Teaching Fractions to Young Children

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For many of us—adults and children alike—math is a boring subject at best and a scary one at worst. Unfortunately, our society's overall negativity toward math is reflected in student achievement. For example, in the 2015 Trends in International Mathematics and Science Study (TIMSS), fourth-graders in the United States were outperformed by students in 10 other educational systems, ranging from Singapore to Northern Ireland (Provasnik et al. 2016).

To improve math knowledge, skills, and achievement, one critical area to focus on is children's understanding of fraction magnitudes (i.e., the sizes of fractions). Standardized tests such the National Assessment of Educational Progress (NAEP; results reported as the Nation's Report Card) have shown that inadequate grasp of fraction magnitude persists at least through middle school. For example, on the 2013 NAEP, only 55 percent of fourth-graders chose the correct answer to 2/5 + 3/5 + 4/5, and 39 percent chose 9/15, which can be obtained by adding the numerators and the denominators of the three addends separately (National Center for Education Statistics). This insufficient understanding of fraction magnitude is concerning because better fraction magnitude knowledge is associated with better fraction arithmetic skills as well as higher general math achievement (Siegler, Thompson, & Schneider 2011; Torbeyns et al. 2015). In this article, we first review research on proportional reasoning, as this skill is closely related to understanding fraction magnitudes. We then present evidence from brief and effective interventions teaching children how to show fractions on a number line. Finally, we share research-based strategies for improving children's fraction magnitude understanding in the classroom.

Proportional reasoning

Very young children—even infants—have strong intuitions about proportions without any countable units. These are called *continuous visual proportions*, such as those in figure 1. These particular examples are adaptations of doughnut-shaped spinners from a study in which researchers showed children pairs of spinners and asked them to choose the one that was more likely to have the arrow land on red (Jeong, Levine, & Huttenlocher 2007). Children as young as 6 years old chose the correct spinner more often than the incorrect spinner. In another study, 3- and 4-year-olds showed success on a similar task (Hurst & Cordes 2018). There is also evidence that infants as young as 6 months old recognize differences between proportional lengths (Huttenlocher, Duffy, & Levine 2002) and that 10-month-olds have intuitive knowledge of probabilities (Denison & Xu 2014).

As children develop proficiency with counting, however, they may begin to make mistakes in proportional reasoning. They tend to erroneously use counting to match or compare whole numbers even when they need to focus on proportions instead. For example, in figures 1 and 2 on this page, the spinners shown in figure 1 have the same proportions of blue and red as the spinners in figure 2. The difference is that there are now countable segments. So instead of showing continuous visual proportions, these show *discrete proportions*.

With the discrete spinners, the results were very different than with the continuous spinners: even 10-year-olds failed to choose the correct spinner (i.e., the spinner more likely to have the arrow land on red). Their reasoning was likely thrown off by counting the number of red segments, which would lead to an incorrect choice (Jeong, Levine, & Huttenlocher 2007).

Another way researchers have examined children's proportional reasoning is with pictures of marbles. In a recent study, children were shown pictures of two groups of marbles, as in figure 3 (adapted from O'Grady & Xu 2019). They were asked to look at the two groups of marbles and compare the probability of getting a particular color of marble (researchers told half of the children to do so for the color red and told the other half to do so for the color white). Here, too, counting and comparing the number of desirable colored marbles could lead to an answer that was sometimes correct and sometimes incorrect. The researchers found that most 7- to 9-year-old children did not focus on the

Figure 1

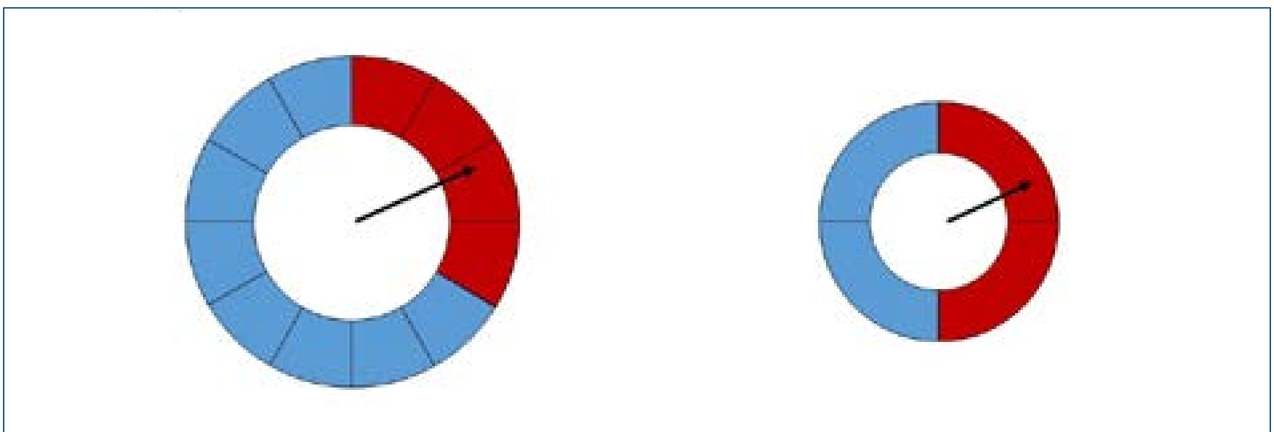

Figure 2

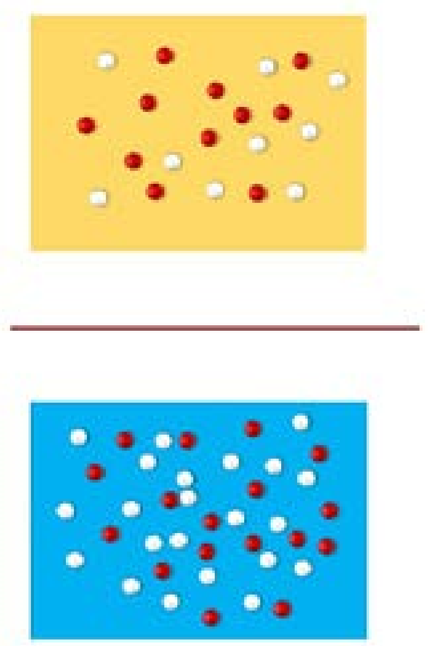

Figure 3

proportion of red and white marbles and, therefore, did not accurately assess the probability of getting a particular color.

In contrast, 10-month-old infants tend to be successful on a conceptually similar probability task (Denison & Xu 2014). It is likely that infants, who are not able to count, are able to reason based on proportions. As children learn to count, they seem to fixate on the numbers without thinking about proportions.

Even though children tend to make (predictable) mistakes with discrete proportions, recent evidence has shown that activating and building on children's estimation skills leads to better performance. In one study, asking fourth-graders to first complete a continuous proportional reasoning task improved their performance on the discrete proportional reasoning task that followed (Boyer & Levine 2015). Similarly, teaching 5- and 6-year-olds proportional strategies by beginning with continuous proportions improved their performance on discrete proportional reasoning tasks afterward (Hurst & Cordes 2018). Children's tendency to fixate on number counting instead of proportions can be overcome by first activating their knowledge of continuous proportions.

Teaching fractions with number lines

The challenges children face when learning about fractions are similar to their difficulties with reasoning about discrete proportions. In both cases, children erroneously fixate on whole numbers when they need to consider the integrated proportion or magnitude. Prior to learning fractions, children's experience with numbers is limited to whole numbers, which leads many children to believe that all numbers are like whole numbers (Ni & Zhou 2005). For most children, it is especially challenging to understand that the two numerals in a fraction are not separate whole numbers; the two numerals must be considered together because they express an integrated magnitude.

To help young children develop the understanding that a fraction expresses an integrated magnitude, visual models that show the relation between a part (the numerator) and a whole (the denominator) are commonly used in US schools. Elementary math textbooks primarily use two-dimensional area models to introduce fractions (Alajmi 2012). These area models typically involve shapes, such as circles or squares, like those shown in figure 4. They usually are

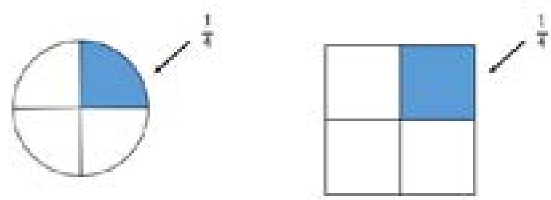

Figure 4

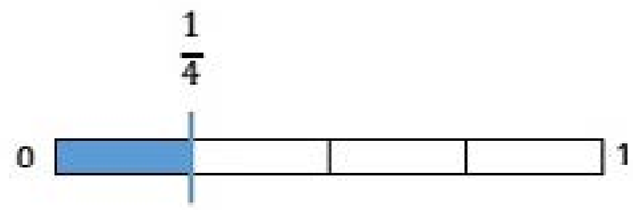

Figure 5

partitioned into the number of equal segments that correspond to the denominator of a fraction, and the number of segments that correspond to the numerator of the fraction are shaded.

Using shaded, segmented parts within a larger shape, area models powerfully represent the part–whole meaning of fractions (Cramer & Wyberg 2009). However, area models have some important limitations. The models' emphasis on part–whole relations may lead children to focus on the part and disregard the whole, or they may disregard the relationship between the part and the whole. Either way, children do not grasp the key concept of the fraction expressing an integrated magnitude (Hamdan & Gunderson 2017). Additionally, area models are awkward for showing fractions larger than one because more than one identical shape (each representing one whole unit) must be involved (Behr, Wachsmuth, & Post 1988).

The number line may be more beneficial than area models. A number line, like the one in figure 5 (Gunderson et al. 2019), clearly shows children: 1) all fractions have magnitudes (just as whole numbers do), and 2) all fractions can be ordered (along with whole numbers) on the same number line (Wu 2009; Siegler, Thompson, & Schneider 2011).

Moreover, number lines take advantage of children's mental representations of numbers: extensive behavioral and neuroimaging evidence suggests that numbers are represented in our minds in a manner similar to a number line (Dehaene, Bossini, & Giraux 1993; Ansari 2008; Toomarian & Hubbard 2018).

Several large-scale interventions with fourth- through sixth-graders have highlighted that students acquire fraction magnitude knowledge better when they are taught using number lines (Saxe et al. 2007; Fuchs et al. 2013; Dyson et al. 2018). Aligned with these findings, US curricula frequently use number lines to represent fractions in the upper elementary grades (National Governors Association Center for Best Practices 2015). However, early elementary instruction has not yet followed suit: the primary model for fractions remains the area model. Findings from the two studies (Hamdan & Gunderson 2017; Gunderson et al. 2019) demonstrate that young children (second- and third- graders) could benefit from learning fractions with number lines and suggest that number line models should be used earlier on in schools.

The first study (Hamdan & Gunderson 2017) involved randomly assigning second- and third-grade students to three groups. Two of the groups received a 15-minute lesson on fractions, while the third group worked on a crossword puzzle (i.e., non-numerical control group). Among the children who received instruction, one group learned about fractions through a lesson with a number line, while the second group learned about fractions through a lesson with area models made with circles (Hamdan & Gunderson 2017). Both groups were taught the idea that a fraction (e.g., 1/4) has a number on the top (the numerator; in this case, 1) and a number on the bottom (the denominator; in this case, 4).

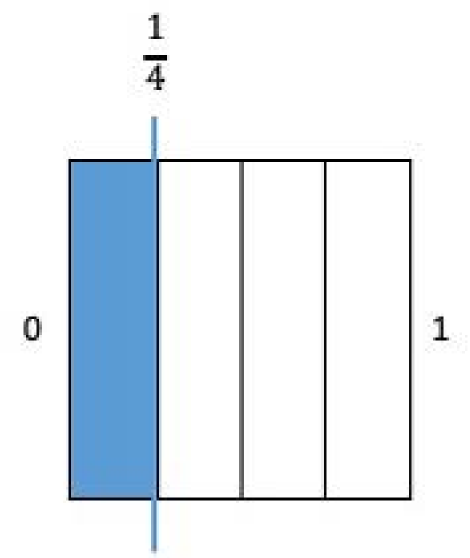

Figure 6

However, instruction differed in how this fraction was visually represented based on group assignment (i.e., dividing the number line vs. dividing the area model). In both conditions, the experimenter demonstrated the fractions 1/2, 1/4, and 1/5, and children practiced modeling these fractions as well as the fractions 2/4, 3/4, 2/5, 3/5, and 4/5. Immediately after the training, children completed an assessment of their fraction magnitude knowledge.

Not surprisingly, children taught to use the number line improved at showing fractions on a number line, and children taught to use a circle area model improved at showing fractions on the circle. More importantly, children who learned about fractions using the number line were more accurate at comparing fractions (for example, *Which is more: 2/4 or 1/5?*) than children who learned fractions using the area model. These results are especially striking because none of the children were directly taught how to compare fractions.

In a recent follow-up study (Gunderson et al. 2019), researchers not only replicated these findings, but they also explored *why* number lines were more effective than area models. They found that, even when a square was designed to be similar to the number line, like the one in figure 6 (partitioned from left to right and with end points like a number line) (Gunderson et al. 2019), learning fractions through a typical number line still led to better performance on fraction comparison. Children who learned fractions with squares like in figure 6 did not show any evidence of better fraction learning than those who learned with other area models (e.g., like the square shown in figure 4). These results suggest that an essential feature of number lines is that they are one-dimensional (Gunderson et al. 2019). In sum, across both studies, second- and third-graders learned the most when they were taught how to show fractions using a one-dimensional number line.

Teaching fractions to young children

Given that understanding fractions is so important for children's future math achievement, substantial effort has been put into developing effective approaches to teaching fractions (e.g., Gabriel et al. 2012; Fazio, Kennedy, & Siegler 2016; Fuchs et al. 2016). Based on the key findings highlighted in this article, we suggest teachers use the following strategies to successfully teach fractions to young children:

- **Recognize and expand upon what children already know.** The close relation between proportional reasoning skills and fraction knowledge (Möhring et al. 2016) suggests that building children's fraction knowledge on their existing understanding of proportions might be promising. A practice guide on fraction instruction by the Institute for Education Sciences recommends that educators "build on students' informal understanding of sharing and proportionality to develop initial fraction concepts" (Siegler et al. 2010, 12).

- **Use continuous rather than discrete visual models.** When introducing fractions to young children, teachers can capitalize on children's intuitive understanding of proportions and avoid the distraction of numerical information by using continuous models.
- **Utilize the number line.** The number line represents fractions as integrated magnitudes. It may encourage children to think of fractions as continuous proportions and to use their intuitive knowledge. As found in the training studies described earlier, the number line is more helpful to teach young children fraction magnitudes than the area model (Hamdan & Gunderson 2017; Gunderson et al. 2019). Moreover, the ability to accurately estimate fractions on number lines is associated with higher performance on other fraction tasks and higher general math achievement (Siegler & Pyke 2013; Fazio, Dewolf, & Siegler 2016; Schneider et al. 2018).

About the authors

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