Developmental Cognitive Science Goes to School

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Dedication

This book is dedicated to Tom Trabasso, whose lifelong interest in learning and understanding served as a guideline for addressing issues related to developmental psychology, learning, and schooling. Tom's interest in learning was apparent in his earliest work in the 1960s, with Gordon Bower and Rochel Gelman on discrimination and concept learning, in his work with Peter Bryant on transitive inferences, learning and development, in his work with Peter Ornstein on organizing, learning, and remembering, and in all of his work on models of understanding, thinking, and development. His later work on narrative and causal understanding was fueled by an attempt to account for language, memory, and thinking that went beyond simple word and sentence understanding. That is, he wanted to study complexity, and the ways in which complex systems impacted people on a daily basis.

The feature that characterized Tom the most was his passionate quest for answers and discoveries that would lead to a better understanding of how children think, reason, and remember. He was a consummate scientist and always sought evidence, no matter what the issues were, and no matter whose theories were being tested, to explain and account for data. The pursuit of answers and evidence often got him into trouble, especially with those whose theories were being tested. Tom persevered, however, until he got the answers he was seeking, often at variance with current belief systems related to how children learn.

Tom was as good at teaching as he was at doing research. His tenacity, goal directedness, and razor-sharp intellect enabled him to impart a sense of discovery and delight to students and colleagues who were engaged in studying learning and memory. Tom was faster than just about anyone in discerning confounds and problems with an approach, devising ways of explicating and testing an issue, advancing a theory that was far more robust than the one with which he started, and pointing to broad implications that theories of learning had in regard to developmental issues.

Had Tom survived, he would have been an integral part of the efforts to become more intimately involved in science and school learning. Although Tom was mathematically gifted, he was never trained in the physical sciences, and so his new venture required that he take time out to actually learn the content of physics, chemistry, and earth sciences.

Several of the chapters in the present volume do what he would have done, if he could: make developmental and cognitive science relevant and understandable to those who teach young children on a daily basis.

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13 Number Development in Context

Variations in Home and School Input During the Preschool Years

Susan C. Levine, Elizabeth A. Gunderson, and Janellen Huttenlocher

The ability to think mathematically is a central aspect of human cognition, and the development of this ability has been a topic of intense study. This literature, in large part, paints a general picture of the development of number knowledge. Much less attention has been paid to individual differences in number knowledge, to variations in number-relevant input, or to the relation between number knowledge and input. A confluence of research findings and societal goals is fueling an increased interest in understanding individual differences in early mathematics knowledge. First, it is now clear that individual differences in math knowledge emerge early, and that these early differences predict later achievement (e.g., Duncan et al., 2007). Second, early differences in mathematics achievement are associated with socio-economic status (SES) (e.g., Jordan, Huttenlocher, & Levine, 1992; Jordan, Levine, & Huttenlocher, 1994; Lee & Burkam, 2002; Saxe, Guberman, & Gearhart, 1987), which is an impediment to diversifying the workforce in the science, technology, engineering, and mathematics (STEM) disciplines (e.g., Arnold, Fisher, Doctoroff, & Dobbs, 2002). Finally, workforce demands for people with high levels of mathematical skill are increasing while American children lag children in other countries in math achievement (e.g., Gonzales et al., 2004; OECD, 2007). Research aimed at increasing our understanding of the factors that contribute to individual differences in mathematics achievement is central to addressing these issues.

In this chapter, we review research on early variations in numerical development, and the relation of these differences to variations in number-related input. These research efforts are grounded in socio-cultural views of development, which emphasize the impact of adult support in propelling children's development (e.g., Fluck, 1995; Rogoff, 1990; Vygotsky, 1978; Wertsch & Tulviste, 1992). We begin with a brief discussion of evidence supporting universal starting points. We then discuss the slow and effortful process through which children acquire an understanding of the verbal, symbolic number system. Second, we review evidence that children have already diverged in their level of mathematical knowledge by the start of school, and that this divergence predicts later academic achievement. Third, we review the evidence that parents and teachers vary widely in the number-related inputs they provide young children, and that these variations impact children's number knowledge. Finally, we consider directions for future research and implications for policy and practice.

Infants Show an Early Number Competency

Infants' sensitivity to variations in set size, for both large and small sets, has been demonstrated through habituation studies (e.g., Antell & Keating, 1983; Brannon, 2002; Starkey & Cooper, 1980; Strauss & Curtis, 1981; Wood & Spelke, 2005; Xu & Spelke, 2000). Various models have been proposed to account for the pattern of results obtained. Gallistel and Gelman (1991, 2000) proposed the accumulator model, based on Meck and Church (1983), which provides approximate, ratio-limited numerical representations for small and large sets. Thus, six-month-old infants can discriminate 8 vs. 16 but not 8 vs. 12 (Xu & Spelke, 2000). A different model, proposed by Feigenson, Dehaene, and Spelke (2004), posits that infants begin life with two core systems for representing numbers: a small number system and a large number system. The small number system is characterized by exact representation of set sizes up to three and offers an explanation for young infants' ability to discriminate between small set sizes such as 2 vs. 3 but not between larger set sizes with the same ratio such as 4 vs. 6 (e.g., Starkey & Cooper, 1980). The large number system, for numbers greater than or equal to 4, is characterized by ratio-dependent performance (2:1 ratio at six months), consistent with the accumulator model.

This early competency with numbers might suggest that young children need only map the verbal count words onto their underlying representation of numbers. However, this mapping develops in a slow and gradual manner. Wynn (1990) found that the majority of three-year-old children were able to count objects, but if the same children were asked "How many are there?" they seldom produced a response corresponding to the last number in their count. Instead, they typically re-counted the objects. Although it is possible that children simply misinterpreted the "how many" question (Zur & Gelman, 2004), young children also fail when asked to give a puppet a certain number of objects (known as a "give-a-number" task) (e.g., Wynn, 1990, 1992). In a longitudinal study, Wynn (1992) found that it takes about a year from succeeding in giving a set of "one" to being able to produce sets of four and above, at which time children become "cardinal principle knowers," as they can map all the numbers in their count list onto corresponding cardinal values.

Understanding the cardinal principle has important implications for understanding numerical relations (e.g., Carey, 2004; Le Corre, Van de Walle, Brannon, & Carey, 2006; Sarnecka & Carey, 2008). For example, it is related to attaining the concept of cardinality, knowledge that all sets with a given cardinal value (e.g., three candies, three jumps, three clouds) form an equivalence class. Children who understand the cardinal principle are able to make numerical matches for perceptually dissimilar sets, whereas those who have not yet acquired this understanding are able to make numerical matches only for perceptually similar sets (e.g., Mix, 2008). In addition, cardinal principle knowers have connected counting to determining the cardinal value of sets, as they are much more likely to count when asked to produce sets of four or more on the give-a-number task than children who have not reached this milestone (Le Corre et al., 2006). Further, only cardinal principle knowers understand the successor function as they know that adding exactly one item to a set means moving forward exactly one count word (Sarnecka & Carey, 2008).

These studies suggest that children's number concepts do not come "for free" based on their early numerical sensitivity (although see Gallistel & Gelman, 1992, for an argument that counting principles are present from birth). Rather, children gradually construct the natural numbers (Wynn 1990; Carey, 2004), raising the possibility that the number-related input they receive plays a substantial role in this accomplishment.

Children's Math Abilities Vary Widely by the Start of School

Preschool children show marked variation in their numerical knowledge, and these differences are especially apparent for children from different SES groups (e.g., Ginsburg & Russell, 1981; Lee & Burkam, 2002). At the start of preschool, low-income children score significantly lower than middle-income children on some mathematical tasks, but not others. For example, low-income preschoolers show less knowledge of the ordering of the numbers 1 through 10 in a number line task, correctly ordering only 61 percent of the numbers compared with 81 percent by middle-income children (Siegler & Ramani, 2008). However, Saxe et al. (1987) report SES differences among preschoolers only on tasks that require higher-level mathematical reasoning skills, such as knowledge of the cardinal principle and arithmetic calculations, but not on less complex tasks involving counting or the ability to read number symbols. Further, Ginsburg and Russell (1981) report SES-related differences among kindergarteners on complex but not simple addition problems.

The nature of the presentation format also impacts whether SES-related differences are found on calculation problems (Jordan et al., 1992, 1994). Thus, low- and middle-SES kindergarteners performed similarly on calculation problems administered nonverbally (children were asked to reproduce the resulting set after watching the experimenter lay out and cover a set of disks, and then remove or add disks without revealing the result). In contrast, low-SES kindergarteners performed worse than middle-SES kindergarteners when the same calculation problems were presented as story problems (e.g., "Beth has m balloons, Steve gives her n more balloons. How many balloons does Beth have altogether?") or number-fact problems (e.g., "How much is m and n?").

At the start of school, children from middle- and low-SES backgrounds also differ in their calculation strategies and error patterns on verbal calculation problems. Middle-income kindergarteners were more likely than lower-income children to use a finger-counting strategy, which at that age is associated with more accurate performance. Further, low-income children were more likely to make errors in the wrong direction than middle-income children (e.g., $4-2=5$), suggesting that they have a weaker conceptual understanding of numerical relations (Jordan et al., 1992).

These early differences in numerical skills are of course of concern only if these differences are related to long-term achievement patterns. In an important meta-analytic study of six longitudinal data sets, Duncan et al. (2007) have shown that that this is the case. In particular, children's math skills at the time of school entry predicted subsequent mathematics and reading achievement through the third grade.

What Do We Know about Numerical Input Provided by Parents During the Preschool Years?

The finding of early individual differences in numerical knowledge associated with SES raises the question of whether differential exposure to math input in the home environment may be an important factor in children's learning trajectories. This hypothesis is supported by evidence showing that cognitive and social factors in the home environment are more predictive of children's mathematics achievement than either SES or mothers' math test scores (although all three factors predict variance in children's mathematics achievement) (Crane, 1996). For reading and literacy, we have learned much about the kinds of parental inputs that support strong foundations for school achievement by examining early home environments (e.g., Sénéchal & LeFevre, 2002; Snow, Burns, & Griffin, 1998; Whitehurst & Lonigan, 1998). However, much less is known about the nature or frequency of early mathematically relevant parent–child interactions, or about the extent to which these interactions predict children's later math achievement. Nonetheless, we are beginning to build a knowledge base about variations in early parent–child interactions about number based on studies using questionnaires, checklists, structured observations, and naturalistic observations.

Questionnaires and Checklists

Several studies of the mathematical input that preschool children receive at home have relied on parental interviews or activity checklists. In a study that used a structured parental interview with mothers of two- and four-year-olds, Saxe et al. (1987) found that middle-SES mothers reported mathematics activities with more complex goal structures than lower-SES mothers. Complex activities included calculation and comparing the cardinal values of multiple arrays, whereas simpler activities involved labeling the cardinal values of single arrays, reciting numbers, and recognizing number symbols. The activities engaged in by middle-SES mother–child dyads also spanned a greater range of complexity than those engaged in by dyads from lower-SES backgrounds, suggesting that middle-SES families do not abandon simpler number activities in favor of more complex ones, but rather engage in both simple and more complex activities.

In another study, using an activity checklist, Blevins-Knabe and Musun-Miller (1996) asked parents to estimate the frequency of their kindergarten children's engagement in a large set of number-related activities during the previous week (e.g., using the concept "more" with the child, singing a number song, and encouraging the child to write numbers). They found that the frequency with which the parent or child used the words "one," "two," or "three" and the frequency with which the parent or child mentioned number facts (e.g., "1 + 1 = 2") were positively correlated with children's scores on the TEMA-2, a standardized test that focuses on number-related knowledge (Ginsburg & Baroody, 1990). Interestingly, the reported frequency of teaching children to recite the numbers 1 to 10 was *negatively* correlated with the child's performance on the TEMA-2, suggesting that rote counting activities may be less useful than activities that relate number words to set size.

Finally, Starkey et al. (1999) surveyed low- and middle-SES American parents about the types and frequency of math activities that their four-year-old children engaged in at home. Middle-SES parents reported that their children engaged in more types of math-related activities and received more parental math support than lower-SES parents. This was true both for math activities that required outlays of money (e.g., math books, math software, and purchased games) and for those that

did not (e.g., math activities that were part of the home routine and made-up games). Starkey et al. hypothesized that these SES differences may be related to parents' education levels, exposure to math courses, and the value they place on their children's mathematical competence.

As noted by Starkey and Klein (2008), more research is needed to understand the causal factors underlying SES-related differences in mathematical input. The effects of poverty on parents' involvement in math learning are likely to be similar to those impacting their involvement in early literacy. These include financial strains, time constraints, and inadequate education. For mathematics, these issues are likely to be compounded by additional factors including parents' discomfort with their own math skills, lack of awareness about the importance of early math input, and inadequate knowledge about how to support children's math development (e.g., Barbarin et al., 2008; Clements & Sarama, 2007; McLoyd, 1990).

Structured Observations

Other studies have examined the kinds of number-related input parents provide during prescribed numerical activities. Saxe et al. (1987) observed mothers assisting their two- and four-year-old children in two tasks: one that involved counting an array of five or 13 dots, and one that involved producing a set of three or nine pennies to match the number of Cookie Monster cards in a given set. In both tasks, most mothers introduced the problem with a relatively high level of complexity, then adjusted their level of instruction to their child's ability level, with mothers of less knowledgeable children structuring less complex goals. Children in turn modified their behavior appropriately in response to their mothers' instruction, and many children who were unable to complete the tasks when unassisted were successful when scaffolded by their mothers' instruction. Furthermore, although the complexity of maternal instruction was highly related to the child's ability level, SES differences in mothers' instruction were found even after the child's age and ability were controlled. For example, on the task of producing a set of three pennies, working-class mothers were more likely to simplify the goal structure of the task in their initial instructions, even after controlling for child's age and ability. These findings suggest that mothers' instructional complexity is not completely dependent on children's ability levels, and therefore may play a causal role in shaping number development.

Fluck (1995) also observed the kinds of support mothers provide when asked to help their two- and three-year-old children count sets of objects. The results showed that the most common prompt mothers used included the word "count" (e.g., "Can you *count* them?"). Less commonly, mothers asked their children how many there were, which explicitly refers to the cardinal value of the set (e.g., "*How many* animals are there?") or used both "count" and "how many" (e.g., "Are you going to *count how many* there are?"). Regardless of the parents' prompt, children tended to count rather than respond with a one-word answer corresponding to the set size. The results also showed that mothers were more likely to state the cardinal value of a set (e.g., "There's five bricks there") after the child counted correctly than after the child counted incorrectly. Fluck hypothesized that this type of input may help the child connect counting and cardinality. In fact, maternal repetitions of the child's final count word were positively correlated with the child's counting competence. Although these findings suggest that this type of input is important, it is not possible to conclude that it is causally related to children's number understanding because of the correlational nature of the study.

Naturalistic Observations

Durkin, Shire, Riem, Crowther, and Rutter (1986) carried out a longitudinal, observational study of 10 mother–child dyads when children were 9 to 36 months of age that described the use of number words by mothers and children. The results showed that parents used number words throughout this developmental period, which overlaps extensively with the period during which preschoolers' counting skills are commonly studied (e.g., Gelman & Gallistel, 1978; Wynn, 1990). Mothers' number word uses were largely confined to the first four numbers, and frequency of use increased from 9 to 27 months and then leveled off. Common contexts in which number words were used included nursery rhymes, stories and songs, recitation of number strings, alternation between mother and child while counting, repetition and clarification of cardinality, and routines such as "one, two, three, go!"

Durkin et al. (1986) point out that many of these uses may be confusing to children. For example, parents sometimes ask their child to repeat each count word after them, resulting in a combined mother–child utterance of "one, one, two, two, three, three," and at other times ask their child to say the next number in the sequence, so that the combined utterance is the typical count string. Parents' number input also frequently contains elements such as "one, two, three, tickly" rather than "one, two, three, four." Findings also show that much of the child's use of number words occurs in joint dialogues with the mother. Given the noise in parents' number input, and children's difficulty learning the cardinal meaning of number words, it is likely that high amounts of exposure to number talk are helpful to the child in figuring out these meanings.

In a longitudinal study examining parent number talk in naturalistic parent–child interactions, we found such frequency effects (Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010). Forty-four parent–child dyads were visited every four months at home beginning at child age 14 months. At each visit, parent–child interactions were videotaped for 90 minutes. Parent and child speech from the 14- to 30-month visits was transcribed in the laboratory, and all parent and child number talk was coded. At 46 months, children were given a test assessing their knowledge of the cardinal values of the numbers 1 through 6, commonly known as the Point-to-X task. On each item, the child was shown two cards, each with a set of dots on it, and was asked to, for example, "point to 3."

Our findings show dramatic variations in the amount of number talk that parents engage in with their children. For example, during the 7.5 hours of observation that occurred with children between ages 14 and 30 months, parental input varied from a low of four number words to a high of 257 number words. Further, we found that this variation in number talk was significantly related to children's talk about numbers and to their later knowledge of the cardinal values of the number words, even when we controlled for SES and for parents' overall amount of talk to their children. These findings indicate that it is specifically talk about numbers, not talk in general, that predicts children's later knowledge. Interestingly, the frequency of number word

usage during this early period was not related to parents' SES; however, the nature of the input does vary with SES, as parents from low-SES backgrounds provided more number input involving counting and parents from high-SES backgrounds provided more number input about the cardinal value of sets.

What Do We Know about Numerical Input that Occurs in Preschool Classrooms?

As for research on numerical input at home during the preschool years, various approaches have been used to study the numerical input at school during this developmental period.

Survey and Questionnaire Studies

A survey of public and private preschool teachers in California about their beliefs and practices concerning children's mathematical development showed that they regarded the preschool learning environment as more important to children's mathematical development than the home environment (Starkey et al., 1999; Starkey & Klein, 2008). However, they also believed that general classroom enrichments rather than focused mathematical experiences were needed to support children's development in this domain. Despite their belief that the preschool learning environment was important for mathematical development, few preschool teachers had knowledge of the mathematics goals of the kindergarten curriculum used in their local schools. Consistent with these findings, in a survey of public preschool teachers in North Carolina, teachers generally expressed a lack of knowledge about how to support the development of children's numerical skills and concepts (Farran, Silveri, & Culp, 1991).

Naturalistic Studies

Observational studies of preschool classrooms report that activities supporting this development are rare (e.g., Graham, Nash, & Paul, 1997). Starkey and Klein (2008) report that time spent on math in preschool classrooms varies as a function of children's SES. On average, teachers in programs serving middle-income children provided 21 minutes of mathematically relevant support per day whereas teachers in programs serving low-income children provided less than 10 minutes per day. Most of this time was in the context of group activities such as calendar time. Moreover, based on survey data, Copley (2004) reports that prospective early childhood teachers feel uncomfortable with mathematics, find it difficult to teach, and generally ignore the subject except for counting and simple arithmetic operations. In contrast, these prospective teachers feel much more comfortable teaching reading and language skills.

In research in our laboratory, we have observed preschool classrooms to examine naturally occurring variations in teacher talk about number during circle time, and the relation of this variation to the growth of children's mathematical knowledge over the school year. Our studies included Head Start as well as tuition-based preschool classrooms. Our results provide an important piece of the puzzle concerning the relation of early input to the development of children's mathematical knowledge. That is, the studies involving parent input, although suggestive, leave open the possibility that the finding of a relation between parent input and child skill is attributable to the biological relation of the parent and child. The finding of a relation between teacher input and children's mathematical knowledge strengthens the argument that this type of input is causally related to the growth of children's knowledge, as teachers are not biologically related to their students. Further, by assessing children at the beginning and end of the school year, we were able to determine whether there was a relation of teacher input to children's start levels or only to the growth of their knowledge over the school year. If teacher input is related to the growth of children's mathematical knowledge but not to children's levels at the beginning of the school year, it eliminates many alternative explanations for the finding of teacher input effects on children's mathematical knowledge. Notably, this pattern of results suggests that input effects are not attributable to parents with higher levels of mathematical ability selecting preschools that emphasize mathematics.

The first school study carried out in our laboratory (Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006) examined the growth of mathematical knowledge in 198 four-year-old children attending 26 different preschool and daycare classrooms in the Chicago area, which served children from a variety of SES backgrounds. Children were administered a short assessment of mathematical knowledge at the beginning and end of the school year. The items assessed knowledge of ordinality, cardinality, calculation, shape names, understanding "half," and recognizing conventional number symbols. We found main effects of Time of Test [F (1, 137) = 33.25, $p < .01$] and SES [F (2, 137) = 28.91, $p < .01$] on children's scores. High- and middle-SES children performed significantly better on the math assessment than those from low-SES families at both time points ($p < .01$).

This study went beyond the typical finding of SES differences by examining teachers' talk about numbers. One hour of teacher talk, which included circle time and the time surrounding circle time, was audiotaped and all speech was later transcribed and coded for number-related talk. Teacher number talk varied widely, from 1 to 104 instances. The most common type of teacher number talk referred to cardinal value of the number words (e.g., "What's the difference between these two beads?"; 48 percent of all instances). This was followed by labeling number symbols (e.g., "Can you get the blue 9?"; 17 percent of all instances), and counting (e.g., "Now we counted 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 teeth to go in the top of your mouth"; 13 percent of all instances). Together, these three types of number input account for 78 percent of all the number instances teachers produced. Interestingly, there was no significant difference in the amount of mathematically relevant input provided by classrooms serving children from different SES backgrounds. Using hierarchical linear modeling (HLM) we showed that teacher number talk was related to the growth of children's mathematics knowledge over the school year, controlling for complexity of teacher syntax, classroom quality, and SES. Moreover, none of these other variables predicted math growth when math input was controlled. Effect size of teacher input was such that an additional 25 number words would lead to a gain of .21 standard deviations in children's scores and an additional 50 number words would lead to a gain of .42 standard deviations.

A second study, carried out by Ehrlich (2007) in her dissertation, examined teacher input in more detail. Ninety children attending eight tuition-based classrooms and nine Head Start classrooms were given the Test of Early Mathematics Achievement (TEMA-3) (Ginsburg & Baroody, 2003) in the fall and spring. Ehrlich also looked in greater detail at the kinds of number concepts teachers talked about and examined whether variations in this input made a difference in students' growth patterns.

Replicating prior research findings, Ehrlich found that children attending Head Start classrooms performed significantly lower than children attending tuition-based preschool on the TEMA-3 (82 vs. 102 at pretest and 85 vs. 105 at post-test, $p < .001$ at both time points). As in the Klibanoff study, teachers varied dramatically in the amount of number talk they engaged in. Because Ehrlich observed each teacher on two occasions during the school year, she could examine whether this variation was stable. In fact, she found that the frequency of their number talk was correlated across time, such that teachers who engaged in a lot of number talk during the first visit were likely to engage in number talk during the second visit ($r = .60$, $p < .02$).

An HLM analysis showed that children's growth in TEMA-3 scores was related to the frequency of teachers' number words (number words per minute) but not to their overall verbosity (overall words per minute). In addition, teachers in classrooms in which students' TEMA-3 standard scores increased over the school year (12 classrooms) showed more number word utterances and more number elicitations than teachers in classrooms in which students' TEMA-3 standard scores decreased over the school year (five classrooms). Teachers in TEMA-increase classrooms were also more likely to elicit cardinal values (How many are there?), name number symbols (read the numeral "3"), match numbers (e.g., show me four fingers), and to elicit calculations from children (e.g., $2 + 1 = 3$) than teachers in TEMA-decrease classrooms. Thus, TEMA-increase and TEMA-decrease classrooms appear to differ not only in the frequency of number related input but also in the nature of this input.

Enrichment and Intervention Studies

Many studies of mathematical input in preschool classrooms have examined the impact of enrichment efforts on children's mathematical learning, and have generally found positive effects on children's mathematical knowledge. Some studies have provided children with high-quality educational programs without focusing on mathematical instruction per se. For example, Campbell, Pungello, Miller-Johnson, Burchinal, and Ramey (2001) found that children enrolled in high-quality child-care from infancy through five years of age had higher math achievement than control children who did not experience this intervention but who did receive the same nutritional supplements and social services as the intervention group. Moreover, these gains were long lasting, as they were still apparent during young adulthood. On the other hand, the Head Start Impact Study (U.S. Department of Health and Human Services, Administration for Children and Families, 2005) does not show differential gains in mathematics for children who attended Head Start programs compared with an SES-matched control group that did not. Thus, attending preschool does not appear to be enough to positively impact the mathematical development of low-SES children (Starkey & Klein, 2008).

Other studies have specifically focused on enriching mathematics instruction in preschool and kindergarten classrooms. The Rightstart program was developed to teach kindergarten children the "central conceptual structures" that underlie formal mathematics, such as the concepts of relative magnitude and continuous quantity

(Griffin, Case, & Siegler, 1994). In this program, children participated in 30 different number games for 20 minutes per day for a three- to four-month period. As an example, in the number line game a child rolls a die, determines the quantity represented, asks the banker for that number of chips, places the chips on each space on the board (which is labeled with the number symbols), and moves his or her playing piece until it rests on the last chip. Thus, the child's understanding of the concept "four" is reinforced by the visual array of chips, the distance moved along the number line, the amount of time it takes to move the piece, and the visual symbol "4." Children who participated in the Rightstart program showed gains in number concepts relative to control children. Impressively, these gains transferred to topics that were never mentioned in the Rightstart curriculum (e.g., counting money, telling time) and were present one year after the intervention had ended.

Another intervention that focused on integrating math activities into the daily routine of Head Start classrooms had similar success (Arnold et al., 2002). In this program, teachers were asked to implement many different math activities for six weeks. Relative to a control group that engaged in their typical activities, children in intervention classrooms showed significant gains in math scores (TEMA-2) and in math interest.

A final intervention example included school and home components for preschool children (Starkey, Klein, & Wakeley, 2004). The Pre-K mathematics curriculum (Klein, Starkey, & Ramirez, 2002) was implemented in preschool classrooms serving children from low- and middle-SES backgrounds. Control children attended the same classrooms a year before the intervention was implemented. The curriculum consisted of units linked to the Pre-K to second-grade standards specified by the National Council of Teachers of Mathematics (2000) as well as to research on early mathematical development. The school component included small-group activities, computer-based mathematics activities, and a math learning center. Teachers were provided with intensive professional development, including workshops and on-site training. The home component complemented the school mathematics units and included workshops for parents and children. The results showed typical SES differences, but both SES intervention groups showed significant gains in mathematics knowledge and performed better than their corresponding control groups. Moreover, children in the low-SES intervention group performed as well as children in the middle-SES control group in the spring of the preschool year.

Although these intervention programs are clearly effective, the inclusion of many different activities and the "business-as-usual" control groups makes it difficult to determine which aspects of the program are most effective. Intervention studies focusing on specific activities can shed light on this question. One such effort has found that a simplified version of the number line game, played for only four 15- to 20-minute sessions over the course of two weeks, can have a positive impact on low-income children's number concepts (Ramani & Siegler, 2008). The number line game consisted of a linear row of 10 colored squares, labeled with the number symbols above each square. Children were asked to use a spinner to determine whether to move one or two spaces, and to say out loud the space number as they moved (for example, if they were on "3" and spun a two, they would say "four, five"). In the control condition, the board was identical except that it was not labeled with number symbols, and children used colors instead of numbers to determine where to move their piece. Playing the number line game improved low-income preschoolers' ability to count to 10, recognize number symbols, compare magnitudes, and represent numbers linearly. Moreover, this improvement was present at a nine-week post-test.

Conclusions

Existing research supports several strong conclusions concerning the early development of numerical knowledge. First, it is clear that there are marked individual differences in number knowledge as early as the preschool years, and these differences are associated with children's SES backgrounds. Second, there are large differences in the number input children receive at home and preschool that are associated with early differences in math knowledge. These differences involve talk about numbers as well as number-related activities such as board games involving numbers.

Early differences in number knowledge are of concern because levels of math knowledge at school entry predict later mathematical achievement. Fortunately, early number development is highly malleable. Number-relevant input at home and at preschool is associated with children's number concepts and skills. Further, intervention studies show substantial improvement in children's developmental trajectories when supports for mathematical learning are put into place in preschools and home environments. Thus, children who fall behind in math do not have to stay behind if they receive effective instruction.

Key Research Questions and Policy Implications

Although existing research indicates that variation in input accounts for at least some of the individual differences in young children's numerical knowledge, several questions remain. A key question concerns the kinds of input that are most effective in teaching children important mathematical concepts. We know that mathematics curricula in preschool classrooms are effective in increasing children's mathematical knowledge and that amount of input is an important predictor of growth. However, we do not know which kinds of input are most effective. There are some hints in the literature that children who engage in more complex mathematical problem solving, such as calculation, may show greater growth than those who do not. There are also hints that number talk that explicitly connects counting to set size may promote development. However, these hints are provided by correlational data, and thus are only hypotheses. In order to identify the kinds of number-related input that are most helpful to children at certain ages and knowledge states, we need to carry out carefully designed experimental studies using pretest/post-test designs and then test these findings in classroom settings.

Consistent with the recommendation of the National Council of Teachers of Mathematics (2000), existing data support the importance of concerted efforts to augment the mathematical experiences of preschool children in their home and school environments. This will involve large changes in preschool curricula and a commitment to preservice and in-service teacher training. In addition, it will involve efforts to increase public awareness about the importance of early mathematical talk and activities, paralleling efforts that have been made to increase awareness about the importance of early language and literacy to children's academic achievement. Such efforts to increase both the frequency and the quality of children's early mathematical experiences have great potential to impact children's academic success in elementary school and beyond.

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14 Analogy and Classroom Mathematics Learning

Lindsey E. Richland

A young child sits down with blocks to solve a new problem the teacher has given her as a follow-up to earlier instruction on addition. The child exclaims: "Oh, I can do this one, this is sort of like that problem we did before."

This child's simple statement reflects a sophisticated recognition of analogical similarity between the mathematical structure of two instances, separated by time and context. Supporting the flexible, generative understanding reflected in this child's analogy lies at the heart of high-quality mathematics instruction. The domain structure of mathematics creates an epistemology of necessary classroom mathematical knowledge that is quite different from retention of verbatim details, as might be privileged in other academic domains such as geography or spelling. In fact, information taught in mathematics classrooms is rarely instructed with the intention that children retain the verbatim details (e.g., the context or numbers used in problem 4). Rather, mathematical proficiency is more directly related to learners' ability to draw inferences from prior knowledge and instruction to represent and solve previously unseen problems [National Research Council (NRC), 2001; National Mathematics Advisory Panel (NMAP), 2008].

Mathematics is a system for rule-based manipulation of numbers, or "anything that plays by the rules" (Gallistel & Gelman, 2005), that is accessible to even very young children (Gelman & Gallistel, 1978/1986). The rules themselves combine into structured systems that can be instantiated in widely varied representations. Once the structured systems have been instantiated into varied representations, however, recognizing their similarity is not a trivial cognitive act. Varied representations may include multiple mathematical problems, abstract concepts and a problem context, graphical or physical manipulatives as representations. Some of these representations appear quite similar at a surface level, using similar-sized numbers and mathematical form (e.g., "3 + 4 = ?" and "5 + 3 = ?"), whereas others appear different at a surface level (e.g., an equation and a word problem with the same mathematical composition). Many novice learners are misled by surface, or featural, characteristics of mathematical representations, and tend to either fail to notice commonalities between representations or draw false parallels between them (e.g., using the same procedure to solve two mathematically different problems about trains).

In spite of these difficulties, recognizing commonalities in mathematical structure across contexts is a critical skill, and is a key element of mathematical proficiency (NMAP, 2008; NRC 2001). The ability to notice commonalities between