e.g., “four tenths and two hundredths”), and common-unit labels (e.g., “forty-two hundredths”) on children’s processing and representation of decimal magnitudes. We randomly assigned 162 5^{th}- and 6^{th}-graders to briefly learn decomposed, common-unit, or point labels. Children then completed measures of decimal magnitude processing and representation. We found that the place-value labels (i.e., decomposed and common-unit labels) each showed unique advantages in reducing the whole-number bias, and common-unit labels also reduced componential processing. No difference was found in the ratio effect – which served as an index of the precision of decimal magnitude representation - among children from the three conditions. These findings add to our understanding of the role of verbal labels in math learning and have important implications for instructional practices.

Keywords: verbal labels, decimal magnitude, whole-number bias, componential processing, ratio effect

Introduction

Proficiency in mathematics predicts success in school and beyond (Duncan et al., 2007; Ritchie & Bates, 2013). Competence with math is essential for learning many other school subjects, such as chemistry, physics, and engineering. Beyond school, strong math skills open the door to many sought-after jobs (Koedel & Tyhurst, 2012). In particular, knowledge of rational numbers, which includes fractions, decimals, and percentages, is often a stumbling block and gatekeeper to more advanced mathematics (Booth & Newton, 2012; DeWolf et al., 2015; Siegler et al., 2012; Wong, 2020). For example, fifth and sixth graders' decimal knowledge predicts general math achievement after controlling for whole number knowledge (Schneider et al., 2009). However, mastering rational numbers poses great challenges to many children. After years of instruction, even many community-college students and pre-service teachers still struggle with the topic (Siegler & Lortie-Forgues, 2015; Stigler et al., 2010). Therefore, it is important to understand the cognitive processes that are involved in learning rational numbers.

The current study focuses on one factor that may impact the processing of rational numbers – the verbal labels of these numbers in our language. Verbal labels for math concepts play an important role in math learning. Labels that precisely describe the mathematical structure of a concept can facilitate children's learning of that concept (Laski & Yu, 2014; Miller & Stigler, 1987; Miura et al., 1999; Paik & Mix, 2003). For example, all Chinese number labels have transparent base-ten structures, even for the numbers 11 and 12, whereas the English labels do not explicitly show the base-ten structures (e.g., the Chinese word for 11 translates to "ten one"). The transparency of Chinese number labels likely facilitates Chinese children's learning of the base-ten system and contributes to their more advanced number knowledge than English-speaking children (Laski & Yu, 2014; Miller & Stigler, 1987). To cite another example, Korean fraction labels emphasize the part-whole relationship between the numerator and the denominator (e.g., the Korean word for 2/3 translates to "of three parts, two"). Possibly due to fraction labels being more mathematically meaningful in Korean than in English, Korean children demonstrate stronger competence with fractions than their English-speaking US counterparts (Miura et al., 1999; Paik & Mix, 2003). Paik and Mix (2003) further showed that English-speaking US children's fraction performance improved after learning fraction labels that explicitly referred to the part-whole relations like those in Korean.

Here, we examined the influence of verbal labels on children's processing and representation of decimal magnitudes. Understanding decimal magnitudes is critical for learning more advanced math topics, such as algebra, and for overall math achievement (DeWolf et al., 2015; Hurst & Cordes, 2018b; Wong, 2020). Although in the U.S., decimals are formally introduced in math class in fifth grade (National Governors Association Center for Best Practices, 2010), many middle school students and even adults lack competence with decimals (DeWolf et al., 2015; Lortie-Forgues & Siegler, 2017; Tirosh et al., 1999). Therefore, examining factors that can influence the processing and representation of decimal magnitudes is of particular interest.

Verbal Labels for Decimals

One decimal can be labeled in several different ways. Adults usually label decimals using point labels, which label the individual digits in a decimal without referring to their place values. For example, the decimal .428 can be labeled as "point four two eight". However, place-value labels are recommended in formal decimal instruction (National Governors Association Center for Best Practices, 2010). Place-value labels specify the place value of each digit in a decimal (i.e., decomposed labels) or the place value of the right-most digit in a decimal (i.e., common-unit labels). For example, for .428, the decomposed label is "four tenths, two hundredths, and eight thousandths", and the common-unit label is "four hundred and twenty-eight thousandths".

The two types of place-value labels, decomposed labels and common-unit labels, have been treated as interchangeable in prior research (Loehr & Rittle-Johnson, 2016; Malone et al., 2017; Rittle-Johnson et al., 2001). Indeed, both of these place-value labels share properties that may facilitate decimal processing. On a conceptual level, both decomposed labels and common-unit labels provide an explicit connection between decimals and fractions. Given that fractions are introduced before decimals (National Governors Association Center for Best Practices, 2010), this verbal connection to fractions may facilitate children's understanding of decimals. On a more perceptual level, in both types of place-value labels, the decimal digits (i.e., digits to the right of the decimal point) all have "th" at the end, whereas labels for whole number digits (i.e., digits to the left of the decimal point) do not. As compared to the decimal point that separates whole-number and decimal digits and is highlighted by point labels, the "th" at the end of decimal digits may be more helpful for children learning to differentiate between whole-number and decimal digits.

However, decomposed labels and common-unit labels also have important differences. We argue that, rather than being interchangeable, these two types of place-value labels have strengths and weaknesses that may lead to unique effects on different aspects of decimal processing. We focus on two key aspects of decimal processing that have been identified in prior literature as showing individual and developmental differences - whole-number bias and componential versus holistic processing - and examine whether some labels lead to more advanced decimal processing (i.e., weaker whole-number bias and stronger holistic processing) than others. We also explore whether decimal labels improve the mental representations of decimal magnitudes. Below, we describe each aspect of decimal processing and decimal magnitude representation. We discuss the properties of decomposed labels and common-unit labels that we expect to facilitate or impede children's processing and representation of decimal magnitudes, compared to each other and compared to point labels.

Decimal Labels and Whole-Number Bias

Whole-number bias is the tendency to overgeneralize whole number knowledge to decimals, which leads to confusion about decimal magnitudes (Ni & Zhou, 2005). Likely due to children's extensive exposure to whole numbers before learning decimals, errors yielded by whole-number bias are very common on decimal tasks among children (Desmet et al., 2010; Durkin & Rittle-Johnson, 2015; Ren & Gunderson, 2019, 2021). One type of error concerns the role of zero in decimals. Adding a zero to the end of a whole number (i.e., a trailing zero) increases the magnitude by ten times (e.g., 2 vs. 20) whereas adding a zero at the beginning of a whole number (i.e., a leading zero) does not change its magnitude (e.g., 2 vs. 02). In contrast, adding a trailing zero to a decimal does not change its magnitude (e.g., .2 vs. .20) but adding a leading zero after the decimal point reduces the magnitude by ten times (e.g., .2 vs. .02). Misconceptions about the role of zero in decimals are prevalent among children (Desmet et al., 2010; Durkin & Rittle-Johnson, 2012, 2015; Nesher & Peled, 1986; Sackur-Grisvard & Léonard, 1985). For example, in one study, 40% of fifth graders ignored the zero in the tenths place (e.g., treated .07 as .7) when estimating decimals on number lines (Rittle-Johnson et al., 2001).

We expected decomposed labels to reduce role-of-zero misconceptions, as compared to common-unit labels and to point labels. Decomposed labels emphasize the place value of each digit, including zeros. For example, the decomposed label for .070 is “zero tenths, seven hundredths, and zero thousandths”. These labels may help children understand the meaning of both leading and trailing zeros and thereby reduce errors produced by ignoring them. In contrast, common-unit labels, which do not label leading zeros in decimals and only implicitly label trailing zeros (e.g., .070 as “seventy thousandths”), and point labels, which do not specify any place values, may not have this benefit. One study, in which children played a decimal-comparison card game, provides evidence consistent with this view (Loehr & Rittle-Johnson, 2016). In this study, third and fourth graders, who did not have much decimal knowledge, were randomly assigned to play the card game using decomposed labels, point labels, or no labels. Compared to children who used point labels or no labels, children who used decomposed labels made fewer errors on trials assessing role-of-zero knowledge (e.g., comparing 0.3 and 0.30). However, to our knowledge, no study to date has directly compared the effect of decomposed labels and common-unit labels on reducing role-of-zero misconceptions.

Another type of error driven by the whole-number bias is to assume that numbers with more digits are inevitably larger, a rule that is always correct for whole numbers, but not for decimals (e.g., assuming .53 is larger than .7 because the former has two digits and the latter has one). In decimal comparisons, this tendency yields a *string-length-congruity effect*: individuals are faster and more accurate at comparing decimals where the larger decimal has more digits (i.e., string-length congruent pairs such as .76 versus .5) than comparing decimals where the larger decimal has fewer digits (i.e., string-length incongruent pairs such as .53 versus .7). This effect is present both among adults (Huber et al., 2014; Varma & Karl, 2013) and children (Ren & Gunderson, 2019). For instance, in Ren and Gunderson (2019), sixth to eighth graders' accuracy on string-length-congruent items was 86%, compared to only 74% accuracy on string-length-incongruent items. Similarly, college students in Huber et al. (2014) had higher accuracy and shorter reaction times when comparing string-length-congruent pairs than string-length-incongruent pairs.

We expected decomposed labels to weaken the string-length-congruity effect, as compared to common-unit labels and point labels because decomposed labels are the least similar to whole-number labels. As compared to point labels, decomposed labels specify the place value of each decimal digit, which highlights the differences between decimal digits and whole number digits. Although common-unit labels also specify decimal place values, they are composed of a whole-number label and the place value of the smallest decimal digit (e.g., .53 is labeled "fifty-three hundredths"), which may reinforce the whole-number interpretation of decimals (e.g., that .53 is similar to "fifty-three"). Therefore, decomposed decimal labels are the most distinctive from whole-number labels and are likely to be the most helpful for children to distinguish decimals from whole numbers, thereby weakening the string-length-congruity effect. In line with this view, Loehr and Rittle-Johnson (2016) found that third and fourth graders who played the decimal comparison card game using decomposed labels exhibited weaker string-length-congruity effect than those who used point labels or no labels. Again, no study to date has compared the effect of decomposed and common-unit labels.

Decimal Labels and Componential vs. Holistic Processing

The three types of decimal labels may also influence the extent to which decimal magnitudes are processed componentially (i.e., separately processing the magnitude of each digit) versus holistically (i.e., processing the overall magnitude of the decimal). Prior research suggests that individuals process the magnitudes of each digit in whole number and decimal comparisons, even when they are irrelevant (Huber et al., 2014; Nuerk et al., 2001; Varma & Karl, 2013). People tend to respond faster and more accurately when comparing whole number and decimal pairs where each digit in the larger number is larger than all digits in the smaller number (e.g., 64 vs. 52 or .64 vs. .52; digit-compatible pairs) than comparing pairs where some of the digits in the larger number are smaller than digits in the smaller number (e.g., 64 vs. 58 or .64 vs. .58; digit-incompatible pairs). This effect is called the *digit-compatibility effect* (Nuerk et al., 2001). When the comparisons involve two decimals to the hundredths place, the effect is also called the *tenths-hundredths-compatibility effect* (Huber et al., 2014). The fact that the magnitudes of the hundredth digits influence decimal comparison speed and accuracy suggests that decimal digits are processed individually, and decimals are not processed solely as holistic magnitudes. The tenths-hundredths-compatibility effect in decimal comparison has been shown among adults (Huber et al., 2014), and the digit-compatibility effect in whole number comparison has been documented among both adults and children (Mann et al., 2012; Nuerk et al., 2001, 2004).

We expected that decomposed and point labels would lead to stronger componential processing than common-unit labels. Decomposed labels and point labels both explicitly label each digit in a decimal and are therefore likely to draw equal attention to all the decimal digits, even though digits with larger place values bear greater weight in determining decimal magnitudes. In contrast, common-unit labels only refer to the smallest unit and emphasize holistic magnitudes. Thus, we expected decomposed labels and point labels to result in more componential processing of decimals and yield a stronger tenths-hundredths-compatibility effect than common-unit labels. However, no prior research, to our knowledge, has tested how verbal labels affect the tendency to process decimals componentially.

Decimal Labels and the Ratio Effect

Finally, we also explored whether decimal labels influence the precision of decimal magnitude representations, as measured by the *ratio effect*. In decimal comparison, people tend to have higher error rates and longer response times as the ratio of the smaller number over the larger number increases, and this effect is referred to as the ratio effect (Moyer & Landauer, 1967). For example, comparing .6 and .8 (a ratio of 3:4) is more prone to errors and takes longer than comparing .6 and .9 (a ratio of 2:3).[1] The ratio effect has been documented both among children (Hurst & Cordes, 2018b) and adults (Hurst & Cordes, 2018a), and the existence of the ratio effect is often viewed as evidence that decimal magnitudes are represented in an ordered manner on a mental continuum (Hurst & Cordes, 2018a, 2018b; Wang & Siegler, 2013). Individual differences are present in the strength of the ratio effect – compared to others, some people’s response times and error rates increase at a greater rate as the ratio of the smaller number over the larger number increases. The strength of the ratio effect is viewed as reflecting the precision of the individual’s mental representation of numerical magnitudes - a more precise representation means less overlap with nearby numbers and a weaker ratio effect.

Although we expect decimal labels to influence children’s decimal magnitude processing tendencies, it is less clear whether and how different decimal labels would affect the precision of decimal magnitude representations, i.e., the strength of the ratio effect. It is possible that place-value labels, by directing attention to place values, would enhance the precision of magnitude

[1] Distance effect, the effect that the error rates and reaction times of comparing two numbers increase as the distance between the two numbers decreases, is closely related to the ratio effect (Lyons et al., 2015). In addition to numerical distance, the ratio effect also concerns the numerical magnitudes, and therefore, we focus on the ratio effect here.

representations and yield a weaker ratio effect. Some indirect evidence consistent with this possibility comes from research on decimal number line estimation, which, like the ratio effect, is argued to reflect the precision of magnitude representations (Berteletti et al., 2010; Dehaene et al., 2008; Kim & Opfer, 2017). Specifically, fifth graders' correct use of decimal place-value labels was associated with how much they improved on decimal number line estimation accuracy after a number line estimation intervention (Rittle-Johnson et al., 2001). Alternatively, the precision of decimal magnitude representations may be resistant to change via brief exposure to decimal labels. Consistent with this, the development of precise linear mental representations of large whole numbers (e.g., 0-1,000), as measured by number line estimation, takes years in the absence of direct feedback on the representation. In one study, only 9% of second graders exhibited evidence for a linear representation of whole numbers between 0-1000; this proportion was 38% among fourth graders, 72% among sixth graders, and reached 97% among adults (Siegler & Opfer, 2003). We explored these alternative possibilities by examining whether each type of decimal label influenced the strength of children's ratio effects in decimal comparisons.

The Current Study

In summary, the current study investigated how different decimal labels (decomposed labels, common-unit labels, and point labels) influence decimal magnitude processing and representation. To examine these effects, we randomly assigned children to briefly learn to use either decomposed labels, common-unit labels, or point labels. Children then completed measures of whole-number bias (i.e., the role of zero and the string-length-congruity effect), componential processing (i.e., the tenths-hundredths-compatibility effect), and precision of magnitude representation (i.e., the ratio effect). Knowledge of the role of zero was measured using a multiple-choice task. The string-length-congruity effect, the tenths-hundredths-compatibility effect, and the ratio effect were measured using decimal comparison tasks. Table 1 summarizes our predictions for how decimal labels would influence each aspect of decimal processing. We tested these predictions among fifth and sixth graders in the U.S. Children in these grade levels have received some formal instruction on both fractions and decimals - according to Common Core State Standards (National Governors Association Center for Best Practices, 2010), fractions are introduced in third grade and decimals are introduced in fifth grade. We therefore expected them to have some understanding of the fraction words “tenths”, “hundredths”, and “thousandths” in the place-value labels without additional instruction, while still being in the process of learning about decimal magnitudes. At the same time, most of the effects of interest (i.e., role-of-zero errors, string-length-congruity effect and ratio effect) have been documented among children of similar ages (Durkin & Rittle-Johnson, 2015; Hurst & Cordes, 2018b). Although to our knowledge, no study has examined the tenths-hundredths-compatibility effect (a type of digit-compatibility effect in which the comparison involves two decimals to the hundredths place) among children, we expect children to show this effect because they show the digit-compatibility effect with whole numbers (Mann et al., 2012). Findings of the current study can contribute to our theoretical understanding of how verbal labels influence cognitive processing, and at the same time, inform educational practice in decimal instruction.

Table 1

Summary of Predictions and Results

Measure	Prediction	Result
Whole-number bias: Role-of-zero	*Prediction 1.* Decomposed labels will lead to more correct answers than common-unit labels	✓
	Prediction 2. Decomposed labels will lead to more correct answers than point labels.	✓
	Prediction 3. Decomposed labels will lead to fewer leading-zero errors than common-unit labels.	×

	Prediction 4. Decomposed labels will lead to fewer leading-zero errors than point labels.	✓
Whole-number bias: String-length-congruity effect	*Prediction 5.* Decomposed labels will lead to a weaker string-length-congruity effect than common-unit labels.	!
	Prediction 6. Decomposed labels will lead to weaker string-length-congruity effect than point labels.	×
Componential vs. holistic processing: Tenths-hundredths-compatibility effect	*Prediction 7.* Common-unit labels will lead to a weaker tenths-hundredths-compatibility effect than decomposed labels.	✓
	Prediction 8. Common-unit labels will lead to a weaker tenths-hundredths-compatibility effect than point labels.	×
Magnitude representation: Ratio effect	*Exploratory.* Decomposed and common-unit labels will lead to a smaller ratio effect than point labels.	×

Note. "✓" indicates that a significant effect in the predicted direction is found ($p < .05$). "×" indicates that there was no significant difference between the two conditions. "!" indicates a significant effect in the opposite direction of what was predicted.

Methods

Participants

Fifth- and sixth-grade students were recruited from nine schools (12 classrooms) in a large city in the northeastern US ($N = 177$; 115 fifth graders and 62 sixth graders; 95 girls and 82 boys; $M_{age} = 11.20$ years, $SD_{age} = 0.68$). A power analysis indicated that a sample size of 156 (52 in each condition) would be sufficient to detect a medium-sized effect ($\eta^2 = .06$; Loehr & Rittle-Johnson, 2016) of differences between conditions, with $\alpha = .05$ and power = .80. Because we allowed all children with parental consent and child assent from the participating classrooms to participate in the study, the number of participants exceeded the target sample size. Children were randomly assigned within classroom to a decomposed-label condition (*N* = 59), a common-unit-label condition (*N* = 57), or a point-label condition (*N* = 61).

Due to an experimenter error (incorrectly administering basal and/or ceiling rules), 15 children did not complete the reading achievement measure. Because this measure was used as a covariate in all the inferential analyses, we excluded these children from our analytic sample, resulting in an analytic sample of 162 (*N* = 55 in the decomposed-label condition, *N* = 50 in the common-unit-label condition, and *N* = 57 in the point-label condition). As a robustness check, we ran parallel analyses without including the covariate of reading achievement, allowing us to include all 177 children. These analyses yielded similar results as reported below (see Supplementary Materials, Section A for results of these analyses).

Based on parents' reports (*N* = 136), 60% of the children were Black or African American, 11% were multiracial, 12% were White, 10% were Hispanic, 4% were Asian or Asian American, 1% were American Indian or Alaskan Native, and 1% were of another race or ethnicity. Based on parents' reports of language(s) spoken at home (*N* = 137), 66% of the families spoke only English at home, 1% spoke only a language other than English at home, 31% spoke two languages, and 1% spoke three languages. English was the primary language spoken at home in 91% of the families. All children in the study spoke English at school and were able to speak and understand English during this study. The study procedures were approved under Temple University Institutional Review Board (IRB) protocol 21935, "Cognitive and Emotional Bases of Math, Reading, and Spatial Development."

Procedure

Each child worked with a trained experimenter for one 20- to 30-minute session in a quiet space at their school. At the beginning of the session, children completed a standardized reading achievement measure, specifically a decoding measure, as a control. Decoding is essential for recognizing the meaning of words (Perfetti, 2010). Controlling for decoding allows us to parse out at least some influence of verbal skills from the effects of verbal labels on decimal processing. Children then received approximately 10 to 15 minutes of training on labeling decimals with either decomposed labels, common-unit labels, or point labels, depending on the condition they were assigned to. Finally, children completed four decimal magnitude measures assessing whole number bias in two ways (role-of-zero knowledge and the string-length-congruity effect), componential processing (i.e., tenths-hundredths-compatibility effect), and precision of magnitude representations (i.e., ratio effect). The order of the four measures was counterbalanced using a Latin squares design. The order of the test items within each measure was fully randomized for each participant. To remind students of the trained decimal labels, after completing each of the first three decimal magnitude measures, children were shown two decimals along with the labels that children were trained with. The three sets of reminder decimals were 0.20 and 0.84, 0.6 and 0.02, and 0.63 and 0.49. These reminder decimals appeared in the same order specified here for all participants. The decimal training and assessment stimuli were presented on a laptop using jsPsych on JATOS (de Leeuw, 2015; Lange et al., 2015).

Training

During training, the experimenter explained to the child how to label three types of decimals: decimals with no leading zeros or trailing zeros, such as 0.2 and 0.57; decimals with leading or trailing zeros, such as 0.04 and 0.700; and decimals with both leading and trailing zeros, such as 0.030. See Supplementary Materials, Table S1 for the complete list of decimals used in the training. The training procedure and stimuli were the same across the three conditions, except for the verbal and written labels used for the decimals.

In the first phase of the training, children were asked to name 14 decimals by reading the labels presented below the decimals (see Table 2 for examples). The experimenter introduced the first decimal by saying, “The way we can name this decimal is [the presented label]. Can you repeat the name?" On each subsequent training trial, if the child named the decimal with the provided label correctly, the experimenter would confirm and read the label again by saying, "Yes, it is [the presented label]." If the child named the decimal incorrectly, the experimenter would point to the label on the screen, correct the child, and ask the child to name the decimal again by saying, "This is actually [the presented label]. Can you read the name?" If the child provided a correct decimal label that was different than the presented label, the experimenter would encourage the child to use the presented label without commenting on the correctness of the label the child used. Specifically, the experimenter would say, "Another way to name it is [the presented label]. Can you read the name?"

Table 2

Example Training Trials in Each Condition

Type of Decimal	Decimal Example	Decomposed-Label Condition	Common-Unit-Label Condition	Point-Label Condition
Decimals with no leading or trailing zero	0.2	Two tenths	Two tenths	Point two
Decimals with leading or trailing zeros	0.051	Zero tenths, five hundredths, and one thousandth	Fifty-one thousandths	Point zero five one
Decimals with both leading and trailing zeros	0.030	Zero tenths, three hundredths, and zero thousandths	Thirty thousandths	Point zero three zero

In the second phase of the training, children practiced naming a new set of 14 decimals without any labels presented. After the child responded on each trial, the experimenter provided corrective feedback similar to that in the first phase.

Measures

Whole-Number Bias: Role-of-Zero Knowledge

This measure was designed to examine students' knowledge and misconceptions regarding the role of zero in decimal magnitudes (all items were created by the authors, adapted from "role of zero" items in Durkin & Rittle-Johnson, 2015). Children were shown a target decimal and four choices and were asked to choose all the choices that were equal to the target (see Figure 1A for an example test trial). On each trial, the four choices included one correct choice and three incorrect choices. The correct choice was created by either adding or deleting a trailing zero from the given decimal. For example, on the trial where 0.020 was the target decimal, the correct choice was 0.02. The three incorrect answers (foils) were designed to capture specific misconceptions. The leading-zero foil was created by adding or deleting a leading zero from the target (e.g., for the target 0.020, the leading zero foil was 0.20). The whole-number foil was created by ignoring the decimal point and any leading zeros in the target decimal (e.g., for the target 0.020, the whole-number foil was 20). The random-string foil was created by arranging the same digits in the given decimal differently (e.g., for the target 0.020, the random string foil was 2.000).

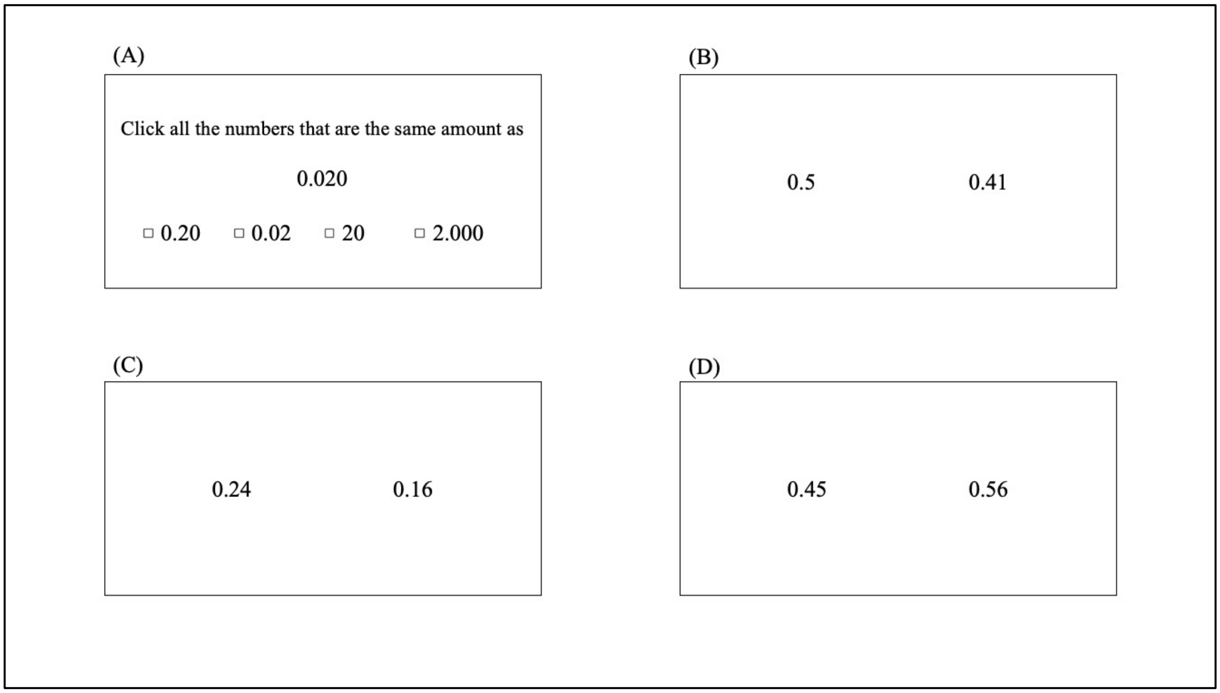

Figure 1. Example test items in measures of (A) whole-number bias: role-of-zero knowledge, (B) whole-number bias: string-length-congruity effect, a string-length-incongruent trial, (C) componential processing: tenths-hundredths-compatibility effect, a tenths-hundredths-incompatible trial, and (D) precision of magnitude representation: ratio effect. On the decimal comparison tasks (B-D), children were instructed to choose the larger number in each pair.

At the beginning of the task, children completed four practice trials where the target decimals were presented with labels corresponding to the child's assigned condition. Children were asked to first read the label for the target decimal and then click all the choices equal to the target. Children then completed nine test trials without labels (see Supplementary Materials, Table S2 for a complete list of stimuli).[2] No feedback was provided on practice trials or test trials.

[2] A tenth trial was administered during the role-of-zero task but was excluded from our analyses due to an error in the item's design.

Because the predictions relevant to this task concerned choosing the correct answer or the leading zero foil, we categorized the response to each trial as a correct answer (when *only* the correct answer was chosen), a leading zero error (*whenever* the leading zero foil was chosen; e.g., choosing the correct answer and the leading zero foil), or other. Reliability of accuracy on the measure was high (Cronbach's alpha = 0.92). Reliability was also high for leading-zero errors (Cronbach's alpha = 0.86).

Whole-Number Bias: String-Length-Congruity Effect

As a measure of the string-length-congruity effect produced by whole-number bias, children completed a decimal comparison task designed to assess their tendency to choose the longer string of digits as the larger number (items were created by the research team following similar prior work, e.g., Huber et al., 2014; Ren & Gunderson, 2019; Varma & Karl, 2013). In this task, each pair of decimals included a one-digit decimal and a two-digit decimal (see Figure 1B for an example test trial). None of the decimals had leading or trailing zeros. Half of the trials were string-length-congruent trials, where the one-digit decimal was smaller than the two-digit decimal (e.g., 0.2 vs. 0.95). The other half of the trials were string-length-incongruent trials, where the one-digit decimal was larger than the two-digit decimal (e.g., 0.5 vs. 0.41). Within each trial type, we counterbalanced the side of the screen (left vs. right) on which the correct answer appeared. To control for tenths-hundredths compatibility, each of the decimal digit(s) in the larger decimal were larger than each of the decimal digit(s) in the smaller decimal.

Children first completed four practice trials on which decimals were presented with the trained labels according to the child's assigned condition. Children were asked to read the labels for the two decimals in each pair and indicate which decimal was larger using that label. Children then completed 16 test trials without decimal labels by pressing the "A" key if the number on the left was larger or the "L" key if the number on the right was larger (see Supplementary Materials, Table S3 for a complete list of stimuli). No feedback was given on practice trials or test trials. Children were asked to respond as quickly as possible without sacrificing accuracy on test trials. Trials on which the reaction times (RT) were shorter than (<) 200 ms or longer than (≥) 10,000 ms (0.4% of trials) were excluded from the analyses on this and other tasks involving decimal comparison. The excessively short or long reaction times likely indicated children not paying attention on the trial, and excluding these trials is a common practice in prior studies with magnitude comparison tasks (e.g., Hurst & Cordes, 2018a; Nuerk et al., 2001; Ren & Gunderson, 2021). Accuracy on the remaining trials were used in the analyses. On this task, a larger difference in accuracy favoring string-length-congruent trials over string-length-incongruent trials indicates a greater whole-number bias. Reliability was high for both congruent (Cronbach's alpha = 0.97) and incongruent (Cronbach's alpha = 0.97) items.

Componential Processing: Tenths-Hundredths-Compatibility Effect

To assess students' componential processing, we asked children to complete a decimal magnitude comparison task designed to measure the tenths-hundredths-compatibility effect. Items were created by the research team following similar prior work (Huber et al., 2014; Nuerk et al., 2001). In this task, all decimals had two decimal digits and no leading or trailing zeros (see Figure 1C for an example test item). On tenths-hundredths-compatible trials (half of all trials), the tenths digit and the hundredths digit of the larger decimal were both larger than the corresponding digit of the smaller decimal (e.g., 0.35 vs. 0.23). On tenths-hundredths-incompatible trials (half of all trials), the tenths digit of the larger decimal was larger than the tenths digit of the smaller decimal, while the hundredths digit of the larger decimal was smaller than that of the smaller decimal (e.g., 0.24 vs. 0.16). The side of the screen on which the correct answer appeared was counterbalanced within trial types. Because all decimals had two decimal digits, string length should not affect magnitude judgements. There were four practice trials on which children read the decimal labels corresponding to their assigned condition and verbally responded using the labels. Children then completed 16 test trials without labels (see Supplementary Materials, Table S4 for a complete list of stimuli). No feedback was provided on the practice or test trials. Children were asked to respond as quickly as possible without sacrificing accuracy on the test trials. Trials with RTs were shorter than (<) 200 ms or longer than (≥) 10,000 ms (0.2% of trials) were excluded from the analyses. On this task, a stronger tenths-hundredths-compatibility effect (i.e., a larger difference in accuracy favoring tenths-hundredths compatible trials over tenths-hundredths incompatible trials) indicates stronger componential processing. Reliability was good for compatible items (Cronbach's alpha = 0.76) and high for incompatible (Cronbach's alpha = 0.91) items.

Precision of Magnitude Representation: Ratio Effect

To assess children's precision of decimal magnitude representations, we assessed the ratio effect in decimal comparison. The research team created the items by systematically varying the ratio of the magnitude between the decimals compared, following prior work (see Figure 1D for an example test item; Hurst & Cordes, 2018a; Wang & Siegler, 2013). Four pairs of two-digit decimals were chosen in each of four ratio bins: 1.25, 1.5, 2, or 2.5 (16 test trials in total). To avoid the influence of string length, we used only two-digit decimals in this task. Additionally, all decimal pairs were tenths-hundredths compatible. The side of the screen on which the correct answer appeared was counterbalanced within ratio bin. As in the other decimal comparison tasks, there were four practice trials with decimal labels and 16 test trials without labels (see Supplementary Materials, Table S5 for a complete list of the stimuli). Children did not receive any feedback on the practice or the test trials. On the test trials, children were asked to respond as quickly and accurately as possible. Reliability of the measure was calculated using accuracy and was good (Cronbach's alpha = 0.80).

Analyses of the ratio effect focused on RTs. We excluded trials on which the RTs were shorter than (<) 200 ms or longer than (≥) 10,000 ms (0.5%) from the analyses. We computed a ratio effect index for each child using RTs of that child's accurate responses. We further excluded trials on which the RTs were beyond three standard deviations away from the child's mean RT (1%; all above the mean). For each child, we fit a linear regression model using the ratio of the decimal pairs predicting RTs of accurate responses. Children needed to have data in at least three of the four ratio bins to be included in this analysis (*N* = 161). Beta coefficient estimates of the effect of ratio on RTs were used as the index for the ratio effect. A larger beta coefficient estimate indicates greater ratio effects and less precision of magnitude representations.

Reading Achievement

We assessed students' reading achievement using the Letter-Word Identification subtest of the Woodcock-Johnson IV (Schrank, Mather, & McGrew, 2014). In this test, children were asked to identify letters and read words of increasing difficulty. Basal was met when children answered correctly on the six lowest-numbered items that were administered. Ceiling was met when children answered incorrectly on the six highest-numbered items that were administered. The test ended when both basal and ceiling were met. W scores were used in the analyses.

This study was not preregistered. All testing scripts, data, and analysis code have been made publicly available on the OSF and can be accessed at https://osf.io/qfby8/.

Results

All the analyses were conducted using R (R Core Team, 2018) and the *lme4* package (Bates et al., 2014). There was no missing data on any of the four decimal knowledge measures other than trials excluded because of RTs as described above.

Descriptive Analyses

Table 3 shows descriptive statistics of performance on each measure, separately for children in each condition. Because grade level did not correlate with any measure, we combined children from both grade levels in all subsequent analyses.

Table 3

Demographic Information and Mean Performance on Each Measure by Condition

Measure	Decomposed-Label Condition	Common-Unit-Label Condition	Point-Label Condition

	(*N* = 55) *M* (*SD*)	(*N* = 50) *M* (*SD*)	(*N* = 57) *M* (*SD*)
Demographics			
Gender	32 girls; 23 boys	29 girls; 21 boys	29 girls; 28 boys
Grade	32 fifth graders; 23 sixth graders	34 fifth graders; 16 sixth graders	35 fifth graders; 22 sixth graders
Years of age	11.27 (0.63)	11.14 (0.70)	11.25 (0.72)
Role-of-Zero (frequency)			
Correct answer (when only the correct answer was chosen)	0.43 (0.38)	0.29 (0.35)	0.30 (0.36)
Leading zero error (whenever the leading zero foil was chosen)	0.32 (0.31)	0.43 (0.29)	0.53 (0.36)
String-Length-Congruity Effect (accuracy)			
Congruent items	0.97 (0.14)	0.90 (0.29)	0.98 (0.13)
Incongruent items	0.16 (0.34)	0.22 (0.40)	0.19 (0.36)
Tenths-Hundredths-Compatibility (accuracy)			
Compatible items	0.95 (0.15)	0.95 (0.10)	0.95 (0.14)
Incompatible items	0.87 (0.29)	0.98 (0.06)	0.95 (0.14)
Ratio Effect (accuracy)	0.98 (0.04)	0.96 (0.07)	0.96 (0.14)
Ratio Effect (RT slope index)	-113.78 (208.92)	-108.68 (345.11)	-47.24 (144.33)
Reading Achievement (W Score)	500.42 (19.31)	501.02 (17.31)	503.14 (13.36)

Whole Number Bias: Role-of-Zero Knowledge

To test whether decomposed labels led to more correct answers than common-unit labels and point labels (Predictions 1 and 2), we fit a generalized linear mixed-effects model (GLMM) on the likelihood of choosing only the correct answer on each trial. Training condition was entered as a fixed effect, participant was entered as a random effect, and reading achievement was entered as a covariate. Because W scores on the reading achievement measure were on a much larger scale (ranging from 433 to 541 in our sample) than the dependent variable (i.e., 0 or 1), we scaled the reading achievement W score by dividing each child's score by the maximum score among all children, to improve model convergence.

The model yielded a significant effect of condition, $X^2(2) = 6.29$, $p = .043$. Figure 2A shows the probability of choosing only the correct answer among children in each condition based on the model estimates. Parameter estimates with the decomposed-label condition as the reference level showed that the results were consistent with both Predictions 1 and 2. After controlling for reading achievement ($B = 23.67$, $p = .004$), the likelihood of choosing only the correct answer was higher among children in the decomposed-label condition than the common-unit-label condition ($B = -1.44$, $p = .028$) and the point-label condition ($B = -1.33$, $p = .034$). Setting the common-unit-label condition as the reference level, parameter estimates showed no significant difference in the likelihood of choosing only the correct answer in the common-unit-label versus the point-label conditions ($B = 0.12$, $p = .858$).

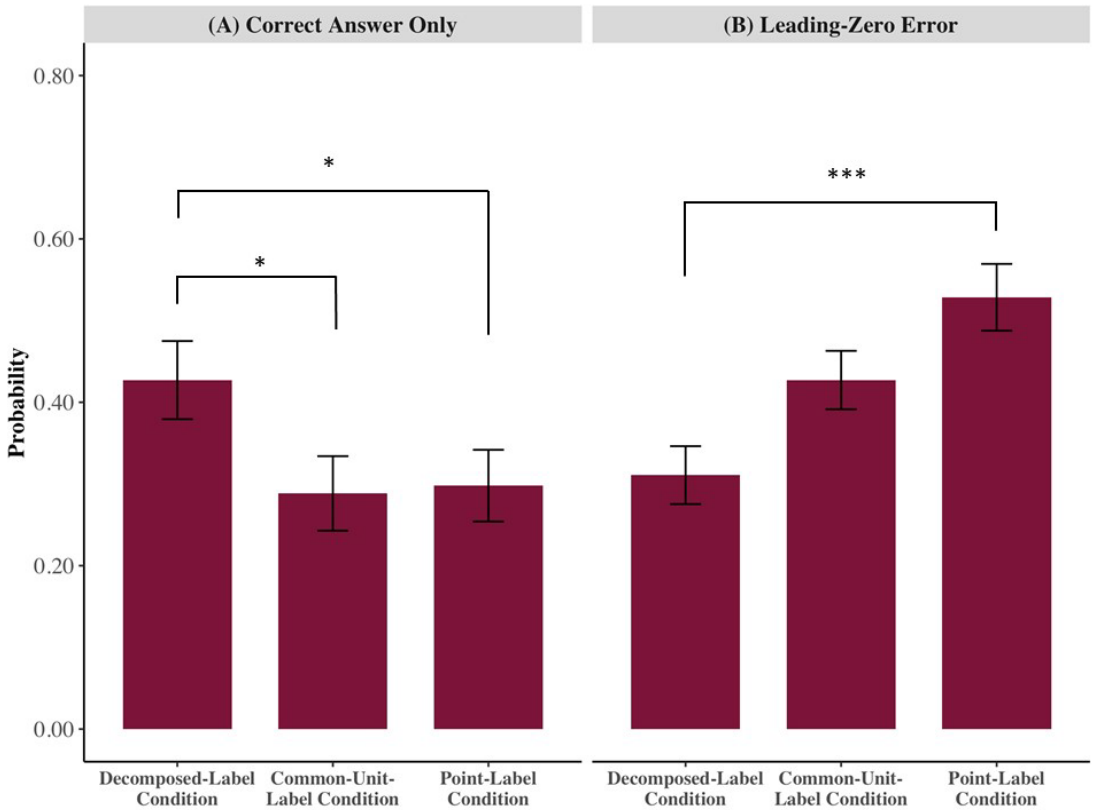

Figure 2. Probability of (A) choosing only the correct answer and (B) making a leading-zero error among children in each training condition based on predicted values in each GLMM. Error bars represent one standard error. Correct answer: only the correct choice was selected. Leading-zero error: whenever the leading zero foil was selected. * $p < .05$, *** $p < .001$.

To test whether decomposed labels led to fewer leading-zero errors (i.e., choosing the leading zero foil either alone or along with other choices) compared to common-unit labels and point labels (Predictions 3 and 4), we fit a similar GLMM on the likelihood of making a leading-zero error on each trial. [3] The model yielded a significant effect of condition, $X^2(2) = 12.00$, $p = .002$. Figure 2B shows the probability of making a leading-zero error among children in each condition based on the model estimates. The effect of reading achievement in this and all the

[3] We also ran a parallel model on the likelihood of choosing *only* the leading zero error on each trial. The model did not yield a main effect of condition, $X^2(2) = 3.424$, $p = .181$.

other models was not significant unless reported otherwise. The model estimates with the decomposed-label condition as the reference level showed no significant difference in the likelihood of making leading-zero errors among children in the common-unit-label condition than the decomposed-label condition, $B = 0.80$, $p = .053$, providing no evidence for Prediction 3 (although the non-significant trend was descriptively in line with Prediction 3). Consistent with Prediction 4, children in the point-label condition were significantly more likely than children in the decomposed-label condition to make leading zero errors, $B = 1.40$, $p < .001$. Setting the common-unit-label condition as the reference level, parameter estimates suggested the likelihood of making leading-zero errors was not significantly different among children in the common-unit-label and the point-label conditions ($B = 0.60$, $p = .140$).

Whole-Number Bias: String-Length-Congruity Effect

We next examined Predictions 5 and 6, that decomposed labels would lead to a weaker whole-number bias and therefore a weaker string-length-congruity effect than common-unit labels and point labels. To do so, we fit a GLMM on children's accuracy on the string-length-congruity effect measure.[4] Training condition, item type (with congruent items as the reference group), and the interaction between the two were entered as fixed effects. Participant was entered as a random effect, reading achievement was entered as a covariate, and accuracy on each trial was entered as the dependent variable. To improve model convergence, we divided each child's reading achievement W score by the maximum score among all children.

The model yielded a significant main effect of item type ($X^2(1) = 546.32$, $p < .001$) and a significant interaction between item type and condition ($X^2(2) = 45.58$, $p < .001$). Figure 3 shows the expected values of accuracy on string-length-congruent and string-length-incongruent trials

[4] Analyses of the string-length-congruity measure and the tenths-hundredth-compatibility measure focused on accuracy. Parallel analyses of RTs on these two measures did not yield any significant effects, suggesting that the effects we observed with accuracy were not due to a speed-accuracy trade-off (see Supplementary Materials, Section B for analyses on RTs).

among children from each of the three training conditions based on the model estimates. Estimates of model parameters with the decomposed-label condition as the reference group indicated that children in the decomposed-label condition showed a stronger string-length-congruity effect than children in the common-unit-label condition (i.e., greater difference in accuracy between congruent and incongruent items; $B = 3.12$, $p < .001$), which contradicted our Prediction 5. There was no significant difference in the strength of the string-length-congruity effect between the decomposed-label condition and the point-label condition ($B = 0.10$, $p = .878$), providing no evidence for Prediction 6. Model estimates with the common-unit-label condition as the reference level showed that children in the point-label condition had a stronger string-length-congruity effect than children in the common-unit-label condition ($B = -3.02$, $p < .001$).

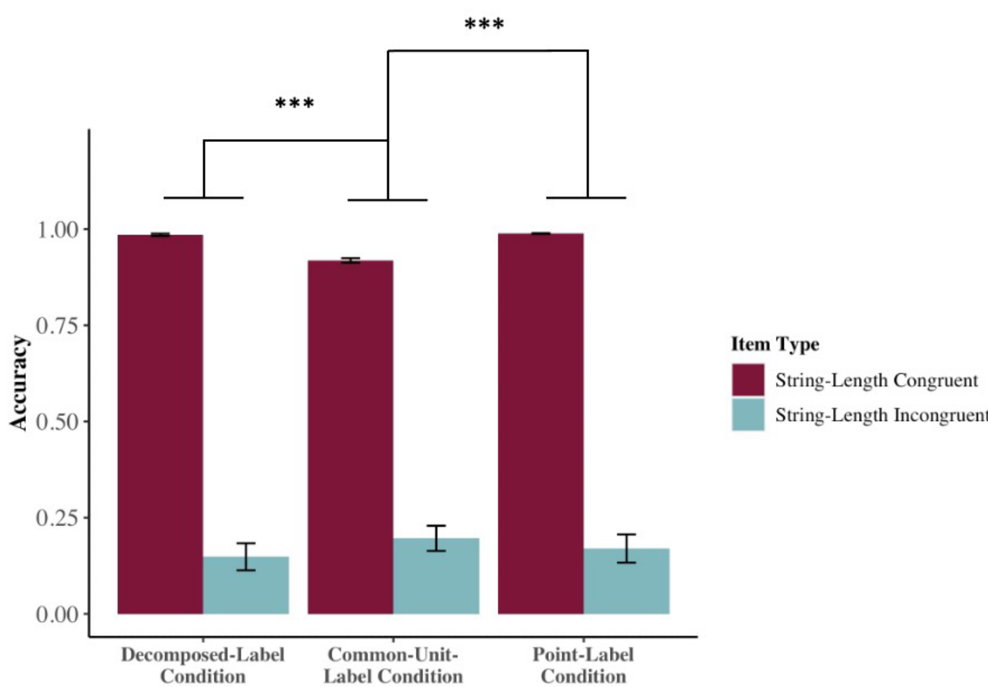

Figure 3. Accuracy on the string-length-congruity effect measure among children from each of the three training conditions. Values are predicted values from the GLMM. Error bars represent one standard error. *** $p < .001$.

Componential Processing: Tenths-Hundredths-Compatibility Effect

Figure 4 shows children's accuracy on tenths-hundredths compatible and tenths-hundredths incompatible trials. To test Predictions 7 and 8, that common-unit labels lead to a weaker tenths-hundredths-compatibility effect than decomposed labels and point labels, we fit a GLMM on accuracy on the tenths-hundredths-compatibility effect measure. Training condition, item type (with tenths-hundredths compatible items as the reference group), and the interaction between the two were entered as fixed effects. Participant was entered as a random effect, reading achievement was scaled and entered as a covariate, and accuracy on each trial was entered as the dependent variable.

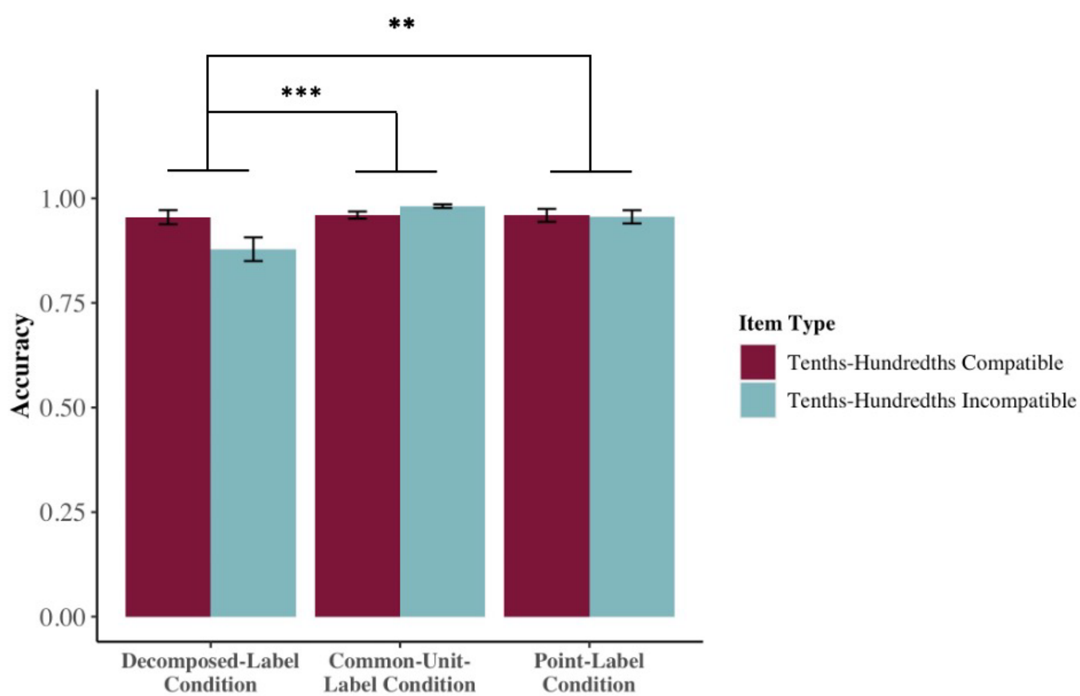

Figure 4. Accuracy on the tenths-hundredths-compatibility effect measure among children from each of the three training conditions. Values are predicted values from the GLMM. Error bars represent one standard error. ** *p* < .01, *** *p* < .001.

The model yielded a significant main effect of item type ($X^2(1) = 4.87$, *p* = .027) and a significant interaction between item type and condition ($X^2(2) = 22.69$, *p* < .001). Estimates of model parameters with the common-unit-label condition as the reference group suggested that consistent with Prediction 7, children in the common-unit-label condition showed a weaker tenths-hundredths-compatibility effect than children in the decomposed-label condition (i.e., a smaller difference in accuracy favoring tenths-hundredths compatible than tenths-hundredths incompatible items; $B = -2.58$, $p < .001$). However, although trending in the expected direction, children in the common-unit-label condition and the point-label condition did not significantly differ in the strength of tenths-hundredths-compatibility effects ($B = -0.98$, $p = .087$), failing to support Prediction 8. Parameter estimates of the model with the decomposed-label condition as the reference group showed that children in the point-label condition showed a weaker tenths-hundredths-compatibility effect than children in the decomposed-label condition ($B = 1.61$, $p = .002$).

Ratio Effect

Figure 5 shows the average RTs of accurate responses on trials in each ratio bin. We used each child's slope of the relation between ratios and RTs as the ratio effect index in subsequent inferential analyses. *T*-tests suggested that the ratio effect index was significantly different from zero among children in all conditions (decomposed-label condition: $M = -113.78$, $SD = 208.92$, $t(54) = -4.04$, $p < .001$, Cohen's $d = 0.54$; common-unit-label condition: $M = -108.68$, $SD = 345.11$, $t(48) = -2.23$, $p = .031$, Cohen's $d = 0.31$; and point-label condition: $M = -47.24$, $SD = 144.33$, $t(55) = -2.45$, $p = .018$, Cohen's $d = 0.33$).

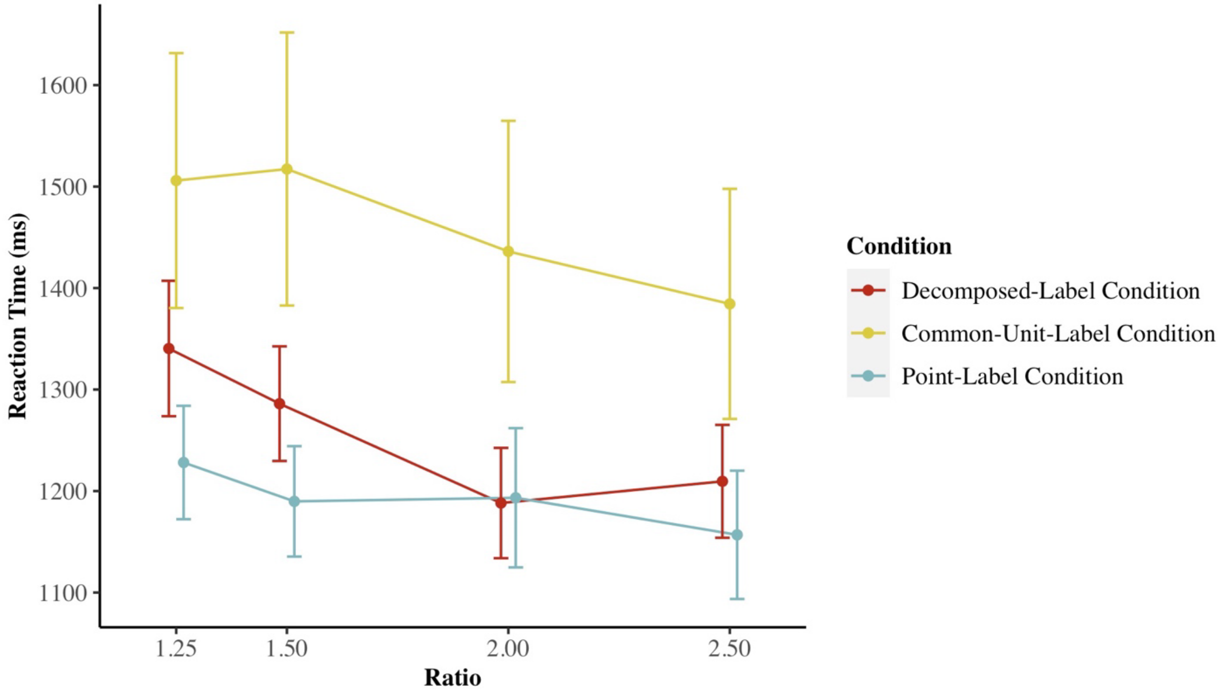

Figure 5. Average reaction times of accurate responses on the ratio effect measure among children from each of the three training conditions. Error bars represent one standard error.

To explore whether the training influenced the strength of the ratio effect, we fit a linear regression model with training condition as a predictor, reading achievement as a covariate, and the ratio effect index as the dependent variable. Reading achievement W score was not scaled in this model because it was on a similar scale as the ratio effect index. The effect of training condition was not significant in the model, F (2, 157) = 1.16, p = .318. Children in the common-unit-label condition did not significantly differ in the ratio effect compared to children in the decomposed-label condition (B = -4.42, p = .926) and the point-label condition (B = 58.89, p = .216).

Discussion

As compared to informal point labels, place-value decimal labels, including decomposed labels and common-unit labels, are believed to scaffold children's understanding of decimal magnitudes (Loehr & Rittle-Johnson, 2016; Malone et al., 2017; National Governors Association

Center for Best Practices, 2010; Rittle-Johnson et al., 2001). We argue that although the two types of place-value labels may both be more beneficial than point labels, each has strengths and weaknesses in promoting children's decimal magnitude processing. By having fifth and sixth graders learn and practice labeling decimals with either decomposed labels, common-unit labels, or point labels, we illustrated the distinctive effects of these labels on decimal magnitude processing. In particular, decomposed labels and common-unit labels each showed unique advantages in reducing the whole-number bias, and common-unit labels also reduced the tendency to process the magnitudes of decimal digits individually.

In the current study, a brief exposure to decomposed labels decreased one type of misconception yielded by the whole-number bias - misconception about the role of zero in decimals - among fifth and sixth graders. Children often incorrectly assume that leading and trailing zeros function similarly in decimals as in whole numbers – that adding leading zeros does not change the magnitude of a decimal (e.g., assuming 0.03 equals 0.3) whereas adding trailing zeros does (e.g., assuming 0.30 does not equal 0.3; Desmet et al., 2010; Durkin & Rittle-Johnson, 2015). We expected that decomposed labels would reduce this misconception by explicitly labeling the place values of zeros in the fractional part of the decimal (e.g., labeling .030 as “zero tenths, three hundredths, and zero thousandths”). Consistent with this expectation, children exposed to decomposed labels were more likely to correctly recognize that deleting or adding trailing zeros in a decimal does not change its magnitude than children in the common-unit-label or point-label conditions. Children who learned and practiced using decomposed labels were also less likely to make errors produced by treating decimals with and without a leading zero as equivalent (e.g., 0.3 and 0.03). These findings suggest that decomposed labels can help reduce children's role-of-zero misconceptions.

However, contrary to our expectations, decomposed labels did not reduce another effect also yielded by the whole-number bias, the string-length-congruity effect. This effect is driven by the tendency to judge numbers with more digits to be larger, an assumption true of whole numbers (e.g., 51 is greater than 7) but not true of decimals (e.g., 0.51 is less than 0.7). Because decomposed decimal labels are the most distinctive from whole-number labels among the three types of labels, we expected them to be the most effective in reducing this tendency. However, children in the decomposed-label and point-label conditions exhibited similar string-length-congruity effects, suggesting that specifying place values of each decimal digit did not influence the strength of this effect. This finding contradicts prior ones where children exposed to decomposed labels exhibited weaker string-length-congruity effects than those exposed to point labels (Loehr & Rittle-Johnson, 2016). One reason for this discrepancy might be that children in our study completed decimal comparison problems without any labels presented. In contrast, children in Loehr and Rittle-Johnson's (2016) study completed half of the problems while naming the decimals with the assigned labels presented below the problems. This manipulation might have resulted in a more substantial influence of decomposed labels on performance than in our study.

Our exploratory analyses showed that common-unit labels weakened the whole-number bias as reflected by a smaller string-length-congruity effect. It is likely that after exposure to common-unit labels, children less often assumed numbers with more digits to be larger. Instead, they more often assumed numbers with fewer digits to be larger - sometimes called the *fraction rule* (Resnick et al., 1989; Sackur-Grisvard & Léonard, 1985). The rule reflects the misconception that numbers labeled as “tenths” are always greater than numbers labeled as “hundredths” because 1/10 is greater than 1/100. Common-unit labels likely encouraged using this rule by signifying the place value of only the smallest digit. For example, when children compare 0.56 and 0.4, common-unit labels make it possible to use the fraction rule and incorrectly conclude that 56 hundredths is smaller than 4 tenths because “hundredths” is smaller than “tenths”. In contrast, the fraction rule cannot be applied with decomposed labels because the decomposed label for 0.56 refers to both “tenths” and “hundredths”. It is worth noting that, even in the common-unit label condition, there was still a strong string-length congruity effect, indicating substantial whole-number bias at the group level. Thus, although the labeling manipulation impacted students' performance, more sustained instruction may be necessary to eliminate whole-number bias on this task.

In the current study, the advantage of common-unit labels was also reflected in a reduction in componential processing (i.e., tenths-hundredths-compatibility effect). Compared to common-unit labels, decomposed labels are expected to draw more attention to the hundredths digit, which is irrelevant in two-digit decimal comparisons when the tenths digit differs. Consistent with this expectation, practicing common-unit labels reduced interference from the hundredths digit, compared to practicing decomposed-labels. Our exploratory analyses further suggested that decomposed labels led to a stronger interference from the hundredths digit than point labels. It is likely that labeling the place values of each decimal digit as in decomposed labels (e.g., "two tenths and four hundredths") encouraged componential processing of decimal magnitudes.

Although decimal labels influenced the processing of decimal magnitudes, as evidenced by the unique effects of the three types of labels on whole-number bias and componential processing, they did not seem to influence the precision of children's decimal magnitude representation. Across the three conditions, we found no significant differences in the size of the ratio effect, which is viewed as an indicator of the precision of numerical representations (Halberda & Feigenson, 2008; Verguts & Fias, 2004). It is possible that the precision of decimal magnitude representations is resistant to change. This would be consistent with the slow development of precision in whole-number magnitude representations (Siegler & Opfer, 2003). The slow progression between second grade and adulthood suggests that developing a precise representation of large whole numbers takes a relatively long time, and this may also be the case for decimals. Although some brief targeted training, such as providing feedback on estimates on the number line and playing linear board games involving numbers, improved the linearity and precision of whole number magnitude representation (Opfer & Siegler, 2007;

Siegler & Ramani, 2008), the brief exposure to decimal labels in our study might be too short and too oblique to lead to changes in decimal magnitude representations.

These findings, in addition to revealing the effects of verbal labels, shed light on the nature of children's knowledge of decimals. Children's performance on the two tasks measuring whole-number bias each benefited from different place-value labels, suggesting that the role-of-zero errors and the string-length-congruity effect are separable components of the whole-number bias. Therefore, whole-number bias is not a single, unified phenomenon and overcoming it may require addressing each component individually. Further, the fact that children's performance changed after only brief training suggests that these aspects of whole-number bias are highly malleable. This may indicate that children had prior knowledge of decimal magnitudes, and the verbal labels activated this knowledge. Alternatively, this malleability to brief training may indicate that the labels led children to attend to certain features of the stimuli and thereby change their task-solving strategies. These possibilities could be distinguished by future research examining the effects of verbal labels on children's performance on multiple tasks with varying characteristics measuring the same aspect of whole-number bias.

Educational Implications

Verbal labels are ubiquitous in teaching and discussing mathematical concepts in the classroom. Prior research has illustrated the power of verbal labels on many aspects of math learning, such as counting (Miller & Stigler, 1987), fraction knowledge (Paik & Mix, 2003), proportional reasoning (Hurst & Cordes, 2019), pattern abstraction (Fyfe et al., 2015), and angle knowledge (Gibson et al., 2015). One reason for some verbal labels to be especially effective in promoting math learning is that those verbal labels direct children's attention to mathematically relevant features. For example, 4-year-olds were more successful in recreating patterns using novel materials based on a model pattern with abstract labels (e.g., A-B-A-B) than with concrete labels (e.g., blue-red-blue-red; Fyfe et al., 2015). The abstract labels facilitated pattern abstraction by directing children's attention to the mathematically relevant relational relationship among the objects in the model pattern. In contrast, concrete labels directed children's attention to the mathematically irrelevant feature - the color of the objects in the model pattern.

In our study, decomposed labels, compared to common-unit labels, likely encouraged children to attend to and process the place values of individual decimal digits. Because decimal magnitudes are sums of the values of each digit, processing individual digits does not necessarily interfere with decimal magnitude processing. In fact, processing the values of the trailing and leading zeros in decimals helps reduce misconceptions about the role of zero in decimals. In the current study, with an emphasis on individual decimal digits, decomposed labels led to higher accuracy in judging decimal equivalence when decimals had trailing or leading zeros than common-unit labels.

However, for decomposed labels to enhance decimal magnitude understanding in general, a solid understanding of place values is needed. This involves knowing that the place value of a digit is ten times the place value of the digit to its right, that the holistic magnitude of a decimal equals the sum of all digits multiplied by their corresponding place values, and that the digits of greater place values are more important in determining a decimal's holistic magnitude. When such knowledge is limited or absent, processing the place values of individual digits could interfere with judgment of the holistic magnitude of decimals, which might be the case in the current study. Compared to children in the common-unit-label condition, children in the decomposed-label condition showed a stronger tendency to process the digits in the hundredths place even when doing so interfered judgement of the decimal magnitudes. Common-unit labels exhibited their advantage in this case, likely by directing children's attention to the holistic magnitudes of the decimals.

The strengths and weaknesses of the two types of place-value labels justify using both labels in math classrooms. It might be particularly beneficial to use the specific labels in cases where they show advantages over the other type, such as using decomposed labels in teaching the role of zero in decimals. Although point labels did not exhibit advantages over the place-value labels on any task in the current study, point labels are more common in daily life than place-value labels, and teaching with point labels may encourage children to utilize their everyday knowledge to learn decimals. However, more research is needed before conclusions about specific uses of decimal labels can be made.

Limitations and Future Directions

Several limitations of the current study suggest potential directions for future research. For example, we did not explicitly assess children's knowledge of fractions or place values. We selected fifth and sixth graders as our sample based on the assumption that they should have some knowledge of place values from recent instruction on the topic in school. However, this might not be the case, and if children had limited place-value knowledge, this might help to explain them not fully benefiting from place-value labels. Future research may benefit from examining children's prior place-value understanding and fraction knowledge to better understand whether and how each type of decimal labels influences children's magnitude knowledge.

Further, we did not assess children's decimal magnitude processing and representation prior to exposing them to specific decimal labels. This posttest-only design was sufficient to examine our main topic of interest, the distinctive effects of the three types of decimal labels on magnitude processing and knowledge. However, this design does not allow us to conclude whether decimal labels would be an effective tool to improve children's decimal magnitude knowledge. Relatedly, the posttest occurred immediately after decimal labeling instruction, leaving open the possibility that the effects of labeling could reflect either short-term changes in strategy use or more durable improvements in conceptual knowledge. For example, the benefits of decomposed labels on the role-of-zero task might result from increased attention to each decimal digit when completing the task, rather than improved conceptual knowledge. However, with sustained use of the place-value labels in math classrooms, we would expect these strategy improvements to lead to better conceptual understanding. Future research should examine these possibilities.

In sum, we have shown that decomposed and common-unit labels both have advantages over point labels, while each has strengths and weaknesses in promoting children's decimal magnitude processing and knowledge. As compared to point labels, brief exposure to decomposed labels reduced fifth and sixth graders' whole-number-bias as reflected by fewer role-of-zero errors. Brief exposure to common-unit labels also reduced a different aspect of students’ whole-number bias, the string-length-congruity effect. Common-unit labels further reduced children's componential processing as compared to decomposed and point labels. These results highlight the power of verbal labels on children's math knowledge and provide a potential avenue for improving students’ decimal magnitude knowledge.

Reference:

Bates, D., Mächler, M., Bolker, B., & Walker, S. (2014). *Fitting linear mixed-effects models using lme4*. *67*(1). https://doi.org/10.18637/jss.v067.i01

Berteletti, I., Lucangeli, D., Piazza, M., Dehaene, S., & Zorzi, M. (2010). Numerical estimation in preschoolers. *Developmental Psychology*, *46*(2), 545–551. https://doi.org/10.1037/a0017887

Booth, J. L., & Newton, K. J. (2012). Fractions: Could they really be the gatekeeper's doorman? *Contemorary Educational Psychology*, *37*, 247–253.

de Leeuw, J. R. (2015). jsPsych: A JavaScript library for creating behavioral experiments in a Web browser. *Behavior Research Methods*, *47*(1), 1–12. https://doi.org/10.3758/s13428-014-0458-y

Dehaene, S., Izard, V., Spelke, E., & Pica, P. (2008). Log or linear? Distinct intuitions of the number scale in western and Amazonian indigene cultures. *Science*, *320*(5880), 1217–1220. https://doi.org/10.1126/science.1156540

Desmet, L., Grégoire, J., & Mussolin, C. (2010). Developmental changes in the comparison of decimal fractions. *Learning and Instruction*, *20*(6), 521–532. https://doi.org/10.1016/j.learninstruc.2009.07.004

DeWolf, M., Bassok, M., & Holyoak, K. J. (2015). From rational numbers to algebra: Separable contributions of decimal magnitude and relational understanding of fractions. *Journal of Experimental Child Psychology*, *133*, 72–84. https://doi.org/10.1016/j.jecp.2015.01.013

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., Pagani, L. S., Feinstein, L., Engel, M., Brooks-Gunn, J., Sexton, H., Duckworth, K., & Japel, C. (2007). School readiness and later achievement. *Developmental Psychology*, *43*(6), 1428–1446. https://doi.org/10.1037/0012-1649.43.6.1428

Durkin, K., & Rittle-Johnson, B. (2012). The effectiveness of using incorrect examples to support learning about decimal magnitude. *Learning and Instruction*, *22*(3), 206–214.

https://doi.org/10.1016/j.learninstruc.2011.11.001

Durkin, K., & Rittle-Johnson, B. (2015). Diagnosing misconceptions: Revealing changing decimal fraction knowledge. *Learning and Instruction*, *37*, 21–29. https://doi.org/10.1016/j.learninstruc.2014.08.003

Fyfe, E. R., McNeil, N. M., & Rittle-Johnson, B. (2015). Easy as ABCABC: Abstract language facilitates performance on a concrete patterning task. *Child Development*, *86*(3), 927–935. https://doi.org/10.1111/cdev.12331

Gibson, D. J., Congdon, E. L., & Levine, S. C. (2015). The effects of word-learning biases on children's concept of angle. *Child Development*, *86*(1), 319–326. https://doi.org/10.1111/cdev.12286

Halberda, J., & Feigenson, L. (2008). Developmental change in the acuity of the "number sense": The approximate number system in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental Psychology*, *44*(5), 1457–1465. https://doi.org/10.1037/a0012682

Huber, S., Klein, E., Willmes, K., Nuerk, H., & Moeller, K. (2014). Decimal fraction representations are not distinct from natural number representations – evidence from a combined eye-tracking and computational modeling approach. *Frontiers in Human Neuroscience*, *8*, 1–14. https://doi.org/10.3389/fnhum.2014.00172

Hurst, M. A., & Cordes, S. (2018a). A systematic investigation of the link between rational number processing and algebra ability. *British Journal of Psychology*, *109*(1), 99–117. https://doi.org/10.1111/bjop.12244

Hurst, M. A., & Cordes, S. (2018b). Children's understanding of fraction and decimal symbols and the notation-specific relation to pre-algebra ability. *Journal of Experimental Child Psychology*, *168*, 32–48. https://doi.org/10.1016/j.jecp.2017.12.003

Hurst, M. A., & Cordes, S. (2019). Talking about proportion: Fraction labels impact numerical interference in non-symbolic proportional reasoning. *Developmental Science*, *March 2018*, 1–12. https://doi.org/10.1111/desc.12790

Kim, D., & Opfer, J. E. (2017). A unified framework for bounded and unbounded numerical estimation. *Developmental Psychology*, *53*(6), 1088–1097. https://doi.org/10.1037/dev0000305

Koedel, C., & Tyhurst, E. (2012). Math skills and labor-market outcomes: Evidence from a resume-based field experiment. *Economics of Education Review*, *31*(1), 131–140. https://doi.org/10.1016/j.econedurev.2011.09.006

Lange, K., Kühn, S., & Filevich, E. (2015). “Just another tool for online studies” (JATOS): An easy solution for setup and management of web servers supporting online studies. *PLoS ONE*, *10*(6), 1–14. https://doi.org/10.1371/journal.pone.0130834

Laski, E. V., & Yu, Q. (2014). Number line estimation and mental addition: Examining the potential roles of language and education. *Journal of Experimental Child Psychology*, *117*(1), 29–44. https://doi.org/10.1016/j.jecp.2013.08.007

Loehr, A. M., & Rittle-Johnson, B. (2016). Putting the “th” in tenths: Providing place-value labels helps reveal the structure of our base-10 numeral system. *Journal of Cognition and Development*, 1–20. https://doi.org/10.1080/15248372.2016.1243118

Lortie-Forgues, H., & Siegler, R. S. (2017). Conceptual knowledge of decimal arithmetic. *Journal of Educational Psychology*, *109*(3), 374–386. https://doi.org/10.1037/edu0000148

Lyons, I. M., Nuerk, H.-C., & Ansari, D. (2015). Rethinking the implications of numerical ratio effects for understanding the development of representational precision and numerical processing across formats. *Journal of Experimental Psychology: General*, *144*(5), 1021–1035. https://doi.org/10.1037/xge0000094

Malone, A. S., Loehr, A. M., & Fuchs, L. S. (2017). The role of domain-general cognitive abilities and decimal labels in at-risk fourth-grade students’ decimal magnitude understanding. *Learning and Individual Differences*, *58*(January 2016), 90–96. https://doi.org/10.1016/j.lindif.2017.05.007

Mann, A., Moeller, K., Pixner, S., Kaufmann, L., & Nuerk, H.-C. (2012). On the development of

Arabic three-digit number processing in primary school children. *Journal of Experimental Child Psychology*, *113*(4), 594–601. https://doi.org/10.1016/j.jecp.2012.08.002

Miller, K. F., & Stigler, J. W. (1987). Counting in Chinese: Cultural variation in a basic cognitive skill. *Cognitive Development*, *2*(3), 279–305. https://doi.org/10.1016/S0885-2014(87)90091-8

Miura, I. T., Okamoto, Y., Vlahovic-Stetic, V., Kim, C. C., & Han, J. H. (1999). Language supports for children's understanding of numerical fractions: Cross-national comparisons. *Journal of Experimental Child Psychology*, *74*(4), 356–365. https://doi.org/10.1006/jecp.1999.2519

Moyer, R. S., & Landauer, T. K. (1967). Time required for judgment of numerical inequality. *Nature*, *215*, 1519–1520.

National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. National Governors Association Center for Best Practices, Council of Chief State School Officers. https://doi.org/10.21890/ijres.61754

Nesher, P., & Peled, I. (1986). Shifts in reasoning: The case of extending number concepts. *Educational Studies in Mathematics*, *17*(1), 67–79. https://doi.org/10.1007/BF00302379

Ni, Y., & Zhou, Y. Di. (2005). Teaching and learning fraction and rational numbers: The origins and implications of whole number bias. *Educational Psychologist*, *40*(1), 27–52. https://doi.org/10.1207/s15326985ep4001_3

Nuerk, H.-C., Kaufmann, L., Zoppoth, S., & Willmes, K. (2004). On the development of the mental number line: More, less, or never holistic with increasing age? *Developmental Psychology*, *40*(6), 1199–1211. https://doi.org/10.1037/0012-1649.40.6.1199

Nuerk, H.-C., Weger, U., & Willmes, K. (2001). Decade breaks in the mental number line? Putting the tens and units back in different bins. *Cognition*, *82*(1). https://doi.org/10.1016/S0010-0277(01)00142-1

Opfer, J. E., & Siegler, R. S. (2007). Representational change and children's numerical estimation. *Cognitive Psychology*, *55*(3), 169–195. https://doi.org/10.1016/j.cogpsych.2006.09.002

Paik, J. H., & Mix, K. S. (2003). U.S. and Korean children's comprehension of fraction names: A reexamination of cross-national differences. *Child Development*, *74*(1), 144–154. https://doi.org/10.1111/1467-8624.t01-1-00526

Perfetti, C. M. (2010). Decoding, vocalbulary, and comprehension. In M. G. McKeown & L. Kucan (Eds.), *Bringing reading research to life.* The Guilford Press.

R Core Team. (2018). R: A language and environment for statistical computing. In *R Foundation for Statistical Computing*. https://www.r-project.org/

Ren, K., & Gunderson, E. A. (2019). Malleability of whole-number and fraction biases in decimal comparison. *Developmental Psychology*, *55*(11), 2263–2274. https://doi.org/10.1037/dev0000797

Ren, K., & Gunderson, E. A. (2021). The dynamic nature of children's strategy use after receiving accuracy feedback in decimal comparisons. *Journal of Experimental Child Psychology*, *202*, 105015. https://doi.org/10.1016/j.jecp.2020.105015

Resnick, L. B., Nesher, P., Leonard, F., Magone, M., Omanson, S., Peret, I., & Peled, I. (1989). Conceptual bases of arithmetic errors: The case of decimal fractions. *Journal for Research in Mathematics Education*, *20*(1), 8–27. https://doi.org/10.2307/749095

Ritchie, S. J., & Bates, T. C. (2013). Enduring links from childhood mathematics and reading achievement to adult socioeconomic status. *Psychological Science*, *24*(7), 1301–1308. https://doi.org/10.1177/0956797612466268

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology*, *93*(2), 346–362. https://doi.org/10.1037/0022-0663.93.2.346

Sackur-Grisvard, C., & Léonard, F. (1985). Intermediate cognitive organizations in the process of learning a mathematical concept: The order of positive decimal numbers. *Cognition and Instruction*, *2*(2), 157–174. https://doi.org/10.1207/s1532690xci0202_3

Schneider, M., Grabner, R. H., & Paetsch, J. (2009). Mental number line, number line estimation, and mathematical achievement: Their interrelations in Grades 5 and 6. *Journal of Educational Psychology*, *101*(2), 359–372. https://doi.org/10.1037/a0013840

Schrank, F. A., Mather, N., & McGrew, K. S. (2014). *Woodcock-Johnson IV Tests of Achievement*. Riverside Publishing Company.

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., Susperreguy, M. I., & Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological Science*, *23*(7), 691–697. https://doi.org/10.1177/0956797612440101

Siegler, R. S., & Lortie-Forgues, H. (2015). Conceptual knowledge of fraction arithmetic. *Journal of Educational Psychology*, *107*(3), 909–918. https://doi.org/10.1037/edu0000025

Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: Evidence for mulitple representations of numerical quantity. *Psychological Science*, *14*(3), 237–250. https://doi.org/10.1111/1467-9280.02438

Siegler, R. S., & Ramani, G. B. (2008). Playing linear numerical board games promotes low-income children's numerical development. *Developmental Science*, *11*(5), 655–661. https://doi.org/10.1111/j.1467-7687.2008.00714.x

Stigler, J. W., Givvin, K. B., & Thompson, B. J. (2010). What community college developmental mathematics students understand about mathematics. *The Carnegie Foundation for the Advancement of Teaching*, *1*, *No. 3*(May), 4–18. https://doi.org/10.1016/j.learninstruc.2014.03.002

Tirosh, D., Fischbein, E., Graeber, A. O., & Wilson, J. W. (1999). Prospective elementary teachers' conceptions of rational numbers. Retrieved September 26, 2016 from http://jwilson.coe.uga.edu/texts.folder/tirosh/pros.el.tchrs.html.

Varma, S., & Karl, S. R. (2013). Understanding decimal proportions: Discrete representations, parallel access, and privileged processing of zero. *Cognitive Psychology*, *66*(3), 283–301. https://doi.org/10.1016/j.cogpsych.2013.01.002

Verguts, T., & Fias, W. (2004). Representation of number in animals and humans: A neural model. *Journal of Cognitive Neuroscience*, *16*(9), 1493–1504. https://doi.org/10.1162/0898929042568497

Wang, Y., & Siegler, R. S. (2013). Representations of and translation between common fractions and decimal fractions. *Chinese Science Bulletin*, *58*(36), 4630–4640. https://doi.org/10.1007/s11434-013-6035-4

Wong, T. T. Y. (2020). Are the acuities of magnitude representations of different types and ranges of numbers related? Testing the core assumption of the integrated theory of numerical development. *Cognitive Development*, *54*(May), 100888. https://doi.org/10.1016/j.cogdev.2020.100888