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The dynamic nature of children's strategy use after receiving accuracy feedback in decimal comparisons



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ABSTRACT

When students start learning decimals, they may incorrectly apply features of their prior numerical knowledge (e.g., whole-number or fraction rules). However, because whole numbers, fractions, and decimals all have their own unique features, these whole-number and fraction strategies do not always lead to correct solutions. We examined whether receiving immediate accuracy feedback while comparing decimal pairs that were either congruent with whole-number rules (e.g., decimals with more digits were larger in magnitude) or incongruent with whole-number rules (e.g., decimals with fewer digits were larger in magnitude) would lead students to change their decimal comparison strategies. We also examined whether students' potential improvement after feedback would generalize to decimal comparisons involving different numbers of digits. We found that sixth- to eighth-grade students' use of the whole-number strategy declined and their use of the normative decimal strategy increased over the course of receiving feedback, whereas no significant strategy change was observed among students who did not receive any feedback. Students who received feedback were also less likely to use a whole-number strategy and more likely to use a decimal strategy in different decimal comparisons in an immediate posttest and a 2-week delayed posttest. Our exploratory analyses found that students' improvement on decimal comparisons did not transfer to decimal arithmetic. Moreover, students' inhibitory control also predicted strategy use in immediate and delayed posttests. Our study provides insights into the mechanisms of rapid strategy change and

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has implications for designing interventions to improve children's understanding of decimal magnitudes.

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Introduction

The understanding of numerical magnitude is not only essential in the development of mathematical cognition but also critical in mastering more advanced topics such as physics and biology (Siegler, 2016). Decimal magnitude knowledge in particular, compared with whole-number and fraction magnitude knowledge, is a unique and positive predictor of seventh-grade students' algebra performance (DeWolf, Bassok, & Holyoak, 2015). However, children often show persistent misconceptions when conceptualizing decimal magnitudes (Durkin & Rittle-Johnson, 2015). For example, when children are asked to compare decimal magnitudes such as choosing the larger number between 0.26 and 0.8, children may choose one of multiple available strategies depending on their current grade in school, how familiar they are with certain strategies, their knowledge level, and other factors. Some children may choose 0.26 as the larger number because they incorrectly treat these two decimals as whole numbers (26 and 8), wherein 26 is larger than 8. Some children may choose 0.8 as the larger number, but the reasons behind the choice vary. One possible reason is that children incorrectly treat two decimals as fractions ($1/26$ and $1/8$), wherein $1/8$ is larger than $1/26$ (e.g., Sackur-Grisvard & Léonard, 1985). Another possible reason is that children understand the decimal place value system and can correctly compare two decimals using normative decimal comparison strategies such as comparing the tenths digits of the two decimals and adding zero to the hundredths place of 0.8.

Children's strategy use in decimal learning

Children's misconceptions or biases when reasoning about decimal magnitudes often come from their prior numerical knowledge about whole numbers and fractions (Desmet, Grégoire, & Mussolin, 2010; Durkin & Rittle-Johnson, 2015; Resnick et al., 1989; Sackur-Grisvard & Léonard, 1985). The *whole-number bias* refers to the misapplication of whole-number rules in decimals (Ni & Zhou, 2005). For example, when reasoning about whole numbers, numbers with more digits are inevitably larger in magnitude than numbers with fewer digits; however, this is not always true in decimals (e.g., 0.26 is smaller than 0.8 even though 0.26 has more digits than 0.8). Similarly, the *fraction bias* represents the tendency to over-generalize fraction rules to decimals (referred to as fraction misconception in Durkin & Rittle-Johnson, 2015). For instance, having learned that, in unit fractions, numbers with more digits in the denominator yield smaller magnitudes, children may incorrectly treat decimals with more digits as being smaller in magnitude (e.g., believing 0.68 is smaller than 0.3 because 0.68 has more digits than 0.3).

Over the course of developing their numerical knowledge, children can have multiple strategies available at one time. As proposed by overlapping waves theory (Siegler, 1996), the relative frequency of each strategy can change with changes in numerical knowledge and experience. With regard to decimals, children may use strategies they learned previously from whole numbers and fractions to solve problems involving decimals, with the relative frequency of each strategy changing over development. This has been supported by empirical studies showing that when third- and fourth-grade students started to learn decimals, around half of the students showed whole-number bias in problems involving decimals, but only 10% to 12% of the students showed fraction bias. However, the fraction bias was much more common among fifth- and sixth-grade students who had gained more experience with fractions (Desmet et al., 2010; Durkin & Rittle-Johnson, 2015; Lai & Wong, 2017; Resnick et al., 1989; Sackur-Grisvard & Léonard, 1985). Moreover, one study followed fourth- and fifth-grade students for a month and found a decrease in the whole-number bias and an increase in the fraction bias over time even though the whole-number bias was always more frequent than the fraction bias (Durkin & Rittle-Johnson, 2015).

With respect to which strategy children use at a given time, the dynamic strategy choice account has proposed that children's strategy choice depends on the saliency and recency of mental representations that are needed to use a certain strategy (Alibali & Sidney, 2015). In addition, strategy choice varies across specific problems, individuals, and contexts. Supporting this theory, researchers found that sixth- to eighth-grade students' strategy choice in decimal magnitude comparisons was affected by the type of numerical comparisons they practiced immediately before decimal comparisons (Ren & Gunderson, 2019). In this study, students were instructed to complete a series of whole-number comparisons, fraction comparisons, or decimal comparisons, or a non-numerical control task, while receiving accuracy feedback after each trial. Results showed that practicing fraction comparisons with feedback increased students' use of an incorrect fraction strategy in subsequent decimal comparisons. (Practicing whole-number comparisons with feedback did not increase students' already high levels of whole-number bias.) Moreover, the researchers found that, surprisingly, after only 56 trials of decimal comparisons with feedback, students showed substantial improvement in an immediate posttest involving decimal comparisons without feedback, manifested as a reduced whole-number bias; only 18.9% of children showed whole-number bias after practicing decimal comparisons with feedback compared with 62.2% of children who showed whole-number bias after a non-numerical control task. These results suggest that students' strategy use is malleable even during a short period of time.

The role of feedback in children's strategy change

The prior study by Ren and Gunderson (2019) raises important questions about the nature of this rapid strategy change in children's decimal comparisons. First, how do children's strategies shift over time while receiving accuracy feedback on each trial? To answer this question, we need to consider how feedback may contribute to children's strategy change. A plethora of research has recorded the beneficial role that feedback plays in facilitating learning (Dihoff, Brosvic, Epstein, & Cook, 2004; Fyfe, Rittle-Johnson, & DeCaro, 2012; Kluger & DeNisi, 1996; Kulhavy, 1977; Smith & Kimball, 2010). For example, a meta-analysis by Alfieri, Brooks, Aldrich, and Tenenbaum (2011) showed that participants' performance was better (with an average effect size of $d = .46$) in feedback conditions than in no-feedback conditions. Ren and Gunderson (2019) further suggested that feedback improved overall performance by potentially generating or switching to correct strategies. Consistently, Fyfe et al. (2012) found that, on average, children used more correct strategies after receiving feedback on math equivalence problems accompanied by reduced preservation on incorrect strategies. However, Ren and Gunderson's (2019) study was not designed to test the strategy change upon receiving feedback, and the authors did not include a condition where which children received no feedback. Therefore, the current study aimed to investigate whether (and, if so, how) receiving feedback in decimal magnitude comparisons, rather than simply exposure and practice, stimulates children's strategy change.

In addition to determining whether feedback itself was responsible for the strategy changes found in Ren and Gunderson (2019), it is also important to understand the nature and time course of these changes. One possible course is that children begin by primarily using a whole-number strategy, and upon receiving feedback the frequency of using the whole-number strategy decreases and the fraction strategy increases, the same pattern demonstrated in a 1-month study involving brief instructions on decimals (Durkin & Rittle-Johnson, 2015). This would be in line with prior work on the impact of immediate accuracy feedback, which suggests that it can encourage children to use more trial-and-error strategies rather than identifying correct strategies (Hattie & Timperley, 2007; Simmons & Cope, 1993). In the decimal comparison context, trial-and-error strategies may lead children to use the fraction strategy rather than a normative decimal comparison strategy. After trying out the fraction strategy, children may discover that this strategy cannot apply to all decimals, which may motivate them to discover the normative decimal strategy. This pattern of change would lead to an initial increase in the fraction strategy followed by a decrease (a curvilinear pattern). Another possibility is that children first use a whole-number strategy but quickly begin using the normative decimal strategy after receiving accuracy feedback because strategy modification can happen abruptly (Alibali, 1999). This would be consistent with prior research showing that accuracy feedback about the task can be beneficial when it helps students to reject erroneous hypotheses and provides necessary

information to form a proper strategy (Harackiewicz, Manderlink, & Sansone, 1984; Hattie & Timperley, 2007). Here, accuracy feedback may help children to reject erroneous assumptions about the inseparable relation between the number of digits and the decimal, and children may gradually deduce the correct strategy of comparing decimal digits at the same place value.

A second question raised from the prior study (Ren & Gunderson, 2019) is whether children's improvement in decimal comparisons reflects learning of a purely procedural strategy or more generalizable knowledge. One possibility is that children merely learned how to compare the specific type of decimal pairs on which they received feedback because they received neither explicit information about the correct strategy nor information about decimal place value. This possibility would be consistent with previous studies showing that feedback primarily affected children's procedural knowledge (Fyfe & Rittle-Johnson, 2016; Fyfe et al., 2012). Another possibility is that children actually mastered the normative decimal comparison strategy while receiving feedback on varied decimal comparisons and would be able to flexibly apply this strategy to other types of decimals regardless of the number of decimal digits. Furthermore, children's learning about decimal comparisons may potentially transfer to other types of decimal problems such as decimal addition and subtraction. Prior research has shown that receiving feedback on a rational number line training game led to transfer, lending support for this possibility (Fazio, Kennedy, & Siegler, 2016; Rittle-Johnson, Siegler, & Alibali, 2001).

A third question we asked was whether the improvement in decimal comparisons that was present immediately after brief feedback is momentary or more durable over time. On the one hand, the strategy children learned after brief exposure to comparing decimals with accuracy feedback may be forgotten over the course of children's exposure to other numbers (e.g., fractions and whole numbers) in their daily lives given that recently formed memories are usually less stable and more vulnerable to interference than older memories (Kang, Duncan, Clements, Sarama, & Bailey, 2019; Wixted, 2004). On the other hand, if children indeed form an understanding of decimal place value, they may be able to distinguish decimal rules from the rules of whole numbers and fractions and may retain this knowledge over a longer period of time.

The last question we were interested in was whether children's strategy change and strategy use during and after decimal comparisons with feedback were affected by their inhibitory control (IC). Prior research has shown that IC may be involved in accurate performance with fractions and decimals (Gómez, Jiménez, Bobadilla, Reyes, & Dartnell, 2015; Lai & Wong, 2017; Meert, Grégoire, & Noël, 2010; Roell, Viarouge, Houdé, & Borst, 2017) and is particularly important when competing strategies create interference (Diamond, 2013; Houdé, 2000). IC may help children to suppress an incorrect and less sophisticated strategy in order to use a more adaptive strategy (Cragg & Gilmore, 2014; Houdé, 2000; Ren, Lin, & Gunderson, 2019). Specific to decimal comparisons, children's responses to items incongruent with whole-number processing were less accurate and slower because these incongruent comparisons require higher IC to suppress using a whole-number strategy (Avgerinou & Tolmie, 2020; DeWolf & Vosniadou, 2015; Roell, Viarouge, Houdé, & Borst, 2019). In the current study, we expected a similar effect such that IC would be related to more normative decimal comparison strategy use. In addition, learners' ability can affect the effect of feedback on learning (Hattie & Timperley, 2007). Therefore, individual differences in cognitive abilities such as IC may affect children's strategy change. Here, we explored whether children with high IC would adopt a decimal strategy more quickly than children with low IC in decimal comparisons. It is also possible that the effect of accuracy feedback would be strong enough that children could learn the normative decimal strategy without requiring high IC.

Preregistered and exploratory hypotheses

To summarize, the current study examined children's strategy change in decimal comparisons. In particular, would children change their incorrect strategies to correct alternatives with the help of immediate accuracy feedback on varied decimal comparison trials? This would potentially contribute in charting children's strategy change in a short-term and microgenetic manner (Siegler, 2007) in comparison with the relatively long-term change that has been identified in prior literature (Durkin & Rittle-Johnson, 2015). We also investigated whether changes in decimal comparison strategies after

receiving feedback would transfer to novel decimal problems and persist until a later time point. Lastly, we explored whether individual differences in IC would affect children's strategy change and strategy use patterns. Pertaining to each question, our preregistered plans (<https://osf.io/fphxw>) are elaborated below.

Our first research question asked how children's strategy use would change over trials while receiving feedback. To address this question, we designed a short-term training study to examine children's dynamic strategy change with and without immediate accuracy feedback in decimal comparisons. Consistent with prior work (Ren & Gunderson, 2019), students completed decimal comparisons between one-digit and two-digit decimals. To identify children's use of a whole-number strategy, fraction strategy, or normative decimal strategy, the decimal comparison task included both whole-number congruent and whole-number incongruent pairs. On whole-number congruent pairs, using the whole-number strategy would always yield correct answers (e.g., 0.49 vs. 0.2); on whole-number incongruent pairs, using the whole-number strategy would lead to incorrect answers (e.g., 0.7 vs. 0.24). To examine how children's strategies shift over time while receiving feedback, we divided the feedback session into a series of time bins, each containing equal numbers of whole-number congruent and whole-number incongruent trials, so that we would be able to track children's strategy use across time. Our preregistered hypothesis was that, over the course of receiving feedback, the percentage of children who used a whole-number strategy (i.e., chose decimals with more digits) would decrease and the percentage of children who used a normative decimal strategy (i.e., chose decimals with larger tenths digits) would increase. Moreover, we also hypothesized that the percentage of children who used a fraction strategy (i.e., always chose decimals with fewer digits) would increase initially after children received feedback on their incorrect use of the whole-number strategy but would decrease later when children realized that the fraction strategy did not work either (a curvilinear pattern). For children in the control group who did not receive feedback, we did not expect to see any strategy change.

Our second and third questions asked whether improvement after training was purely procedural or would generalize to other types of decimal comparisons and whether this improvement would be durable over a 2-week delay. To address these questions, we introduced two additional types of decimal pairs that differed from those in the training in an immediate posttest and a delayed posttest (2 weeks later). One type of items was one-digit versus three-digit decimal pairs (e.g., 0.5 vs. 0.877, 0.8 vs. 0.314); these were expected to increase the temptation for children to focus on the total number of decimal digits as a cue to magnitude. The second type of decimal pairs was two-digit versus three-digit decimals (e.g., 0.14 vs. 0.168, 0.67 vs. 0.253), which required children to attend to the hundredths place on half of the trials. We hypothesized that children who received feedback in the training block would be less likely to use a whole-number strategy and more likely to use a decimal strategy on these different types of decimal comparisons in both immediate and delayed posttests compared with children who did not receive feedback in the training block.

In addition to our preregistered hypotheses, our preregistered plan also included two exploratory questions. First, we explored whether, if children improved on decimal comparisons, this improvement would transfer to other types of decimal problems such as decimal addition and subtraction. The procedures for conducting decimal addition and subtraction are similar to those for whole numbers with the exception that additional attention is needed to correctly align decimals' place values prior to conducting those procedures (for a review, see Lortie-Forgues, Tian, & Siegler, 2015). Children have more difficulties in calculating addition and subtraction problems that have unequal numbers of digits after the decimal point (e.g., $0.15 + 0.6$) than problems that have equal numbers of decimal digits (e.g., $0.15 + 0.62$) (Hiebert & Wearne, 1985). Therefore, it is possible that children who learned to compare same-place digits in decimal comparisons would be able to carry out operations by aligning the same-place digits in decimal arithmetic. Alternatively, children might not be able to connect the correct strategy they learned in decimal comparisons with the procedures that are needed to conduct decimal arithmetic. For example, when comparing decimals, children may learn to start with comparing the first digit after the decimal point and then the second digit if necessary, but when doing decimal addition and subtraction problems, children may still calculate starting from the right-most digit if their knowledge about the role of the decimal point is inconsistent across different problem types.

We explored this question by examining whether children who received feedback in decimal comparisons would have higher accuracy in decimal arithmetic problems.

We also explored individual differences in children's strategy change and strategy use and whether these were associated with children's IC. Feedback may help children to realize their improper use of whole-number and fraction strategies, and high IC may help to accelerate the process of suppressing incorrect strategies and shifting to correct ones. To explore this possibility, children completed an IC task (Hearts and Flowers task) before the training block to examine the role that IC had in strategy change and strategy use in decimal comparisons.

We collected data from sixth- to eighth-grade students because their knowledge about decimals is still developing and a previous study showed that this age group showed significant improvement after receiving accuracy feedback (Ren & Gunderson, 2019). The current study aimed to identify children's patterns of decimal strategy change in decimal magnitude comparisons and to investigate whether such changes are transferable and durable, which could help to answer questions about how learning occurs (Siegler, 2007).

Method

Participants

Children were sixth- to eighth-grade students from seven schools in a large city in the eastern United States ($N = 171$, 90 girls; 56 sixth-graders [28 in experimental condition], 53 seventh-graders [29 in experimental condition], and 62 eighth-graders [30 in experimental condition]; $M_{\text{age}} = 13.09$ years, $SD = 0.88$). All students were tested in the spring semester of their respective grade. These seven schools followed Common Core State Standards, in which students receive instruction on decimals in Grade 4 and decimal operations in Grade 5 (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). Our preregistered power analysis indicated that a sample size of 85 in each condition (170 in total) would be sufficient to detect a difference between conditions with $\alpha = .05$, odds ratio = 4.4, power = .80; this odds ratio was determined based on results from a similar prior study (Ren & Gunderson, 2019). Most parents reported their children's race and ethnicity ($n = 139$): 9.4% Black or African American, 72.7% White, 0.7% Hispanic, 4.3% Asian or Asian American, 9.4% multiple races/ethnicities, and 3.6% other race/ethnicity.

Measures and materials

Decimal magnitude comparisons

We used three types of decimal comparison pairs in the immediate and delayed posttests that varied in the number of digits in the fractional part of each decimal: (a) one-digit versus two-digit decimals (e.g., 0.8 vs. 0.73), (b) one-digit versus three-digit decimals (e.g., 0.4 vs. 0.869), and (c) two-digit versus three-digit decimals (e.g., 0.14 vs. 0.168). Within each type of decimal pair, half of the items were whole-number congruent pairs, where a whole-number strategy (choosing the number with more digits) yielded correct answers (e.g., 0.49 vs. 0.2). Note that the whole-number congruent pairs are also fraction incongruent, where a fraction strategy (choosing the number with fewer digits) yielded incorrect answers. The other half of the items were whole-number incongruent pairs, for which a whole-number strategy (choosing the number with fewer digits) resulted in incorrect answers (e.g., 0.8 vs. 0.24). Note that the whole-number incongruent pairs are also fraction congruent.

We controlled potential confounds that were not the focus of our research questions. For example, we chose the decimal pairs so that the relation between tenths and hundredths places was compatible (e.g., 0.34 vs. 0.7, where both 3 and 4 are smaller than 7) rather than incompatible (e.g., 0.38 vs. 0.5, where 3 is smaller than 5 but 8 is greater than 5) to control the *tenth-hundredth compatibility effect* (Nuerk, Weger, & Willmes, 2001; Varma & Karl, 2013). In the two-digit versus three-digit decimal pairs, half of the decimal pairs shared the same tenths digit and the relation between hundredths and thousandths places was controlled. We also controlled the *distance effect* (Moyer & Landauer, 1967; Schneider, Grabner, & Paetsch, 2009) by choosing decimal pairs in seven distances (1–7) with

respect to the difference between the tenth place of each decimal. In the two-digit versus three-digit decimal pairs that shared the same tenths digit, the distance referred to the difference between the hundredths place of each decimal. Decimal pairs with the same distances appeared equally in congruent and incongruent trials.¹ In addition, except for the zero in the ones place, we did not use zeros in the fractional parts of any decimals because children may hold misconceptions about the role of zero in decimals that differ from misconceptions about other digits (Durkin & Rittle-Johnson, 2012, 2015; Sackur-Grisvard & Léonard, 1985).

We collected children's accuracy and response time (RT) on each trial. We excluded trials with RTs that were either too fast (<200 ms) or too slow (≥ 10 s) from all analyses (0.15% of training block trials, 0.40% of immediate posttest trials, and 0.24% of delayed posttest trials) because these excluded trials are likely to indicate children not paying attention to the task. For analyses of RT, we examined only correct trials and excluded RTs that were 3 standard deviations beyond participants' mean RT (1.59% of all immediate posttest trials and 1.62% of all delayed posttest trials) to reduce the effect of outlying RTs when they were used as the primary outcomes.

Decimal arithmetic

We used four decimal addition problems (e.g., $0.6 + 0.13 = ?$) and four decimal subtraction problems (e.g., $0.84 - 0.6 = ?$) to assess children's understanding of decimal arithmetic (see Appendix A for a full list of arithmetic problems). Each arithmetic problem contained one decimal with one digit in the fractional part and one decimal with two digits in the fractional part. The correct answers to all problems were positive. All the arithmetic problems were presented in a true random order. Children were instructed to type the answers into a laptop computer using the keyboard. If children wanted to skip a trial, the experimenter provided one prompt encouraging children to answer and they were allowed to skip if they still wanted to do so. We used the accuracy of each problem in the analyses. One child was excluded in our analyses of this task in the immediate posttest due to a program error.

Inhibitory control

We used the Hearts and Flowers task (Wright & Diamond, 2014) to measure IC. On each trial, either a red heart or a red flower appeared on the left or right side of the screen. Children were instructed to press the key on the same side as the heart and on the opposite side as the flower. A fixation cross was presented for 500 ms, followed by a 500-ms blank screen at the outset of each trial, and then the stimulus appeared on the left or right side of the screen for 750 ms. Children had up to 1250 ms to respond before the program proceeded to the next trial. Children completed three test blocks, starting with a congruent hearts-only block (12 trials), then an incongruent flowers-only block (12 trials), and finally a mixed hearts and flowers block (32 trials). Trials in each block were presented in a fixed pseudorandom order. At the beginning of each block, children needed to get at least 3 of 4 practice trials correct to be able to proceed to the test trials; if not, the experimenter repeated the instructions and children completed the 4 practice trials again until they got at least 3 trials correct (all children met this criterion on the first or second presentation of the practice trials).

We used children's accuracy on the switch trials within the mixed block in our analyses because these trials require the greatest inhibition of a prepotent rule and require switching to a different rule (Davidson, Amso, Anderson, & Diamond, 2006). We also chose switch trials for the ease of interpretation. One participant was excluded due to having too few trials completed (2 of 32). We excluded trials with RTs that were too fast (<200 ms) from all analyses (0.04% of all trials). After these exclusions, we calculated mean accuracy for switch trials.

¹ Due to an error in stimulus design, the two-digit versus three-digit decimal pairs had one more congruent decimal pair with a distance of 6 (3 in total) and one fewer congruent decimal pair with a distance of 7 (1 in total). We randomly chose one congruent decimal pair with a distance of 6 for each child and compared its accuracy with the congruent decimal pair with a distance of 7 in the immediate posttest. A paired-samples *t* test did not show a significant difference between the performances of the two decimal pairs, $t(170) = -1.89, p = .060$. Therefore, we thought it was reasonable to include all items in our analyses.

Design

Test materials were implemented on JATOS (Lange, Kühn, & Filevich, 2015) using JavaScript and jsPsych (de Leeuw, 2015). Children completed all tasks on a laptop computer in a one-on-one session with an experimenter in their school.

Children were randomly assigned to either the experimental condition (with feedback) or the control condition (without feedback) within classroom. The study comprised two sessions with 2 weeks (12–16 days) in between. In the first session, all children first completed the Hearts and Flowers task and then completed a training block where they compared 84 one-digit versus two-digit decimal pairs (see Fig. 1 for a schematic of the trial sequences). We divided the 84 trials into 14 bins with 6 trials in each bin: 3 whole-number congruent trials and 3 whole-number incongruent trials. Decimal pairs in each bin were presented in a single pseudorandom order, and all children received the same order of bins. On each trial, children were instructed to choose the larger of two decimals by pressing the key on the same side as the larger decimal. Children had unlimited time to respond but were encouraged to respond as quickly and accurately as they could. Children in the experimental condition received accuracy feedback after each trial indicating whether their response was correct and stating the correct relation of each decimal pair (e.g., “Correct! 0.8 is bigger than 0.35,” “Incorrect! 0.8 is bigger than 0.35”). Children in the control condition only saw a screen that read “Press any key to continue” after each trial. These feedback messages remained on the screen until children pressed a key. On completion of the training block, all children proceeded to the immediate posttest blocks where they first completed 28 one-digit versus two-digit decimal comparisons, followed by 56 decimal comparisons that included half one-digit versus three-digit pairs and half two-digit versus three-digit pairs (these types were intermixed). Decimal comparison pairs were presented in a true random order within each posttest block. After completing the decimal comparison posttest, all children completed 8 decimal arithmetic problems. No feedback was given during the posttest blocks. In the second session, the delayed posttest session, all children completed the same three posttest blocks as in the immediate posttest. Session 1 took approximately 15 min in total, and Session 2 took approximately 10 min.

The study procedures were approved under the Temple University institutional review board protocol titled “Cognitive and Emotional Bases of Math, Reading, and Spatial Development.”

Analytic approach

We used IBM SPSS Statistics for Windows (Version 24.0) to analyze our data and used R 3.6.3, the package *ggplot2* (Wickham, 2009), to create graphs. Data files and analysis script are available at <https://osf.io/su2eg>.

Strategy change in the training block

Our first research question was on individual strategy type. We divided the 84-trial training block into 14 bins of 6 trials each. Within each bin, we coded each child’s strategy type based on his or her accuracy on the 3 congruent and 3 incongruent decimal comparison trials. Children who scored high ($\geq 66.7\%$, correctly answered at least 2 of 3 trials) on both congruent trials and incongruent trials were coded as showing a decimal-accurate performance pattern, whereas children who scored low ($\leq 33.3\%$, correctly answered 0 or 1 of 3 trials) on both congruent and incongruent trials were coded as displaying a decimal-inaccurate pattern. Children who had high accuracy on congruent trials ($\geq 66.7\%$) but low accuracy on incongruent trials ($\leq 33.3\%$) were coded as using a whole-number strategy (showing a whole-number bias). Finally, children who performed well on incongruent trials ($\geq 66.7\%$) but poorly on congruent trials ($\leq 33.3\%$) were coded as applying a fraction strategy (presenting a fraction bias). Each child received 14 strategy codes (1 per bin).

Following our preregistered analysis plan, we conducted binary logistic generalized estimating equations (GEEs) on children’s use of the whole-number strategy, decimal-accurate strategy, and fraction strategy separately. Each strategy type was recoded into a dummy variable. The GEE approach allows us to model bins as the nesting item within individuals and to model the binary outcome (Liang & Zeger, 1986). The GEE approach also allows us to model the population-average effect, which tells us on average how children in the experimental condition change differently from children in the

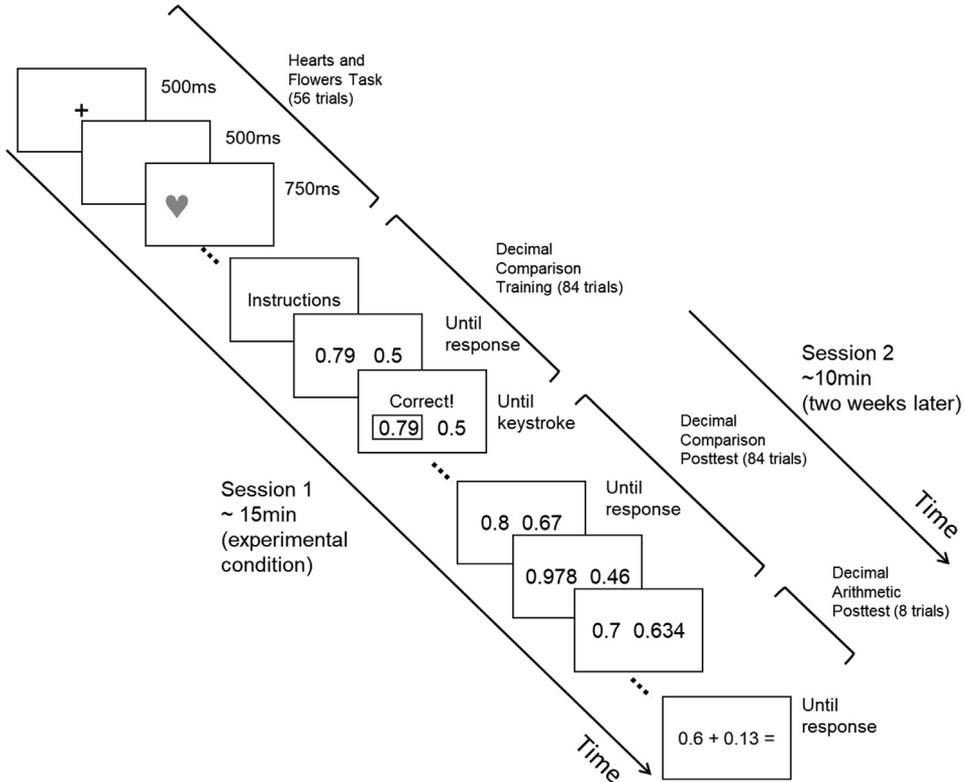


Fig. 1. Example of trial sequences. Session 1 included all tasks shown. Session 2 (2 weeks later) included only the decimal comparison posttest and decimal arithmetic posttest.

control condition, rather than the individual-specific effect (e.g., the effect of the experimental condition on a particular child’s strategy change), which generalized linear mixed models demonstrate (McCulloch & Neuhaus, 2014). In the whole-number strategy and decimal-accurate strategy models, we included bin number as a linear within-participant predictor and condition as a factor to test the prediction that the likelihood of each strategy would decrease and increase, respectively, as a function of bin number. In the model examining fraction strategy, we conducted GEE models separately for the feedback condition and control condition and included bin and bin² as linear within-participant predictors to test the hypothesis that the fraction strategy would change in a curvilinear manner (i.e., increase and then decrease) in the feedback condition only.

Decimal comparisons in posttests

We examined children’s performance on posttest decimal comparisons by first conducting individual-level analyses on children’s strategy use and then conducting group-level analyses on overall accuracy and RT. Individual-level analyses enabled us to examine different performance patterns that might be masked by similar group means. Specifically, for individual-level analyses, we categorized children based on their accuracy on all congruent and incongruent trials within each type of decimal comparison using the same criteria as in the training block. For each type of decimal comparison, we tested whether condition was a significant predictor of children’s strategy category by conducting logistic regressions with condition predicting the percentage of children categorized as using a whole-number strategy or decimal-accurate strategy. However, the categorical strategy codes were based on children’s percentage accuracy for the two types of decimal pairs (congruent and incongruent).

Therefore, to check the robustness of our results, we also conducted group-level analyses of accuracy and RT for all items, which allowed us to retain all the information in the statistical models. We used accuracy of each trial as the outcome variable, with congruency and condition as the factors, separately for each type of decimal comparison. The control condition and incongruent trials were the reference groups. We also conducted the same models with RT as the outcome variable.

Results

Preregistered analyses

Strategy change in the training block

Our analyses were on individual strategy type and the percentage of participants in each strategy category by bin, as shown in Fig. 2. For descriptive purposes, we also present group-level accuracy in Table 1. We also checked the average time children spent on each trial in both the feedback condition and control condition and found that children in the feedback condition ($M = 1412.98$, $SD = 388.13$) on average spent more time on each trial than children in the control condition ($M = 1063.30$, $SD = 298.74$), $t(161.08) = 6.62$, $p < .001$. This suggested that children's decision processes were affected by the feedback information, which was a sign of our manipulation being effective.

We first examined whether condition, bin, and their interaction were significant predictors of the percentage of children who used the whole-number strategy. Consistent with our hypothesis, there were significant main effects of bin, $\chi^2(1, N = 171) = 21.43$, $p < .001$, and condition, $\chi^2(1, N = 171) = 8.81$, $p = .003$, which were qualified by a significant Bin \times Condition interaction, $\chi^2(1, N = 171) = 18.12$, $p < .001$. We interpreted this interaction by first examining the parameter estimate for bin in the GEE model with the feedback condition as the reference group, which can be interpreted as the effect of bin on whole-number strategy use in the feedback group. This effect was significantly negative, showing that in the feedback group the percentage of children who used the whole-number strategy decreased over bins ($B = -0.12$, $SE = 0.03$, $p < .001$). We then examined the same model with the control condition as a reference group, and found that in the control condition, bin was not a significant predictor of whole-number strategy use ($B = -0.01$, $SE = 0.01$, $p = .553$).

Regarding the change in percentage of children who showed the decimal-accurate strategy across bins, we also found a significant main effect of bin, $\chi^2(1, N = 171) = 26.28$, $p < .001$, which was qualified by a significant Bin \times Condition interaction, $\chi^2(1, N = 171) = 20.76$, $p < .001$ [there was not a significant main effect of condition, $\chi^2(1, N = 171) = 2.89$, $p = .089$]. The parameter estimates from these GEE models, varying the reference group, showed that the percentage of children who showed the decimal-accurate strategy increased as a function of bin in the feedback condition ($B = 0.10$, $SE = 0.02$, $p < .001$), whereas there was no significant change in decimal-accurate strategy use in the control condition ($B = 0.01$, $SE = 0.01$, $p = .360$).

Finally, we examined whether fraction strategy use followed a curvilinear pattern in the feedback condition. Contrary to our hypothesis, we did not find a significant main effect of bin, $\chi^2(1, N = 87) = 0.10$, $p = .748$, or bin², $\chi^2(1, N = 87) = 0.34$, $p = .559$, in the feedback condition. In the control condition, we also did not find a significant effect of bin, $\chi^2(1, N = 84) = 0.29$, $p = .593$, or bin², $\chi^2(1, N = 84) = 0.47$, $p = .493$.

Decimal comparisons in posttests

Individual-level analyses. We first examined decimal comparison strategies in the immediate posttest. The percentage of children using each strategy is displayed in Fig. 3. We found that the percentage of children who used a whole-number strategy was significantly smaller in the feedback condition than in the control condition for all three decimal comparison types: one-digit versus two-digit decimal comparisons (feedback condition = 8.05%, control condition = 42.86%, $B = -2.15$, $SE = 0.45$, $p < .001$), one-digit versus three-digit decimal comparisons (feedback = 13.79%, control = 41.67%, $B = -1.50$, $SE = 0.38$, $p < .001$), and two-digit versus three-digit decimal comparisons (feedback = 12.64%, control = 42.86%, $B = -1.65$, $SE = 0.39$, $p < .001$). Furthermore, the percentage of children who showed the decimal-accurate strategy was significantly larger in the feedback condition than in

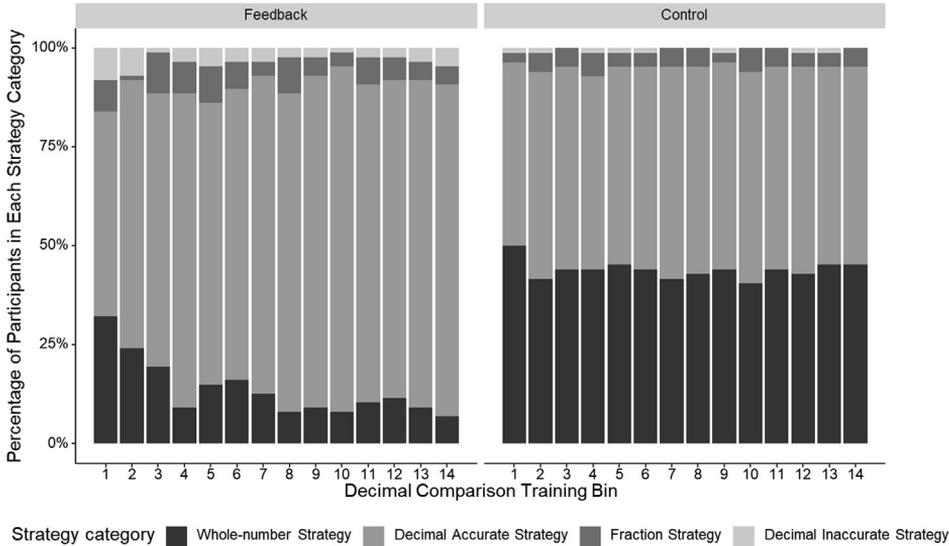


Fig. 2. Percentage of children in each decimal comparison strategy category by bin in the training block, separately for the feedback and control conditions.

Table 1

Mean accuracy in decimal comparisons by condition, congruency, and decimal comparison type separately for training block and posttest blocks.

	One-digit vs. two-digit decimal comparisons		One-digit vs. three-digit decimal comparisons		Two-digit vs. three-digit decimal comparisons	
	Feedback Condition M (SD)	Control Condition M (SD)	Feedback Condition M (SD)	Control Condition M (SD)	Feedback Condition M (SD)	Control Condition M (SD)
Training block						
Congruent items	.88 (.17)	.94 (.18)				
Incongruent items	.76 (.21)	.53 (.42)				
Immediate posttest						
Congruent items	.91 (.17)	.94 (.19)	.87 (.25)	.93 (.21)	.86 (.23)	.93 (.19)
Incongruent items	.80 (.21)	.52 (.42)	.78 (.29)	.55 (.44)	.77 (.27)	.54 (.43)
Delayed posttest						
Congruent items	.92 (.16)	.94 (.18)	.91 (.21)	.94 (.19)	.89 (.21)	.92 (.19)
Incongruent items	.82 (.27)	.57 (.45)	.78 (.31)	.58 (.45)	.78 (.32)	.58 (.45)

Note. Standard deviations are in parentheses.

the control condition for all three decimal comparison types: one-digit versus two-digit decimal comparisons (feedback = 85.06%, control = 52.38%, $B = 1.64$, $SE = 0.37$, $p < .001$), one-digit versus three-digit decimal comparisons (feedback = 74.71%, control = 53.57%, $B = 0.94$, $SE = 0.33$, $p = .004$), and two-digit versus three-digit decimal comparisons (feedback = 73.56%, control = 52.38%, $B = 0.93$, $SE = 0.33$,

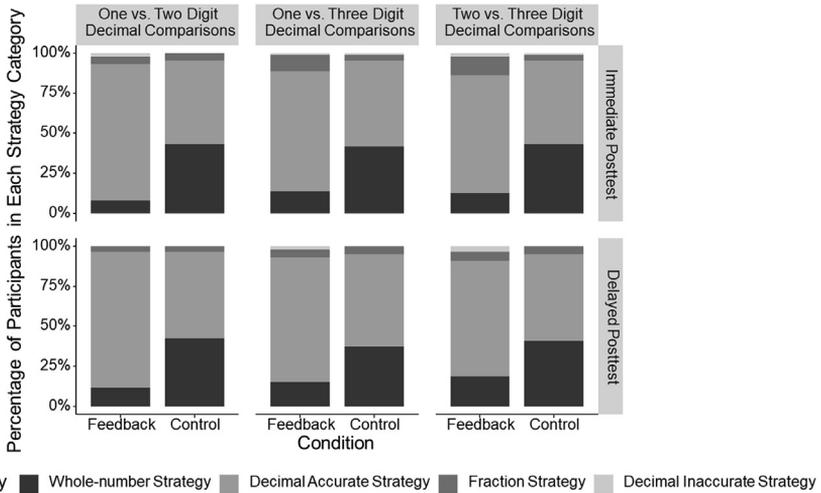


Fig. 3. Percentage of children in each decimal comparison strategy category in posttest decimal comparison tests by condition and decimal comparison type, separately for the immediate posttest and delayed posttest.

$p = .005$). We next examined decimal comparison strategy use in the delayed posttest and found the same pattern of differences between the feedback condition and control condition, with the whole-number strategy being greater in the control condition ($ps \leq .002$) and the decimal-accurate strategy being greater in the feedback condition ($ps \leq .017$) (see Fig. 3, delayed posttest).

Group-level analyses. When examining mean accuracy across all children in the same condition, average performance on congruent and incongruent decimal comparisons may suggest a group-level strategy pattern. For example, when most children in the same group used a whole-number strategy and performed well on congruent trials but failed on incongruent trials, the mean accuracy of congruent trials would be higher than that of incongruent trials. Thus, we interpreted higher accuracy on congruent trials than on incongruent trials as an indicator of using a whole-number strategy at the group level. (See Table 1 for the overall group-level accuracy; for similar results based on RT, see Appendix Table B1 for descriptives and Appendix Table B2 for GEE models.)

We first examined children’s decimal comparison accuracy in the immediate posttest (see Table 1 for descriptive statistics and Table 2 for results of all models). For each of the three types of decimal comparisons, we found significant Condition \times Congruency interactions [one-digit vs. two-digit decimal comparisons, $\chi^2(1, N = 171) = 12.07, p < .001$; one-digit vs. three-digit decimal comparisons, $\chi^2(1, N = 171) = 12.02, p < .001$; and two-digit vs. three-digit decimal comparisons, $\chi^2(1, N = 171) = 15.08, p < .001$]. The main effects of congruency were significant in all types of decimal comparisons ($ps < .001$) such that accuracy on incongruent items was consistently worse than that on congruent items. Post hoc comparisons showed that the performance on congruent items did not differ between conditions ($ps \geq .052$), but the performance on incongruent items in the feedback condition was significantly higher than that in the control condition ($ps < .001$). These results suggested a reduced whole-number bias in the feedback condition than in the control condition.

We then examined children’s decimal comparison accuracy in the delayed posttest (see Table 3 for results of all models). The results showed the same pattern as the immediate posttest; we found significant Condition \times Congruency interactions in each type of decimal comparison [one-digit vs. two-digit decimal comparisons, $\chi^2(1, N = 169) = 8.79, p = .003$; one-digit vs. three-digit decimal comparisons, $\chi^2(1, N = 169) = 6.42, p = .011$; and two-digit vs. three-digit decimal comparisons, $\chi^2(1, N = 169) = 6.93, p = .008$]. The effects of congruency were significant in all types of decimal comparisons ($ps < .001$) such that the performance on incongruent items was consistently worse than that on congruent items. Post hoc comparisons showed that the performance on congruent items did not differ

Table 2

Generalized estimating equation model results predicting accuracy in the immediate posttest, separately by type of decimal comparisons.

	One-digit vs. two-digit decimal comparisons			One-digit vs. three-digit decimal comparisons			Two-digit vs. three-digit decimal comparisons		
	χ^2	df	p	χ^2	df	p	χ^2	df	p
Immediate posttest									
Condition	3.85	1	.050	0.88	1	.349	0.81	1	.367
Congruency	60.34	1	<.001	36.08	1	<.001	44.93	1	<.001
Condition × Congruency	12.07	1	<.001	12.02	1	<.001	15.08	1	<.001

Table 3

Generalized estimating equation model results predicting accuracy in the delayed posttest separately by type of decimal comparisons.

	One-digit vs. two-digit decimal comparisons			One-digit vs. three-digit decimal comparisons			Two-digit vs. three-digit decimal comparisons		
	χ^2	df	p	χ^2	df	p	χ^2	df	p
Delayed posttest									
Condition	2.87	1	.090	0.89	1	.346	1.83	1	.176
Congruency	44.23	1	<.001	38.06	1	<.001	41.25	1	<.001
Condition × Congruency	8.79	1	.003	6.42	1	.011	6.93	1	.008

between conditions ($ps \geq .311$), but the performance on incongruent items in the feedback condition was significantly higher than that in the control condition ($ps < .003$). These results highlight the effectiveness of the feedback condition, compared with the control condition, in reducing whole-number bias 2 weeks later.

Exploratory analyses

Decimal arithmetic in posttests

See Table 4 for descriptive statistics on the decimal arithmetic task. To determine whether children’s improvement on decimal comparisons transferred to decimal arithmetic, we conducted binary logistic regressions with accuracy on each arithmetic problem as the outcome variable, and with condition as the predictor, in separate models for the immediate and delayed posttests. Results showed that children in the feedback condition did not perform significantly better on decimal arithmetic problems than children in the control condition in the immediate posttest ($B = -0.02, SE = 0.12, p = .899$) or in the delayed posttest ($B = -0.11, SE = 0.11, p = .336$).

IC in the training block

See Table 5 for descriptive statistics for the IC task. We first checked whether there were systematic differences in IC between the two conditions. An independent-samples *t* test indicated that children in the feedback condition did not have significantly different IC from children in the control condition, $t(168) = 0.67, p = .505$. We next examined whether individual differences in IC related to children’s strategy change and strategy use in the training block in each condition. We used accuracy on the switch trials within the mixed block in the Hearts and Flowers task as our measure of IC. We included condition, switch trial accuracy, bin, and their interactions as predictors in the GEE models. We examined only models predicting whole-number strategy and decimal-accurate strategy (two separate models); we did not examine fraction strategy use because our main analyses did not show any significant changes in use of this strategy in the training block. In both models, there were no significant three-way interactions (Condition × Bin × IC) or two-way interactions ($ps \geq .148$). IC showed a significant main effect in both models ($ps \leq .003$). To better interpret the effect of IC, we reran the models without interactions involving IC (i.e., adding the IC main effect to our main analyses in the training

Table 4
Mean accuracy on the decimal arithmetic task by condition separately for posttest blocks.

	Feedback condition	Control condition
Immediate posttest	.66 (.39)	.66 (.38)
Delayed posttest	.62 (.39)	.64 (.38)

Note. Standard deviations are in parentheses.

Table 5
Mean accuracy and RT of inhibitory control task by block and condition.

	Feedback condition		Control condition	
	Accuracy	RT (ms)	Accuracy	RT (ms)
Congruent block	.99 (.04)	333.52 (67.09)	.99 (.04)	339.43 (69.24)
Incongruent block	.94 (.10)	420.17 (88.01)	.93 (.14)	430.49 (89.83)
Mixed block–nonswitch trials	.82 (.17)	592.98 (83.71)	.82 (.16)	606.31 (89.33)
Mixed block–switch trials	.81 (.15)	614.55 (87.79)	.79 (.17)	629.67 (84.74)

Note. Standard deviations are in parentheses. RT, response time.

block). Results showed that IC accuracy predicted less frequent whole-number strategy use ($B = -3.51$, $SE = 0.94$, $p < .001$) and more frequent correct decimal strategy use ($B = 4.25$, $SE = 0.93$, $p < .001$) after taking the effects of condition, bin, and their interaction into account.

IC in the decimal comparison posttests

We also examined whether IC related to children’s strategy use in the immediate or delayed decimal comparison posttest. We included switch trial accuracy and condition in the binary logistic regression models.² Results showed that, while controlling for the significant effect of condition, switch trial accuracy significantly predicted whole-number strategy use ($ps \leq .001$) and decimal-accurate strategy use ($ps < .001$) such that better IC (higher accuracy) predicted less frequent whole-number strategy use and more frequent decimal-accurate strategy use on each type of decimal comparison (see Appendix Table B3 for full model results). These effects were also present in the delayed posttest for whole-number strategy ($ps \leq .003$) and decimal-accurate strategy ($ps \leq .001$). The significant effects of IC on strategy use showed that children with high IC were more likely to use the normative decimal comparison strategy and less likely to use the whole-number strategy.

Discussion

Children have multiple strategies to choose from when comparing decimal magnitudes. The current study examined whether children’s strategy use in decimal magnitude comparisons can dynamically change over the course of receiving immediate accuracy feedback. Consistent with our hypotheses, while completing 84 trials of decimal comparisons with immediate accuracy feedback, the percentage of sixth- to eighth-grade students who used a whole-number strategy decreased and that of students who used a correct decimal strategy increased. In contrast, students who did not receive feedback on decimal comparisons showed no change in their strategy use. In terms of the fraction strategy, we had hypothesized that students would switch from a whole-number strategy to a fraction strategy before shifting to a correct decimal strategy over the course of the 84 trials while receiving accuracy feedback, similar to the pattern previously reported in longitudinal studies of numerical development (Durkin & Rittle-Johnson, 2015). This was not the case in our short-term experiment; we did not find any evidence that students’ fraction strategy changed in either condition.

² To check model specification, we also tested binary logistic regression models with an additional Condition \times Switch Trial Accuracy interaction. However, the interaction effect was not part of our questions and it was not significant in the models, so our results focused on models without interactions.

Taken together, these results suggest that students' strategy use shifted dynamically from a whole-number strategy to a normative decimal comparison strategy as a result of receiving immediate accuracy feedback on each trial.

The current study provides further support for the effectiveness of immediate accuracy feedback, which can help children to reject an erroneous strategy (e.g., whole-number strategy) and move on to a correct strategy. Specifically, in both the immediate and delayed decimal comparison posttests when students were no longer receiving feedback, our individual-level analyses revealed that children in the feedback condition not only used the whole-number strategy less frequently, but also applied the correct decimal comparison strategy more frequently than children in the control condition, reinforcing that immediate accuracy feedback about the task helped children to shift to a correct decimal strategy. These results were consistent across different types of decimal comparisons with varied numbers of decimal digits. Children did not simply learn a procedural rule for comparing the one-digit versus two-digit decimal pairs in the training block (such as comparing the first digit); instead, they may have learned to compare decimal digits that were at the same place value after the decimal point (e.g., hundredths place), signifying an understanding of more generalizable knowledge of decimal comparisons.

In addition, we explored whether children's improvement on decimal magnitude comparisons would generalize to other decimal problems, specifically decimal arithmetic. Children on average correctly answered 65% of arithmetic problems, suggesting that they were not perfect on decimal arithmetic and still had room for improvement. However, we did not find evidence for improvement on decimal arithmetic after feedback on decimal magnitude comparisons. This suggests that although children may grasp the correct decimal comparison strategy to compare digits at the same place value starting from the decimal point, they do not spontaneously generalize this to decimal arithmetic. Future studies can examine whether receiving immediate accuracy feedback on different types of decimal arithmetic problems (e.g., decimal pairs with the same number of digits, decimal pairs with different number of digits) would lead to improvement in decimal arithmetic, similar to that found in decimal comparisons here.

We also explored the role of IC in children's dynamic strategy change during the training block and children's strategy use in the decimal comparison posttests. Results showed that IC control did not moderate the rate of children's strategy change in the training block. However, IC did predict children's overall strategy use in the training block and the posttest blocks such that high IC predicted less frequent whole-number strategy use and more frequent correct decimal strategy use. These results are consistent with the theorized role of IC in suppressing the use of a prepotent and well-practiced whole-number strategy and employing a more adaptive and correct strategy in decimal magnitude comparisons.

Limitations and future directions

The focus of the analyses in current study was on children's accuracy because children's strategy use can be straightforwardly identified based on their accuracy pattern across whole-number congruent and incongruent decimal pairs. Despite the ease of interpretation of this approach, by ignoring RT in our main analyses, we might not have fully captured the nuances of students' decision processes during these tasks. Future work should consider analyzing accuracy and RT simultaneously, such as through the diffusion decision model (Ratcliff & McKoon, 2008), to further illuminate individual differences in decimal comparison processes.

In addition, the current study used response patterns on decimal magnitude comparisons to infer children's strategy use (e.g., when students always chose decimals with more digits as larger ones, we categorized them as using a whole-number strategy). Although this approach revealed clear patterns, future research could corroborate and extend these findings using other measures of strategy use such as eye tracking and verbal strategy reporting. This may provide more information about children's strategy use and strategy change. For example, the normative decimal comparison strategy could involve comparing digits in the same decimal place or mentally adding zeros to the shorter decimal so that both decimals have the same number of digits. Future work using other measures of strategy use could disentangle these possibilities.

Conclusions and implications

Altogether, the current study demonstrated the lasting effect of receiving immediate accuracy feedback on children's mastery of the correct strategy in decimal magnitude comparisons. Immediate accuracy feedback helped children to reject an incorrect strategy and gradually recognize the correct strategy of comparing digits at the same place value in decimals. Children's mastery of the correct strategy was evident in their use of normative strategies even for other types of decimal comparisons on which they had not received feedback. This performance increase also carried over to a 2-week delayed posttest. Our study provides empirical evidence regarding the rapid change in children's strategy use in decimal magnitude comparisons after only a few minutes of practice with feedback. Children predominantly started with a whole-number strategy but quickly grasped a normative decimal strategy over the course of receiving feedback without any explicit instruction about this strategy. The fact that in our study children did not first switch to the fraction strategy before using a normative strategy is different from the prior 1-month study in which students received brief instructions using decimal number lines (Durkin & Rittle-Johnson, 2015). The difference is informative in revealing that directly addressing children's misconceptions about decimal comparisons may be an important step toward greater decimal knowledge, particularly in fostering better understanding of how decimals differ from other number types such as whole numbers and fractions.

The current study has clear practical implications for educators, and has the benefit of being easy to deploy in the classroom setting. For example, teachers can assign decimal comparison practice items that are inconsistent with children's prior numerical knowledge in order to address children's misconceptions while providing feedback. Additional translational research is needed to determine whether these improvements last beyond the 2 weeks tested here and how to leverage these improvements to enhance learning of other decimal concepts and procedures. We expect that reducing children's misconceptions, especially while they are in the process of constructing their decimal magnitude knowledge, will be beneficial to their long-term mathematical development.

Acknowledgments

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Appendix A

Addition problems

$$0.6 + 0.13 = ?$$

$$0.35 + 0.8 = ?$$

$$0.3 + 0.97 = ?$$

$$0.74 + 0.1 = ?$$

Subtraction problems

$$0.84 - 0.6 = ?$$

$$0.79 - 0.2 = ?$$

$$0.61 - 0.3 = ?$$

$$0.57 - 0.4 = ?$$

Appendix B

See [Tables B1–B3](#).

Table B1

Mean RT in the decimal comparisons by condition, congruency, and decimal comparison type separately for immediate and delayed posttests.

	One-digit vs. two-digit decimal comparisons		One-digit vs. three-digit decimal comparisons		Two-digit vs. three-digit decimal comparisons	
	Feedback condition	Control condition	Feedback condition	Control condition	Feedback condition	Control condition
Immediate posttest (ms)						
Congruent items	1406.42 (414.92)	1071.92 (391.03)	1432.82 (437.54)	1103.74 (404.11)	1737.23 (637.47)	1331.50 (513.13)
Incongruent items	1580.15 (481.37)	1255.86 (427.54)	1604.25 (477.88)	1358.78 (442.55)	1802.30 (577.07)	1524.22 (554.44)
Delayed posttest (ms)						
Congruent items	1147.51 (314.42)	884.03 (253.92)	1171.94 (308.73)	929.23 (301.66)	1427.89 (372.08)	1087.93 (388.15)
Incongruent items	1230.12 (325.27)	1029.05 (285.62)	1232.60 (333.96)	1073.20 (311.55)	1457.92 (350.66)	1194.10 (382.31)

Note. Standard deviations are in parentheses. RT, response time.

Table B2

Generalized estimating equation model results predicting response time in the immediate and delayed posttests separately for each type of decimal comparisons.

	One-digit vs. two-digit decimal comparisons			One-digit vs. three-digit decimal comparisons			Two-digit vs. three-digit decimal comparisons		
	χ^2	df	p	χ^2	df	p	χ^2	df	p
Immediate posttest									
Condition	26.12	1	<.001	19.10	1	<.001	13.97	1	<.001
Congruency	88.50	1	<.001	55.87	1	<.001	38.53	1	<.001
Condition × Congruency	4.51	1	.034	4.04	1	.045	14.06	1	<.001
Delayed posttest									
Condition	26.69	1	<.001	23.11	1	<.001	28.17	1	<.001
Congruency	62.27	1	<.001	33.56	1	<.001	29.23	1	<.001
Condition × Congruency	7.56	1	.006	3.63	1	.057	7.65	1	.006

Table B3

Binary logistic regression model results predicting strategy use in the immediate and delayed posttests separately for each type of decimal comparisons.

	One-digit vs. two-digit decimal comparisons				One-digit vs. three-digit decimal comparisons				Two-digit vs. three-digit decimal comparisons			
	Whole-number strategy		Decimal-accurate strategy		Whole-number strategy		Decimal-accurate strategy		Whole-number strategy		Decimal-accurate strategy	
	B (SE)	p	B (SE)	p	B (SE)	p	B (SE)	p	B (SE)	p	B (SE)	p
Immediate posttest												
IC	-4.58 (1.31)	<.001	5.10 (1.26)	<.001	-3.94 (1.19)	.001	4.21 (1.13)	<.001	-4.44 (1.23)	<.001	4.13 (1.12)	<.001
Condition	-2.24 (0.48)	<.001	1.75 (0.40)	<.001	-1.51 (0.40)	<.001	0.94 (0.35)	.007	-1.69 (0.41)	<.001	0.92 (0.34)	.007
Delayed posttest												
IC	-4.58 (1.25)	<.001	4.43 (1.21)	<.001	-3.77 (1.17)	.001	4.51 (1.16)	<.001	-3.39 (1.12)	.003	3.75 (1.10)	.001
Condition	-1.78 (0.43)	<.001	1.61 (0.40)	<.001	-1.19 (0.39)	.002	0.94 (0.36)	.009	-1.09 (0.37)	.003	0.76 (0.34)	.027

Note. IC, inhibitory control.

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