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Malleability of Whole-Number and Fraction Biases in Decimal Comparison

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Children and adults often have difficulties comparing decimal magnitudes. Although individuals attempt to reconcile decimals with prior whole-number and fraction knowledge, conceptual and procedural differences between decimals and prior knowledge of whole numbers and fractions can lead to incorrect strategies. The dynamic strategy choice account has proposed that saliency, recency, prior knowledge, and other factors contribute to strategy use when reasoning about decimals. We experimentally tested this theory using a priming technique to manipulate the saliency of different strategies prior to completion of a decimal magnitude comparison task. We hypothesized that whole-number priming (practicing whole-number comparisons with feedback) would increase whole-number bias in decimal comparisons (treating decimals with more digits as larger in magnitude), whereas fraction priming would increase fraction bias (treating decimals with fewer digits as larger in magnitude). We also explored participants' performance in decimal comparisons after being primed by decimal comparisons with feedback. Sixth to eighth graders (N = 149) and adults (N = 175) were randomly assigned to 1 of 4 priming conditions: whole-number, fraction, decimal, or a control (flanker) task. Participants first completed numerical magnitude comparisons according to their priming condition (or control task) with feedback, then all conditions completed decimal comparisons without feedback. In both children and adults, fraction priming significantly reduced whole-number bias compared with the control. Among children, fraction priming significantly increased fraction bias. Moreover, children's performance in the control and whole-number-priming conditions was characterized by strong whole-number bias, but in the decimal-priming condition, relatively brief feedback substantially improved decimal comparison performance.

Keywords: whole-number bias, fraction bias, decimal learning, numerical cognition, dynamic strategy choice

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In the development of numerical knowledge, decimals share the same important status as whole numbers and fractions (DeWolf, Bassok, & Holyoak, 2015; Hurst & Cordes, 2018; Siegler, 2016). People use decimals every day in dealing with continuous numbers that cannot be represented by exact whole numbers, such as in situations involving distance, weight, and currency. For example, in order to buy a bottle of milk for $3.79, you have to know that this refers to 3 dollars and 79 cents. The understanding of decimal magnitude is not only important in daily life but also a critical predictor of later mathematical achievement such as algebra. In one study, only performance on decimal number line estimation significantly and positively predicted seventh-grade students' performance on algebra when taking number line estimation of other number types into account (DeWolf et al., 2015). Given the indispensable role of decimals in life and in academic success, there has been a growing interest in studying conceptual change and children's concomitant misconceptions when reasoning about decimals (Desmet, Grégoire, & Mussolin, 2010; Durkin & Rittle-Johnson, 2012, 2015; Peled & Awawdy-Shahbari, 2009; Resnick et al., 1989; Vamvakoussi & Vosniadou, 2010).

In most current curricula, children learn whole numbers and fractions prior to decimals (Tian & Siegler, 2017). By the time they begin to learn decimals, they already have a considerable amount of knowledge of whole numbers and some limited knowledge of fractions (Hiebert, 1992). According to the conceptual change approach (Vamvakoussi & Vosniadou, 2004, 2010), children first form number concepts that encompass whole numbers and then fractions. When children subsequently learn decimals, these prior number concepts impede their learning when some aspects of decimals are incompatible with their existing number knowledge. Children attempt to assimilate and reconcile decimals with whole numbers and fractions rather than constructing an independent and radical reorganization (Desmet et al., 2010; Van Hoof, Degrande, Ceulemans, Verschaffel, & Van Dooren, 2018). However, this can lead to misconceptions as a result of conceptual and procedural differ-

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ences between decimals and previously learned number types, namely, whole numbers and fractions.

Whole numbers and decimals are similar to each other superficially—they both comply with the base-10 place-value structure, in which the value of each digit is determined by its position within the numerical string (e.g., tenths, units, 10s, 100s). However, whole numbers and decimals are notationally, conceptually, and procedurally different from one another (Johnson, 1956; Siegler, 2016). For example, notationally, adding a "0" to the left of a whole number does not change the number's magnitude, whereas adding a "0" to the right of a whole number increases the magnitude by 10 times. However, in decimals, for digits to the right of the decimal point, adding a "0" to the left indicates a number 10 times smaller in magnitude, and adding a "0" to the right represents no change in magnitude. Conceptually, each whole number has a unique predecessor and successor, whereas there is an infinite set of numbers between any two decimals. Procedurally, multiplying two whole numbers with the same sign always yields a larger number, but multiplying two decimals can yield a larger or smaller number.

In addition to their knowledge of whole numbers, children often approach decimal learning with some prior knowledge of fractions. Fractions and decimals share certain properties; for example, for both fractions and decimals, there are infinite numbers between any two numbers of each type. However, fractions and decimals are notationally dissimilar and require different procedures to carry out calculations. For unit fractions, fewer digits in the denominator results in a larger fraction in magnitude, but the number of digits in a decimal is not consistently related to its magnitude. Besides, fractions have a bipartite a/b structure in their written notation, whereas decimals have a one-dimensional format, like integers. In addition, fractions can represent any number of parts of a whole (e.g., thirds, fourths, fifths), whereas decimals follow base-10 structure and therefore can only denote parts of a whole that are powers of 10 (e.g., tenths, hundredths, thousandths).

These fundamental differences between students' prior knowledge of whole numbers and fractions and the to-be-learned knowledge of decimals creates obstacles for children's conceptual understanding of decimals, leading to systematic errors and misconceptions (Resnick et al., 1989; Sackur-Grisvard & Léonard, 1985). Two major systematic errors, or biases, in learning decimals are the whole-number bias and fraction bias. The *whole-number bias* refers to a strong tendency to apply whole-number rules to decimals (Ni & Zhou, 2005). Children often show interference from prior knowledge of whole numbers when whole-number properties conflict with decimal properties (Vamvakoussi, 2015). Notably, students tend to think that decimals with more digits are larger in magnitude than decimals with fewer digits (e.g., believing that 0.32 is larger than 0.7 because it has more digits than 0.7), a property that is always true of whole numbers but not always true of decimals. For example, a large number of fifth and sixth graders in one study believed that .274 is larger than .83 (Rittle-Johnson, Siegler, & Alibali, 2001). Even educated adults show an influence of prior whole-number knowledge when reasoning about decimals, for example, showing a longer reaction time when comparing the magnitudes of decimals whose properties were incompatible with whole numbers (e.g., comparing 1.4 and 1.198) than those that were compatible with whole numbers (e.g., comparing 1.3 and 1.859; Vamvakoussi, Van Dooren, & Verschaffel, 2012). We note that prior research has referred to this difficulty as the *semantic interference effect* (Varma & Karl, 2013) and the *string length congruity effect* (Huber, Klein, Willmes, Nuerk, & Moeller, 2014). For simplicity, we will refer to this as an example of *whole-number bias* in decimal comparison.

The *fraction bias*, on the other hand, involves generalizing fraction rules to decimals, including treating decimals that are shorter in length as larger in magnitude (e.g., believing that 0.64 is smaller than 0.3 because 1/64 is smaller than 1/3). Third- and fourth-grade students were found to be less affected by the fraction bias at the beginning of learning decimals, but the fraction bias increased among fifth- and sixth-grade students who had more experience with fractions than younger children (Desmet et al., 2010; Lai & Wong, 2017; Resnick et al., 1989; Sackur-Grisvard & Léonard, 1985). In a study of fourth and fifth graders, the fraction bias was less frequent than whole-number bias, but the fraction bias increased over time, whereas whole-number bias decreased over time (Durkin & Rittle-Johnson, 2015). Tenth graders consistently chose shorter numbers as larger decimals more often than choosing longer numbers, and this misconception was also predominant in preservice teachers (Stacey et al., 2001; Steinle & Stacey, 1998) and other adult samples, albeit with a low incidence (Vamvakoussi et al., 2012). Thus, although less common than the whole-number bias, a fraction bias in learning decimal concepts may represent a persistent misconception for some individuals.

Several theoretical accounts can explain how whole-number and fraction rules are misapplied in dealing with decimals. According to overlapping waves theory, children can have multiple strategies available at one time in the face of novel problems (Siegler, 1998). As new strategies are added, the relative frequency of using old strategies decreases and children shift toward using more adaptive strategies over time. Relating to decimals, children may use both whole-number and fraction approaches when handling decimals, with the frequency of each strategy changing over time. As they learn, children are expected to add new rules specific to decimals to their strategy repertoire.

Further, the dual-processing account provides explanations for how biases based on prior knowledge manifest in magnitude comparisons (Epstein, 1994; Kahneman, 2000). This account states that people have intuitive and analytic modes of processing. Intuitive processing is fast, automatic, and less demanding in working memory resources, whereas analytic processing is slow, effortful, and claims more working memory resources. According to this account, whole-number knowledge reflects intuitive processing (i.e., is quickly and automatically activated), leading to whole-number bias (Vamvakoussi et al., 2012).

Building on the dual-processing account, the dynamic strategy choice account proposes a mechanism for understanding how biases based on prior knowledge arise and how people choose a particular strategy among all the strategies available to them at a given time (Alibali & Sidney, 2015). The dynamic strategy choice account proposes that strategy choice varies across specific problems, individuals, and contexts, and that the activation of a particular strategy in a certain context depends on the saliency and recency of the mental representation needed to utilize that strategy. Concerning number processing specifically, children have access to both a whole-number strategy and a fraction strategy, and may also have access to some primitive decimal strategies when attending to decimal magnitudes. When the mental representation of decimal strategies is not highly activated, children tend to choose a whole-number strategy or a fraction strategy, depending on which mental representation is more highly activated at the moment. However, no study has directly and experimentally tested this account. Therefore, we designed the current study to test the dynamic strategy choice theory by manipulating the saliency of each strategy prior to completion of a decimal magnitude comparison task.

We hypothesized that experience with comparing whole numbers or fractions would strengthen the mental representation of the practiced number type, resulting in the activation and utilization of a strategy based on that number type in subsequent decimal comparisons. More specifically, we predicted that compared with non numerical priming, whole-number priming (i.e., practicing whole-number comparisons with feedback) would increase whole-number bias in subsequent decimal comparisons, whereas fraction priming (i.e., practicing fraction comparisons with feedback) would increase fraction bias.

In addition to these main hypotheses, we also explored what individuals' performance would look like after being primed by completing decimal comparisons with feedback. One possibility (proposed, but not directly tested, by Durkin and Rittle-Johnson, 2015) is that after receiving feedback that their initial strategy of choosing the decimal with more digits as the larger was incorrect, children would search for another strategy using the number of digits in each decimal, and therefore attempt to use the opposite rule by choosing decimals with *fewer* digits as being larger than those with more digits (a fraction bias). Alternately, children might be able to learn a correct strategy from this feedback (e.g., compare the tenths digit regardless of the number of digits in the decimal) and eventually perform better on both whole-number congruent and whole-number incongruent decimal comparisons.

In order to answer our questions and test our hypotheses, we asked participants to compare numerical pairs of whole numbers, fractions, or decimals in order to activate participants' mental representation and boost the saliency of one of these number types. We examined participants' performance on two types of trials in a subsequent decimal comparison task. One type of decimal pairs were whole-number congruent trials, on which the whole-number bias would lead to a correct decimal comparison (the decimal with more digits was larger in magnitude, e.g., .64 vs. .3). On another type of decimal trials, the whole-number bias would lead to an incorrect decimal comparison (the decimal with more digits was smaller in magnitude, e.g., .32 vs. .7).

At the group level, we considered better performance (higher accuracy and shorter response time [RT]) on congruent than incongruent trials as evidence for a whole-number bias. In contrast, better performance on incongruent than congruent trials would indicate a group-level fraction bias. With respect to our hypotheses, we expected that whole-number priming would lead to a larger congruency effect (i.e., a larger difference in performance by trial type, with congruent better than incongruent trials) compared with the control condition. We also expected that fraction priming would lead to an increase in fraction bias, manifesting as a smaller congruency effect (or possibly a reversal in the congruency effect, with better performance on incongruent than congruent trials) when compared with the control condition. We also examined these biases at the individual level by categorizing each participant based on their accuracy on congruent and incongruent trials, and expected that more individuals would show a whole-number biased response pattern after whole-number priming (compared with the control condition), and that more individuals would show a fraction-biased response pattern after fraction priming (compared with the control condition).

We collected data from both children and adults. Our target age range was children who had some exposure to decimals but were expected to still have substantial room for improvement in decimal concepts. We first piloted the study on students in fourth to fifth grades based on prior work showing developmental change in decimal concepts at this age (Durkin & Rittle-Johnson, 2015). However, fourth and fifth graders in our sample had little knowledge about the concept of decimals, and many stated that they had never seen decimals before. Therefore, we collected data from students in sixth to eighth grades. Finally, we also collected data from adults as a comparison group to establish whether whole-number and fraction biases remain malleable even among adults with greater decimal magnitude knowledge.

Method

Participants

Children were sixth to eighth grade students from eight schools in a large city in the Eastern United States (N = 149, 76 girls; 47 sixth graders, 47 seventh graders, and 55 eighth graders; age: M = 12.80 years, SD = .94, n_{age} = 147). A power analysis indicated that a sample size of 33 children in each condition (132 in total) would be sufficient to detect a difference between conditions, with $\alpha = .05$, $\eta^2_p = .08$, power = .80 (Durkin & Rittle-Johnson, 2015; McNeil, 2008). Parents reported their child's race and ethnicity (n = 120); children were 55.0% Black or African American, 13.3% White, 5.0% Asian or Asian American, 0.8% American Indian or Alaskan Native, 18.3% Hispanic, 6.7% multiple race/ethnicities, and 0.8% other race. Adults (N = 175) were sampled from college students (n = 61, 51 females; age: M = 20.25 years, SD = 1.85; race: 24.6% Black or African American, 50.8% White, 18.0% Asian or Asian American, 4.9% Hispanic, Latino, or Spanish Origin, and 1.6% other race) and Amazon Mechanical Turk (MTurk) workers (n = 114, 54 females; age: M = 34.04 years, SD = 10.09; race: 7.9% Black or African American, 75.4% White, 8.8% Asian or Asian American, 0.9% American Indian or Alaskan Native, 0.9% Hispanic, Latino, or Spanish Origin, 5.3% multiple race/ethnicities, and 0.9% other race). We expected a smaller effect size in adults than in children; a power analysis indicated a sample size of 44 adults in each condition (176 in total) would be sufficient, with $\alpha = .05$, $\eta^2_p = .06$, power = .80 (DeWolf & Vosniadou, 2015).

Materials

For the decimal comparison test block, each decimal pair consisted of two numbers, with zero in the integer part; one decimal had two digits in the fractional part, whereas the other had only one digit in the fractional part. In all numerical comparison tasks, participants were instructed to choose the bigger number. There

Table 1
Examples of Trials in Prime and Test Block of Four Conditions

Block	Trial type	Decimal priming	Whole-number priming	Fraction priming	Flanker (control)
Prime block	Congruent	.4 vs. .56	4 vs. 56	1/4 vs. 1/56	→→→→→
	Incongruent	.3 vs. .21	3 vs. 21	1/3 vs. 1/21	→→←→→
Test block	Congruent		.2 vs. .37		
	Incongruent		.9 vs. .47		

Note. Whole-number pairs in the prime block are all congruent with whole-number bias (number with more digits is the larger in magnitude). Fraction pairs in the prime block are all incongruent with whole-number bias (number with more digits in the denominator is the larger in magnitude).

were 28 whole-number congruent pairs, for which a whole-number strategy (choosing the number with more digits) resulted in correct answers (e.g., 0.59 vs. 0.3), and 28 whole-number incongruent pairs, for which the whole-number strategy led to incorrect answers (e.g., 0.21 vs. 0.7).[1]

We designed the decimal pairs to avoid potential confounds related to other factors that are known to affect processing of decimal comparisons but were not the focus of this study. The *tenth-hundredth compatibility effect* refers to the finding that people respond faster when the relation between tenths and hundredths places is compatible (e.g., 0.68 > 0.3, where both 6 and 8 are greater than 3) than incompatible (e.g., 0.39 > 0.5, where 3 is smaller than 5 but 9 is greater than 5; Nuerk, Weger, & Willmes, 2001; Varma & Karl, 2013). In this study, all decimal pairs were compatible with respect to the tenth-hundredth compatibility effect. The *distance effect* refers to the fact that people respond faster for pairs with larger numerical distances (e.g., 2 vs. 9) than pairs with smaller numerical distances (e.g., 4 vs. 5; Moyer & Landauer, 1967; Schneider, Grabner, & Paetsch, 2009). To control this, we chose numerical pairs in seven distances (2 through 8) with respect to the difference between the tenth place of each decimal. Number pairs with same distances appeared equally in congruent and incongruent trials. Moreover, except for the zero in the ones place that appeared for all decimals, we avoided using zero in any type of numbers, because misconceptions about the role of zero in decimal magnitudes may differ from misconceptions about other digits (Durkin & Rittle-Johnson, 2012, 2015; Sackur-Grisvard & Léonard, 1985).

For the three numerical priming conditions, the prime block also consisted of 56 number pairs. We designed these numbers in the decimal format first, following the same rules as we did for the test block, but the specific decimal pairs were different from those in the test block. Then, we used the digits in the decimal pairs to create fractions and whole numbers (see Table 1 for examples). The whole-number pairs were the same as the fractional parts in the decimal pairs. The fraction pairs were all unit fractions (i.e., had a numerator of 1); the denominators of the fraction pairs were the same as the fractional parts of the decimal pairs. All fraction pairs were whole-number incongruent (e.g., 1/59 vs. 1/3, and 1/21 vs. 1/7). In the control condition, the flanker task was administered as the prime block. In this task, participants decide the direction that the central arrow pointed to; for half the items, the direction of the central arrow was congruent with the flanking arrows, and for half the items, it was incongruent (Eriksen & Eriksen, 1974). We chose the flanker task because it is non numerical but has similar properties as the number comparison tasks, namely, the presence of both congruent and incongruent trials.

We collected participants' accuracy and RTs on the decimal comparison test block as indices of their performance. We excluded trials with RTs that were too fast (<200 ms) or too slow (≥10 s) from all analyses (0.25% in children, 0.27% of trials in adults). We calculated mean accuracy for the decimal comparison test block. We also calculated RT for correct trials, and excluded RTs that were three standard deviations beyond the participants' mean as outliers (0.87% of correct trials in adults, 2.96% in children).

Procedure

The testing program was launched on JATOS (Lange, Kühn, & Filevich, 2015) using JavaScript and JsPsych (de Leeuw, 2015). Child participants completed the study on a laptop computer in a one-on-one session with an experimenter in a quiet place in their school. College students completed the study on a desktop computer in the lab. Participants recruited via MTurk completed the study online. Data collection via MTurk has been shown to be as reliable as other traditional methods of data collection (Buhrmester, Kwang, & Gosling, 2011). The link on MTurk directed participants to JATOS, thus making data from different sources comparable.

Participants were randomly assigned to one of four conditions: a whole-number priming condition, a fraction priming condition, a decimal priming condition, or a control condition (flanker task). Participants first completed numerical magnitude comparisons according to their priming condition (or the control task) with feedback (prime block), then participants in all conditions completed decimal comparisons without feedback (test block). The items were presented on a computer screen and participants had to press one of two buttons on the keyboard (participants were asked to press "A" if the number on the left side was larger, or the arrow pointed to the left in the flanker task; conversely, they were asked to press "L" if the number on the right side was larger, or the arrow pointed to the right in the flanker task). The item disappeared after the participant responded. Participants were asked to respond as quickly and accurately as they could. In the prime block, feedback appeared regardless of the participant's response, and the participant pressed a button on the keyboard to proceed to the next trial (see Figure 1). There was a 30-s break between the prime block and the test block. Participants in all conditions were instructed to

[1] Five college students had a different number of congruent and incongruent trials in the decimal comparison test block (26 congruent trials and 30 incongruent trials) because of a program error. We included them in the main analyses, but we also reran all analyses after excluding them. The pattern of results did not change.

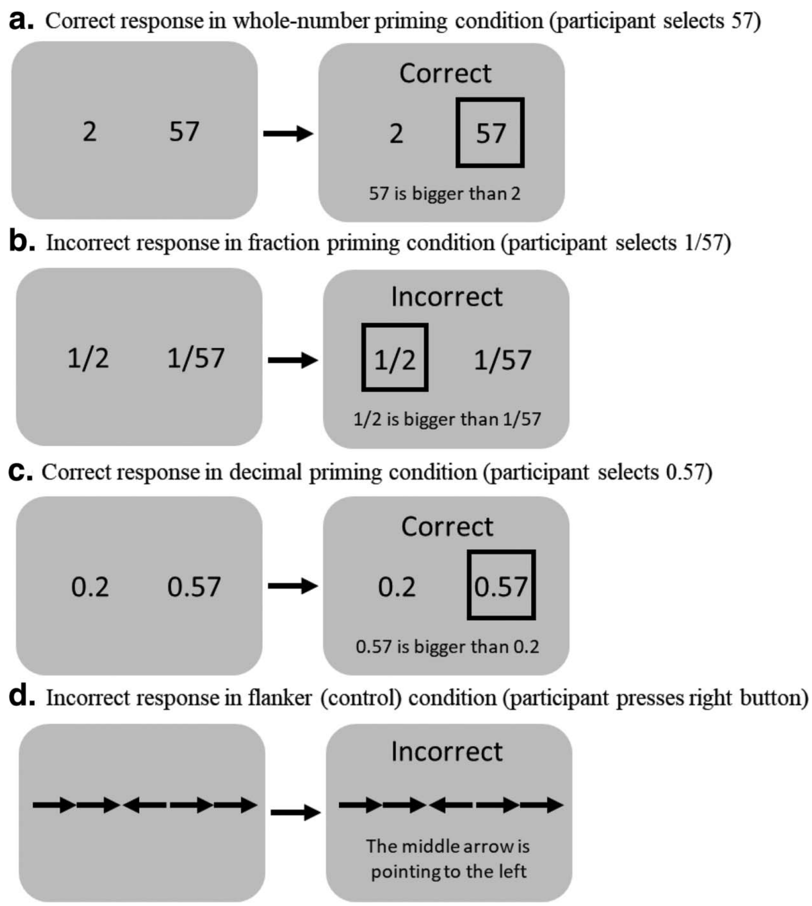

Figure 1. Prime block sample trials with feedback in (a) the whole-number priming condition, (b) the fraction priming condition, (c) the decimal priming condition, and (d) the flanker (control) condition. Each illustration shows the item and the feedback given after the right side was chosen by pressing the “L” key on the keyboard. The square in the feedback indicates the correct answer (the bigger number).

compare decimals in the test block. No feedback was provided in the decimal comparison test block.

The study procedures were approved under Temple University Institution Review Board Protocol 21935, “Cognitive and Emotional Bases of Math, Reading, and Spatial Development.”

Results

Our analyses focus on performance in the decimal comparison test block (see Table S2 in the online supplemental materials for performance in the prime block). Table 2 shows descriptive statistics of RT and accuracy in the decimal comparison test block, by condition and congruency, separately for children and adults (see Table S3 in the online supplemental materials for correlations among key variables). Decimal comparison performance in the test block for children, by condition, is shown in Figure 2.

Data files and analysis script are available at https://osf.io/2jyhp/ (Ren & Gunderson, 2019).

Group-Level Analyses

We conducted generalized estimating equations (GEEs) on accuracy and RT in the decimal comparison test block, with Condition and Congruency as factors, separately for children and adults (see Table 3 for full results of GEE models).[2] We used the GEE approach because it explicitly takes into account the nesting of items within participants and provides a flexible framework for modeling both normally and non-normally distributed data (e.g., binary outcomes such as accuracy on each item; Liang & Zeger, 1986). Within these GEE models, we used a logistic regression approach to analyze accuracy (a binary outcome) and a linear regression approach to analyze RT.

In each model, we included congruent trials and the control condition as the reference groups. Using the control condition as the reference group allows us to interpret a significant parameter estimate for each condition’s interaction effect as a difference in participants’ whole-number bias between that condition and the control condition. These interaction parame-

[2] Our primary research question involved condition differences within children and adults. Although our study was not originally powered to detect effects within grade-level or sample, we explored whether grade-level (sixth, seventh, eighth) or adult sample population (college students, MTurk workers) moderated these effects. These results are reported in Table S1 of the online supplemental materials.

Table 2
Mean Accuracy and RT in the Decimal Comparison Test Block, by Condition and Congruency, Separately for Children and Adults

Measure	Whole-number priming (n = 85) M (SD)	Fraction priming (n = 81) M (SD)	Decimal priming (n = 82) M (SD)	Flanker (control) (n = 76) M (SD)
Demographics				
Child gender	23 female; 15 male	15 female; 22 male	16 female; 21 male	22 female; 15 male
Child years of age	12.86 (1.06)	12.74 (.89)	12.86 (.88)	12.73 (.94)
Adult gender	26 female; 21 male	26 female; 18 male	29 female; 16 male	24 female; 15 male
Adult years of age	29.60 (10.01)	28.89 (10.88)	30.38 (11.76)	27.87 (9.39)
Test block – Decimal Comparison Accuracy				
Children				
Congruent items	.92 (.24)	.62 (.45)	.90 (.19)	.88 (.29)
Incongruent items	.34 (.43)	.60 (.44)	.78 (.28)	.38 (.45)
Adults				
Congruent items	.96 (.14)	.90 (.28)	.99 (.02)	.98 (.05)
Incongruent items	.91 (.17)	.87 (.23)	.94 (.09)	.90 (.20)
Test block – Decimal Comparison RT (ms)				
Children				
Congruent items	943.93 (424.14)	1,037.32 (374.25)	1,310.44 (441.36)	994.58 (453.75)
Incongruent items	1,198.48 (690.49)	967.56 (461.83)	1,422.56 (615.48)	1,330.40 (636.58)
Adults				
Congruent items	778.34 (157.85)	804.39 (248.96)	777.44 (230.72)	727.41 (146.26)
Incongruent items	871.02 (202.61)	905.36 (281.51)	843.28 (219.55)	839.54 (181.02)

Note. RT = response time.

ters therefore serve to test our main hypotheses—that whole-number priming would increase whole-number bias relative to the control condition, and fraction priming would increase fraction bias (i.e., decrease whole-number bias) relative to the control condition, at the group level. In addition, when the overall main effect of condition was significant, we interpreted this by conducting post hoc pairwise comparisons between all pairs of conditions, using Holm's sequential Bonferroni correction for pairwise comparisons.

Children's accuracy during the test block. The GEE on children's decimal comparison test-block accuracy (Table 3, Model 1) showed significant main effects of congruency, $\chi^2(1, N = 149) = 33.83$, $p < .001$, and condition, $\chi^2(3, N = 149) = 19.89$, $p < .001$, and a significant Condition × Congruency interaction, $\chi^2(3, N = 149) = 16.21$, $p = .001$ (Figure 2, Panel a).

In order to interpret the significant overall Condition × Congruency interaction, we examined the GEE parameter estimates for each individual Condition × Congruency interaction term, with the control condition and incongruent trials as the reference groups. Compared with the control condition, whole-number bias was significantly reduced after both fraction priming and decimal priming (parameter estimates for each Condition × Congruency interaction; fraction priming: $B = 2.43$, $SE = .85$, $p = .004$; decimal priming: $B = 1.56$, $SE = .75$, $p = .037$). However, whole-number bias was not significantly increased after whole-number priming ($B = -.52$, $SE = .95$, $p = .583$).

In order to interpret the significant main effect of condition on accuracy, we conducted post hoc pairwise comparisons between each pair of conditions (six different comparisons) using Holm's sequential Bonferroni correction. Overall test-block accuracy was higher in the decimal priming condition (adjusted $M = .86$, $SE = .03$) than in the whole-number condition (adjusted $M = .70$, $SE = .05$; adjusted $p = .047$), fraction condition (adjusted $M = .61$, $SE = .04$; adjusted $p < .001$), and control condition (adjusted $M = .68$, $SE = .05$; adjusted $p = .012$). The other pairwise comparisons were not significant (adjusted $ps > .394$).

Children's RT in the test block. We next examined children's RT in the decimal-comparison test block, with Condition and Congruency as factors (Table 3, Model 2). Similar to the results for accuracy, we found significant main effects of congruency, $\chi^2(1, N = 149) = 18.35$, $p < .001$; and condition, $\chi^2(3, N = 149) = 11.16$, $p = .011$; and a significant Condition × Congruency interaction, $\chi^2(3, N = 149) = 11.33$, $p = .010$ (Figure 2, Panel b).

We interpreted the overall Condition × Congruency interaction by examining the GEE parameter estimates for the individual Condition × Congruency interaction terms. Compared with the control condition, both fraction priming ($B = -271.41$, $SE = 115.85$, $p = .019$) and decimal priming ($B = -180.24$, $SE = 91.80$, $p = .050$) significantly reduced whole-number bias in RT. Whole-number priming did not increase whole-number bias ($B = 87.20$, $SE = 136.69$, $p = .524$).

In examining the main effect of condition on RT, post hoc pairwise comparisons between each pair of conditions (six different comparisons) using Holm's sequential Bonferroni correction showed that overall RT was longer in the decimal priming condition (adjusted $M = 1346.49$, $SE = 65.76$) than in the fraction condition (adjusted $M = 1048.68$, $SE = 64.55$; adjusted $p = .007$). Other comparisons were not significant (adjusted $ps > .185$).

Adults' accuracy in the test block. Adults showed significant main effects of congruency, $\chi^2(1, N = 175) = 25.58$, $p < .001$, and condition, $\chi^2(3, N = 175) = 19.36$, $p < .001$, and a significant Condition × Congruency interaction in accuracy, $\chi^2(3, N = 175) = 7.99$, $p = .046$ (Table 3, Model 3).

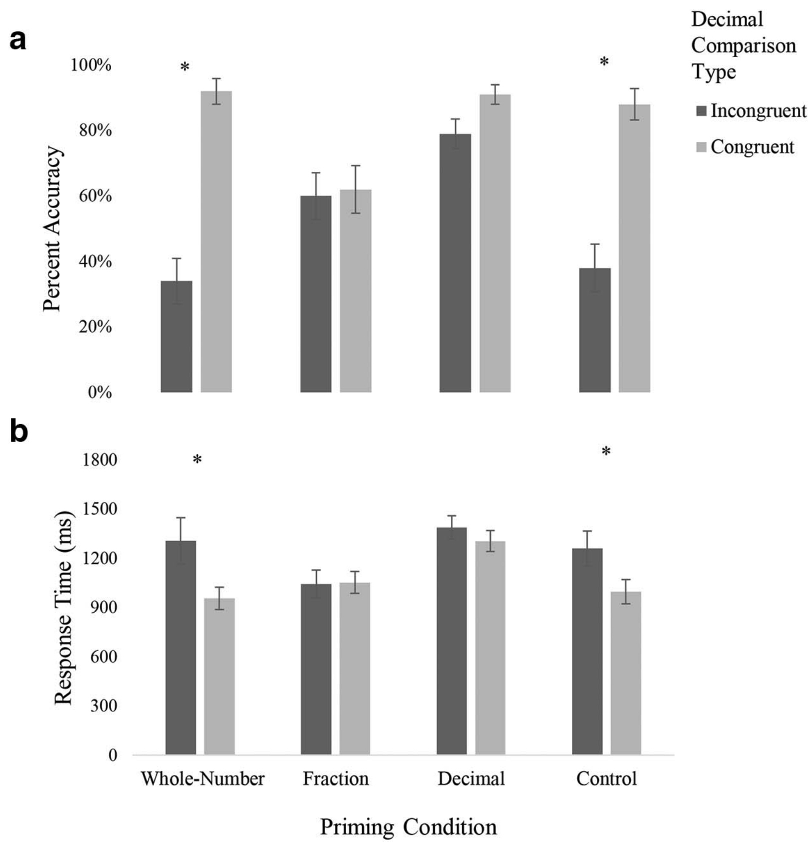

Figure 2. Children's test-block decimal comparison performance by condition in (a) accuracy and (b) response time (RT). Error bars represent one standard error of the mean. * $p < .05$.

Following up on the overall Condition × Congruency interaction, we found that, compared with the control condition, fraction priming significantly reduced whole-number bias ($B = 1.68$, $SE = .79$, $p = .034$). Neither whole-number priming nor decimal priming showed different patterns from the control condition (interaction terms, $p > .178$).

Exploring the main effect of condition on accuracy, post hoc pairwise comparisons between each pair of conditions using Holm's sequential Bonferroni correction showed that overall test-block accuracy was higher in the decimal priming condition (adjusted $M = .98$, $SE = .01$) than in the fraction priming condition (adjusted $M = .89$, $SE =$ 03, adjusted $p = .028$). The other comparisons were not significant (adjusted $ps > .106$).

Adults' RT in test block. For adults, there was a significant main effect of congruency in RT, $\chi^2(3, N = 175) = 108.85$, $p < .001$, indicating significant whole-number bias (Table 3, Model 4). There was no significant main effect of condition, and no Condition × Congruency interaction ($ps > .354$).

Individual-Level Analyses

In our group-level analyses, even though whole-number bias decreased after fraction and decimal priming compared with the control condition (as assessed by the decrease in group-mean differences between congruent and incongruent trials), it was possible that some people still possessed strong whole-number bias, whereas others changed to using a fraction strategy (performed well on incongruent trials but poorly on congruent trials) after fraction priming, which, on average, would produce a smaller group mean difference. However, it was also possible that after receiving feedback of decimal comparisons, children increased in random guessing on both types of trials, which could also result in a smaller group mean difference. Group-level analyses tended to mask those possibilities in each condition, so to probe these possibilities, we chose to examine performance patterns by individual.

We categorized participants according to their pattern of accuracy on the congruent and incongruent items in the decimal comparison test block (see Figure 3 for children's performance by item type and resulting categorizations). We used chance performance (50%) as the cutoff. Participants who had high accuracy on congruent trials (>50%) but low accuracy on incongruent trials (≤50%) were categorized as responding consistently with the whole-number bias. Conversely, participants with high accuracy on incongruent trials (>50%) but low accuracy on congruent trials (≤50%) were categorized as having a fraction bias. Those who performed well (>50% accuracy) on both congruent and incongruent trials were categorized as displaying a decimal accuracy

Table 3

Generalized Estimating Equation Model Results Predicting Accuracy and RT in the Test Block, Separately for Children and Adults

Model parameter	Model 1 Decimal comparison accuracy in children				Model 2 Decimal comparison RT in children				Model 3 Decimal comparison accuracy in adults				Model 4 Decimal comparison RT in adults			
	B (*SE*)	Wald χ^2	*df*	*p*	*B* (*SE*)	Wald χ^2	*df*	*p*	*B* (*SE*)	Wald χ^2	*df*	*p*	*B* (*SE*)	Wald χ^2	*df*	*p*
Overall tests of model effects																
Condition		19.89	3	<.001		11.16	3	.011		19.36	3	<.001		2.79	3	.426
Congruency		33.83	1	<.001		18.35	1	<.001		25.58	1	<.001		108.85	1	<.001
Condition × Congruency		16.21	3	.001		11.33	3	.010		7.99	3	.046		3.25	3	.355
Parameter estimates[a]																
Whole-number priming	.35 (.69)	.26	1	.611	−40.41 (100.49)	.16	1	.688	−1.04 (.74)	1.97	1	.161	48.31 (32.41)	2.22	1	.136
Fraction priming	−1.55 (.56)	7.67	1	.006	56.53 (99.05)	.33	1	.568	−2.03 (.69)	8.65	1	.003	78.67 (44.27)	3.16	1	.076
Decimal priming	.23 (.58)	.15	1	.695	308.76 (97.79)	9.97	1	.002	.29 (.57)	.26	1	.614	44.87 (40.23)	1.24	1	.265
Incongruent trials	−2.52 (.66)	14.43	1	<.001	262.83 (86.11)	9.32	1	.002	−2.02 (.59)	11.68	1	<.001	103.57 (16.34)	40.17	1	<.001
Whole-number Priming × Incongruent Trials	−.52 (.95)	.30	1	.583	87.20 (136.69)	.41	1	.524	1.10 (.82)	1.81	1	.178	−15.19 (22.59)	.45	1	.501
Fraction Priming × Incongruent Trials	2.43 (.85)	8.22	1	.004	−271.41 (115.85)	5.49	1	.019	1.68 (.79)	4.51	1	.034	−22.08 (25.79)	.73	1	.392
Decimal Priming × Incongruent Trials	1.56 (.75)	4.35	1	.037	−180.24 (91.81)	3.86	1	.050	.25 (.64)	.16	1	.694	−36.18 (20.62)	3.08	1	.079

Note. RT = response time; *SE* = standard error; *df* = degrees of freedom.

[a] Results indicate parameter estimates with the control condition and congruent trials as the reference groups.

performance pattern, whereas those who were low on both (≤50%) were categorized as showing decimal inaccuracy.[3]

The percentage of children in each performance category, by condition, is displayed in Figure 4. We conducted Pearson's chi-square test to examine whether the frequency distribution of performing whole-number bias, fraction bias, and decimal-comparison accuracy differed from the control condition, with pairwise comparisons between each of the experimental condition and the control condition using Holm's sequential Bonferroni correction, with $\alpha = .05$. We first examined whole-number bias between conditions (three comparisons in total). The percent of children with a whole-number bias was significantly lower in the fraction priming condition (32.4%) than the control condition (62.2%), $\chi^2(1, N = 74) = 6.56$, adjusted $p = .020$. The percent of children with a whole-number bias was significantly lower in the decimal priming condition (18.9%) than the control condition (62.2%), $\chi^2(1, N = 74) = 14.35$, adjusted $p < .001$. However, the percent of children with a whole-number bias was not significantly different in the whole-number priming condition (63.2%) than the control condition (62.2%), $\chi^2(1, N = 75) = .01$, adjusted $p = .929$. In terms of fraction bias (three comparisons in total), the percent of children with a fraction bias was significantly higher in the fraction priming condition (35.1%) than the control condition (10.8%), $\chi^2(1, N = 74) = 6.19$, adjusted $p = .039$. The comparison was not significant between the decimal priming condition versus control, $\chi^2(1, N = 74) = .73$, adjusted $p = .788$, nor the whole-number priming condition versus control, $\chi^2(1, N = 75) = .19$, adjusted $p = .788$. Turning to decimal-comparison accuracy (three comparisons in total), we found that, strikingly, the percent of children with high decimal-comparison accuracy was significantly higher in the decimal priming condition (75.7%) than the control condition (27.0%), $\chi^2(1, N = 74) = 17.53$, adjusted $p < .001$. The other two comparisons were not significant (adjusted $p = 1.000$).

A parallel set of analyses showed that adults were mostly categorized into decimal accuracy regardless of condition (whole-number priming condition, 93.6%; fraction priming condition, 86.4%; decimal priming condition, 97.8%; control condition, 94.9%), and there were no significant differences between adults' performance category in the priming conditions compared with the control condition ($ps > .097$).

Discussion

Our study, for the first time, directly and experimentally tested the dynamic strategy choice account (Alibali & Sidney, 2015) by showing that the activation of a certain mental representation changed individuals' strategy choice when dealing with relevant decimal magnitude comparison problems. Specifically, we manipulated the recency and saliency of whole-number, fraction, or decimal mental representations and examined performance in subsequent decimal magnitude comparisons, compared with a control group in which no number representations were activated. We found that sixth to eighth graders applied whole-number rules to decimal magnitude comparisons in the control condition, even without any whole-number priming. Compared with the control condition, priming students with whole-number comparisons did not increase this already-strong whole-number bias when completing decimal comparisons. In contrast, priming students with fractions or decimals significantly reduced whole-number bias in completing decimal comparisons, a result that was consistent in both group-level and individual-level analyses.

More specifically, our group-level analyses indicated that both children and adults had strong whole-number bias in the control

[3] To assess the validity of these researcher-defined categories, we also used a data-driven approach (cluster analysis) to categorize individuals' performance and compared these categorizations with our original ones. A hierarchical cluster analysis using average linkage yielded the same categorization as our original categorization for 144 of the 149 participants (97% of participants). Four cases with near-chance performance on both types of items were recategorized as showing decimal inaccuracy, and one case (100% on congruent trials and 44% on incongruent trials) was recategorized as showing decimal accuracy (versus whole-number bias). We conducted the same Pearson's chi-square tests reported in the main text using these new categories. All results remained the same in terms of the direction of the effect and statistical significance.

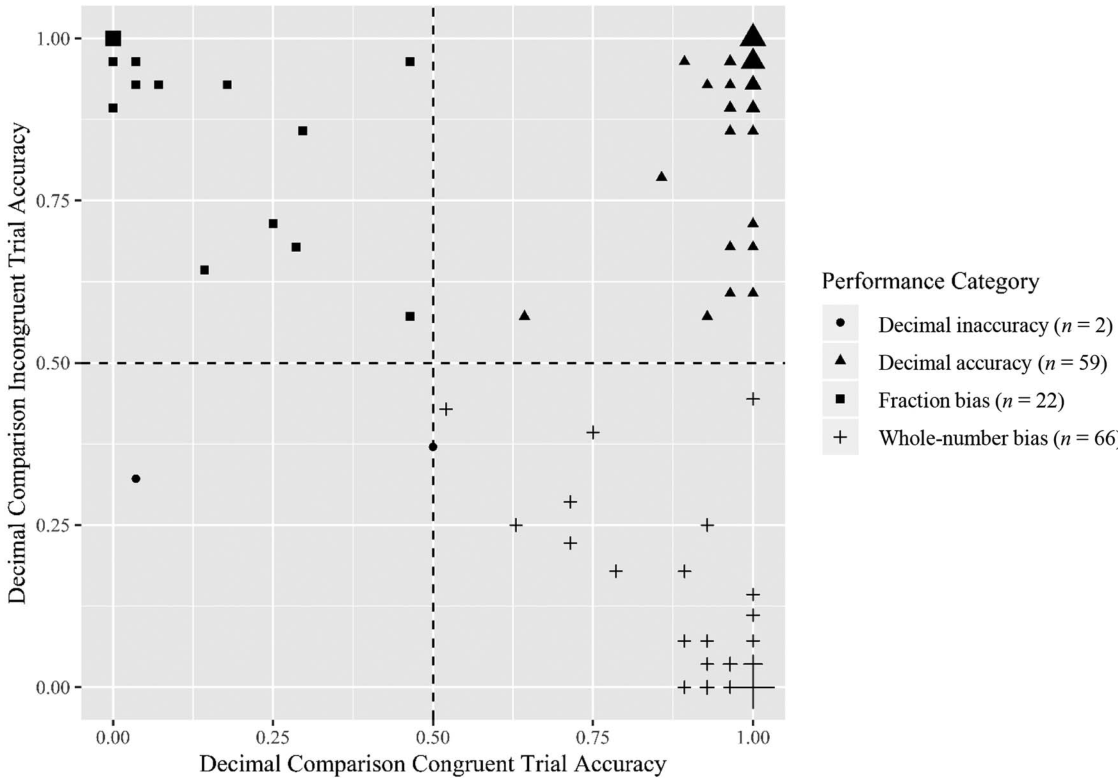

Figure 3. Children's decimal comparison accuracy by item type in the test block. Dashed lines represent chance performance (50%). Dot shapes indicate performance category assigned based on performance relative to chance. Larger dot sizes indicate a larger number of children.

condition, as assessed by higher accuracy and faster RTs on congruent trials versus incongruent trials. However, this tendency declined after fraction priming (in both children and adults) and decimal priming (in children only), although the group-level performance patterns suggested that children's decrease in whole-number bias resulted from different processes in each condition. Tellingly, in the decimal priming condition, but not the fraction priming condition, children had higher overall accuracy than the control condition. Decimal priming increased accuracy on incongruent trials without compromising accuracy on congruent trials, suggesting that decimal priming led children to incorporate an accurate decimal comparison strategy into their repertoire. In contrast, fraction priming increased accuracy on incongruent trials but decreased accuracy on congruent trials, consistent with our hypothesis that children shifted from one inaccurate strategy (whole-number bias) to another (fraction bias), resulting in no overall improvement in accuracy. However, we note that fraction priming did not reverse the group-level performance pattern (i.e., it did not lead to higher overall accuracy on incongruent than congruent trials), and the reduction in the congruency effect could be attributed to other underlying mechanisms, such as increased confusion about the task and random responding. In order to discriminate between these possibilities, we also examined performance at the individual level.

We categorized individual performance into four patterns—displaying whole-number bias (consistently choosing the decimal with more digits as the larger), fraction bias (choosing the decimal with fewer digits as the larger), decimal accuracy (choosing the larger decimal as larger, regardless of the number of digits), or decimal inaccuracy (choosing the smaller decimal as larger, regardless of the number of digits). This analysis allowed us to detect an important difference between fraction priming and decimal priming. Although both conditions reduced the percent of children displaying a whole-number bias compared with the control condition, fraction priming increased the percent of children showing a consistent fraction bias, whereas decimal priming increased the percent of children showing consistent decimal accuracy.

The finding that fraction priming increased fraction bias and reduced whole-number bias compared with the control condition was consistent with our hypothesis. Increasing the salience of the mental representation of unit fractions led students to be more likely to employ the corresponding fraction strategy and to be less likely to draw on the whole-number strategy, consistent with the dynamic strategy choice account (Alibali & Sidney, 2015). It is important to note that our fraction priming was designed to maximize fraction bias by asking participants to compare unit fractions with one-digit or two-digit denominators, such that numbers with fewer digits (in the denominator) were always larger in magnitude. Therefore, future research could examine whether activation of fraction knowledge more broadly would lead to similar effects. It is also important to keep in mind that people's use of a fraction strategy in decimal comparisons after priming is not necessarily a

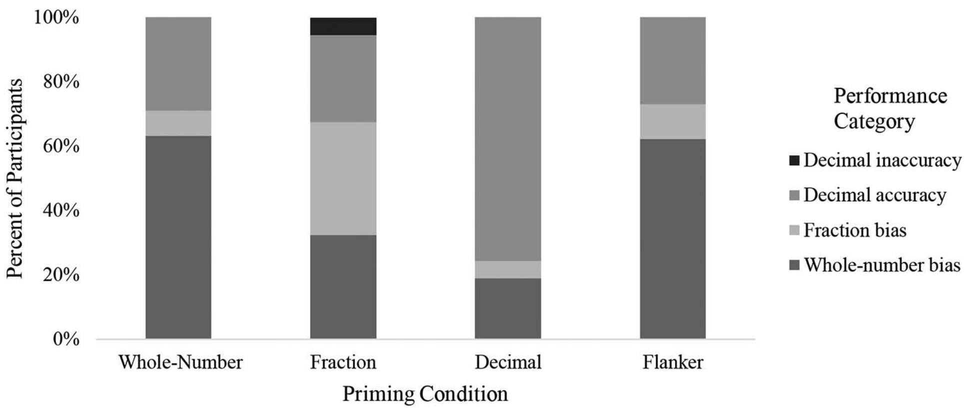

Figure 4. Percent of children in each decimal comparison performance category by priming condition.

sign of poor decimal magnitude understanding. Rather, this pattern signals children's awareness that there are important differences between the number type they are most familiar with (whole numbers) and decimals. In other words, when children attempt to incorporate new information into their existing mental model, they often produce a *synthetic mental model* (in this case, a fraction bias) that differs from both the correct model (accurate decimal magnitude concepts) and their prior model (whole number bias; Vosniadou, 1994). Thus, children's attempts to use strategies that conflict with their prior whole-number knowledge, in this case, use of a fraction strategy, could be regarded as a synthetic conception—an intermediate step toward forming a correct, coherent conception that incorporates properties of all types of rational numbers (Desmet et al., 2010; Vamvakoussi & Vosniadou, 2004, 2010).

Although we had predicted that whole-number priming would strengthen the whole-number bias compared with the control condition, this was not the case. This suggests that the mental representation of whole numbers was highly salient even without whole-number priming. At the group level, the performance pattern in the whole-number condition did not significantly differ from the control condition. At the individual level, over 62.2% of children in both the control and whole-number priming conditions were categorized as responding according to a whole-number bias. This is in line with previous findings showing that children are mainly affected by persistent whole-number bias (e.g., responding that longer numbers are larger) when they lacked formal knowledge of decimals (Desmet et al., 2010; Lai & Wong, 2017).

One of our goals was to explore how completing decimal comparisons (with feedback) would impact subsequent decimal comparisons. Somewhat surprisingly, a relatively small amount of feedback (56 trials of decimal comparisons) led to a large reduction in children's whole-number bias and a large increase in decimal comparison accuracy in our individual analyses, from 27.0% of children showing an accurate response pattern in the control condition to 75.7% in the decimal-comparison priming condition. This improvement after brief feedback is striking, especially given that students in these grade levels have already received formal instruction on decimal magnitudes (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). The increased usage of an accurate decimal strategy also supports the dynamic strategy choice account, in which increasing the saliency and recency of decimal magnitude knowledge led to the use of the strategy aligned with the activated mental representation. However, it is unclear whether students learned a procedural rule from this feedback (i.e., to focus on the tenths digit) or improved their conceptual understanding of decimals in a broader, more durable manner. In either case, these results revealed that sixth to eighth graders were still in the process of developing systematic number knowledge that incorporates normative properties of all types of rational numbers. Future studies could use intervention designs to examine whether practicing decimal magnitude comparisons with feedback, including whole-number congruent and incongruent items, could lead to lasting improvements in conceptual understanding of decimal magnitudes or lead to transfer to arithmetic, algebra, and other more advanced mathematical skills that may require the understanding of rational number properties that decimals entail (DeWolf et al., 2015).

It is also possible that receiving feedback on decimal comparisons may have led children to try out alternate strategies on the decimal comparison task before arriving at a correct strategy. Indeed, one of our exploratory predictions was that children might shift from a whole-number strategy to a fraction strategy after receiving decimal comparison feedback (Durkin & Rittle-Johnson, 2015). Although this was not supported by children's performance in the test block, which reflected decimal comparison accuracy, our task was not designed to capture such strategy shifts during the priming block. Therefore, it may be helpful for future research to examine the dynamic changes in individual's strategy use over time, both on a trial-by-trial basis while receiving feedback and in the long-term over several years of decimal instruction.

In comparison, adults' decimal comparison performance was much more accurate than students. Consistent with prior work (DeWolf & Vosniadou, 2015; Obersteiner, Hoof, Verschaffel, & Dooren, 2016; Obersteiner, Van Dooren, Van Hoof, & Verschaffel, 2013), adults showed an overall whole-number bias in RT (responding more slowly for whole-number incongruent items than congruent items), but this RT pattern was not affected by their priming condition. However, even adults showed reduced whole-number bias (measured by accuracy) after being primed by fraction comparisons compared with the control condition. When viewed at the individual level, nearly all adults presented the decimal accuracy pattern in all conditions. The different results between children and adults suggest that children, who were still in the process of learning decimals, were more susceptible to the influence of other number types than adults. In contrast, adults' considerable knowledge of decimals may have made them less easily affected by priming of other number types.

Altogether, the present study focused on whether the recency and saliency of prior knowledge would affect people's strategy choice in comparing decimal magnitudes. These results provide experimental support for the dynamic strategy choice account by showing that activating children's mental representations of fractions made them more likely to use a fraction-biased strategy in dealing with decimal comparisons. Our results also have practical implications for teaching number concepts. Given that students' misconceptions in learning decimals are common and malleable, it may be beneficial for teachers to provide examples that are incongruent with children's prior knowledge, with which they can clarify the differences among number types. For instance, incorrect worked examples—problems on which students must explain why a fictional students' incorrect answer is wrong—can be used to confront common misconceptions and lead to greater math learning, especially among struggling students (Barbieri & Booth, 2016). Teachers could use such examples to explain that whereas the number of digits in a whole number is associated with whole-number magnitude, this is not necessarily the case for decimal magnitudes. This might be beneficial to students in reducing misconceptions that result from prior knowledge.

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